Step 2: Homogeneous Solution

- v_{CH} : solution to the homogeneous equation (set input drive to zero).

$$v_C(0) = V_0 \text{ given}$$

$$v_I(t) = V_I \text{ for } t \geq t_0$$

$$RC\frac{\mathrm{d}v_C}{\mathrm{d}t} + v_C = V_I$$

✓ $RC\frac{dv_{CH}}{dt} + v_{CH} = 0$

- Guess v_{CH} is in the form of $v_{CH} = A \cdot e^{st}$
  - Discard the trivial solution of $v_{CH} = 0$

$RC \cdot \frac{d(Ae^{st})}{dt} + Ae^{st} = 0 \Rightarrow RC \cdot A \cdot s e^{st} + Ae^{st} = 0 \Rightarrow (RCs+1) \cdot Ae^{st} = 0$

($A \neq 0$ for non-trivial solution), $RCs + 1 = 0 \Rightarrow s = \frac{-1}{RC}$, $v_{CH} = Ae^{-\frac{1}{RC}t}$

Step 3: Total Solution

- The total solution is the sum of the particular and homogeneous solutions.
- Find remaining unknowns from initial condition.

$$v_C = v_{CP} + v_{CH} = V_I + Ae^{-t/RC}, \quad t \geq 0$$

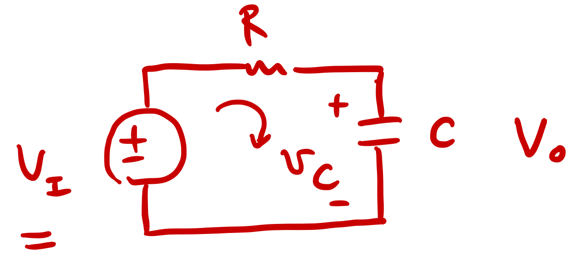
$$v_C(t=0) = V_0 = V_I + A \quad \Rightarrow \quad A = V_0 - V_I$$

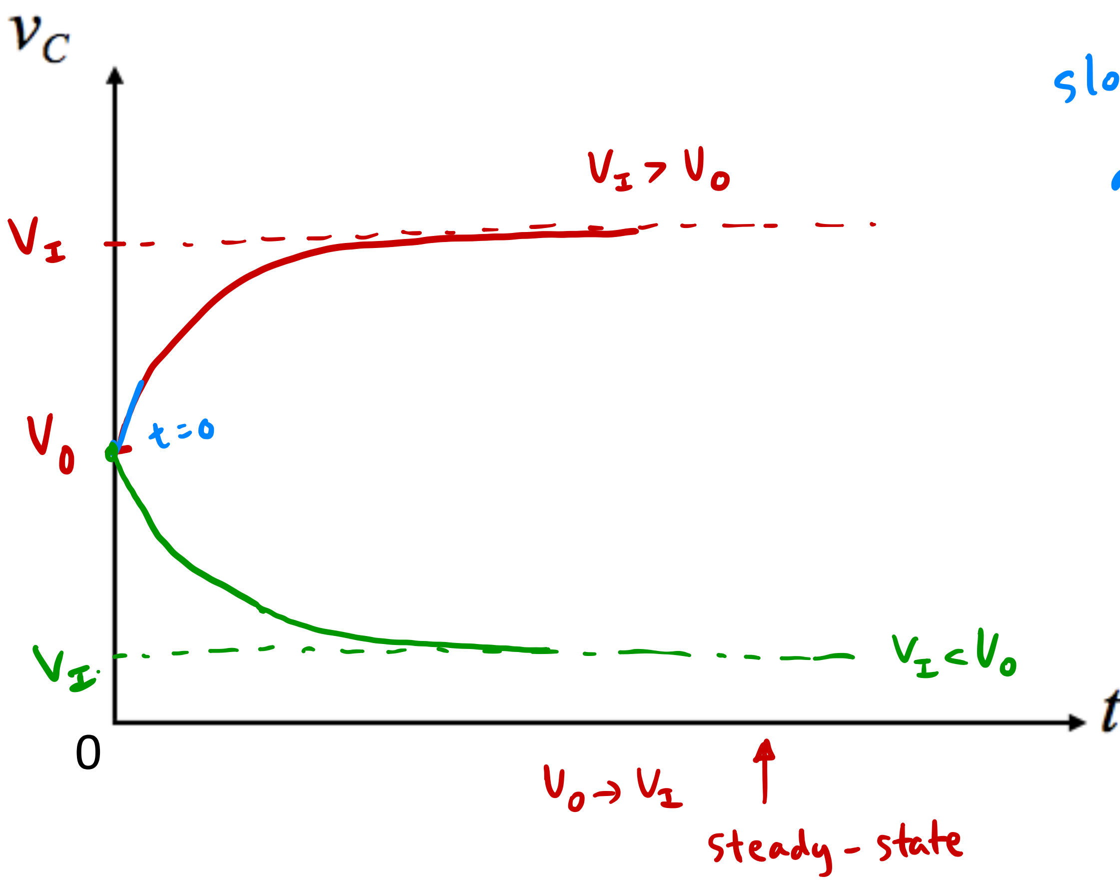
$$\Rightarrow v_C = V_I + (V_0 - V_I) \cdot e^{-t/RC}, \quad t \geq 0$$

$$i_C = C\frac{dv_C}{dt} = \frac{-1}{R}(V_0 - V_I) \cdot e^{-t/RC}, \quad t \geq 0$$

Plot Solution

$$v_C = V_I + (V_0 - V_I)\cdot e^{-t/RC}$$





slope at $t=0$

$$\left.\frac{dv_C}{dt}\right|_{t=0} = \frac{V_I - V_0}{RC}\cdot \left. e^{-t/RC}\right|_{t=0}$$

$$= \frac{V_I - V_0}{RC}$$

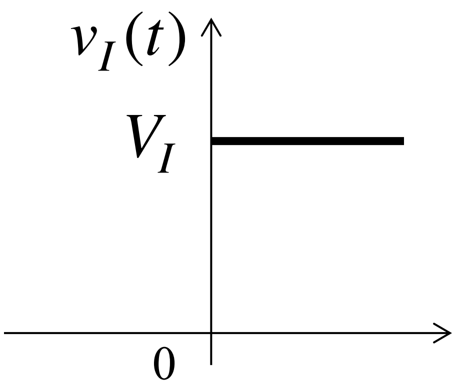
Forced Response

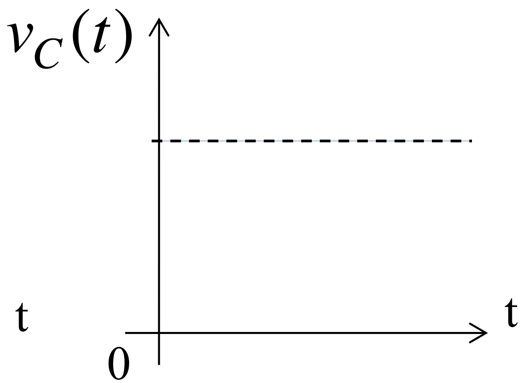
- The forced response of a circuit is its behavior (in terms of voltages and currents) under external sources of excitation.
- The forced response of a circuit depends on
  - Parameters of circuit components.
  - Initial conditions of energy storage components within the circuit.
  - Forms of external excitations.
- The forced response of a circuit can be described by a non-homogeneous differential equation.
- A general solution of the linear non-homogeneous differential equation is the sum of a general solution of the corresponding homogeneous solution and an arbitrary particular solution.

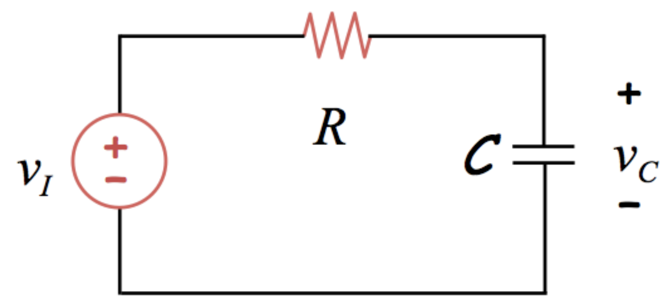
Natural Response

- The natural response of a circuit is the behavior (in terms of voltages and currents) of the circuit itself, with no external sources of excitation.
- The natural response depends on
  - Parameters of circuit components.
  - Initial conditions of energy storage components within the circuit.
- The natural response of a first order circuit can be described by a homogeneous first order differential equation.

For the RC Circuit in the Previous Slide



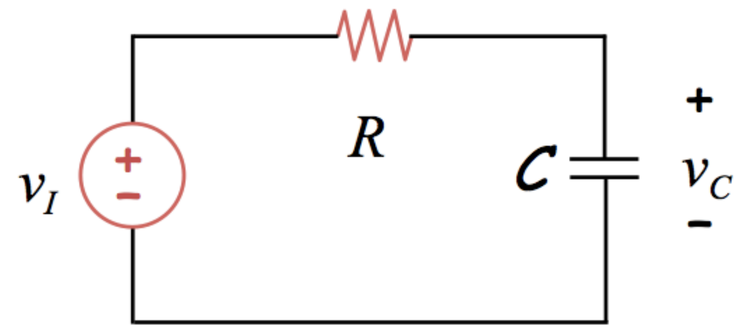




- Notice that the capacitor voltage v_c
  - Independent of the form of the input voltage before $t=0$
- Instead, it depends only on
  - The capacitor voltage at $t=0$
  - The input voltage for $t \geq 0$

State

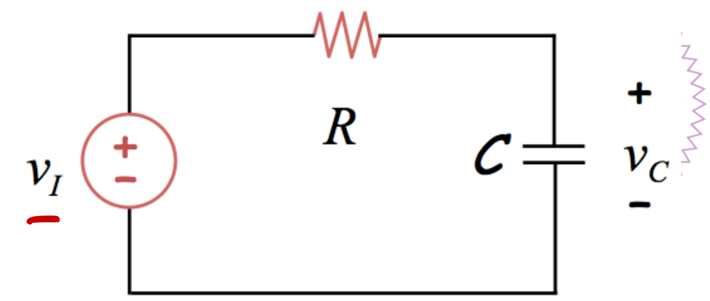
- From our simple RC circuit



  - State: summary of past inputs relevant to predicting the future.

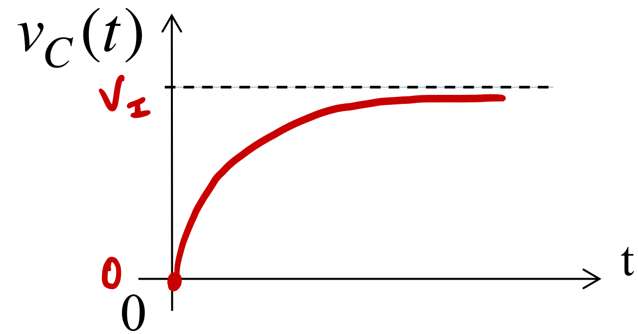
- Actually, the state variable is Q
- For linear capacitor $Q = C \cdot v$
  - Capacitor v_c voltage is also state variable

ZIR and ZSR

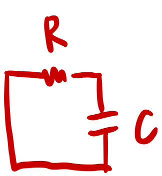


- We are often interested in circuit response for
  - Zero state: zero-state response (ZSR) with $V_0 = 0$

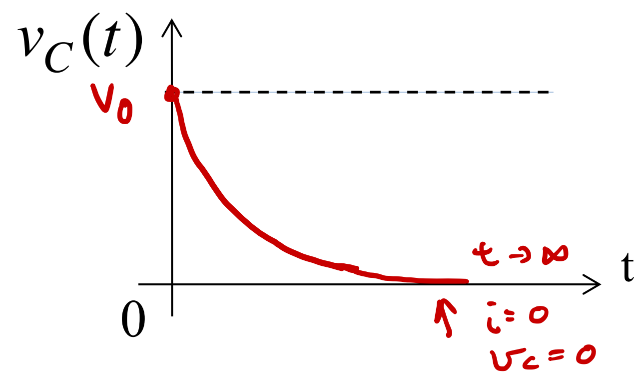
$v_c = V_I\left(1 - e^{-t/RC}\right), \ t \geq 0$



  - Zero input: zero-input response (ZIR) with $V_I = 0$

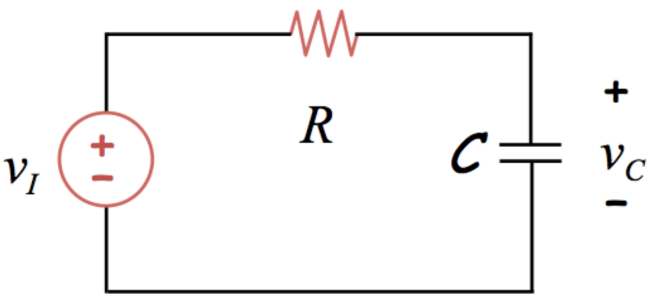


$v_c = V_0 \cdot e^{-t/RC}, \ t \geq 0$



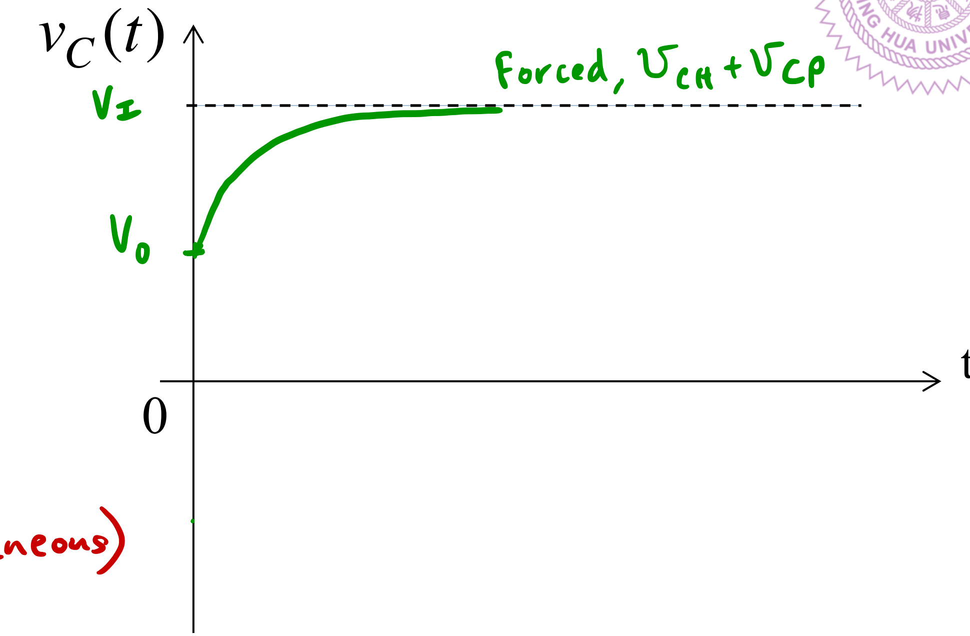
- Total response = ZIR + ZSR

RC Circuit Response

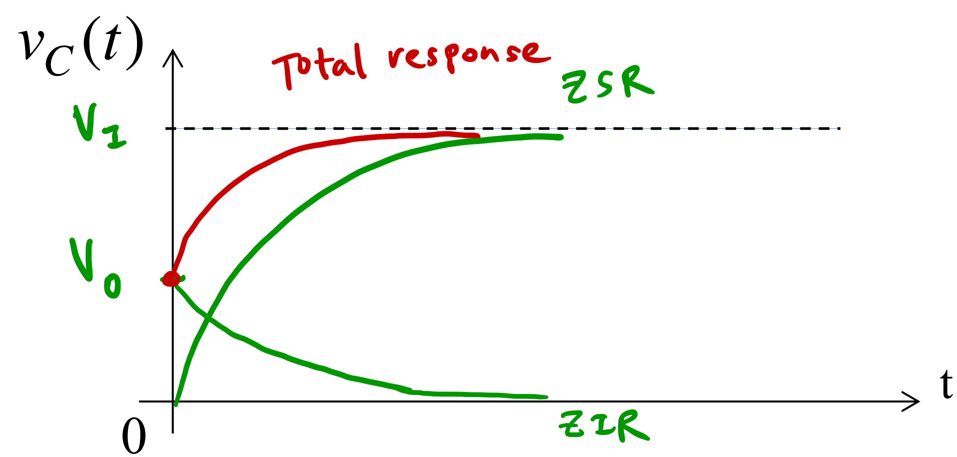


with $v_I(t) = V_I$ for $t \geq 0$ and $v_C(0)$

- ~~Forced~~ response
  particular
  + natural response (homogeneous)

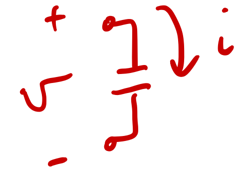


- Zero-state response
  + zero-input response

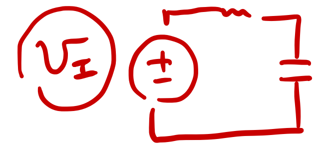


Capacitor *i*-*v* Behaviors

$i = C\frac{dv}{dt}$



- Its current i depends on the time-varying rate of voltage v.



穩態

- Steady state characteristics

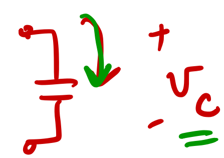
$\frac{dv}{dt} = 0 \Rightarrow i = 0$

  - The capacitor is an open circuit to DC signal at steady state.

  - The capacitor is a short circuit to high frequency signals at steady state.

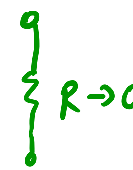
$v_c = A \cdot \sin\omega_0 t$

$i = C\frac{dv}{dt}$ $\quad i \to \infty \quad$ $C\frac{dv_c}{dt} = C \cdot \omega_0 \cdot A \cdot \cos\omega_0 t \to \infty$



$i \to \infty$

v i



- Capacitor voltage v does not change abruptly.
  - It requires an infinite current.