Electric Circuits
Lecture 4 Capacitors and First-Order Circuits

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Lecture Outline

- Review
- Chapter 7 and 8 in the textbook
  - Capacitors
  - RC circuits
  - Inductors
  - RL circuits

Review

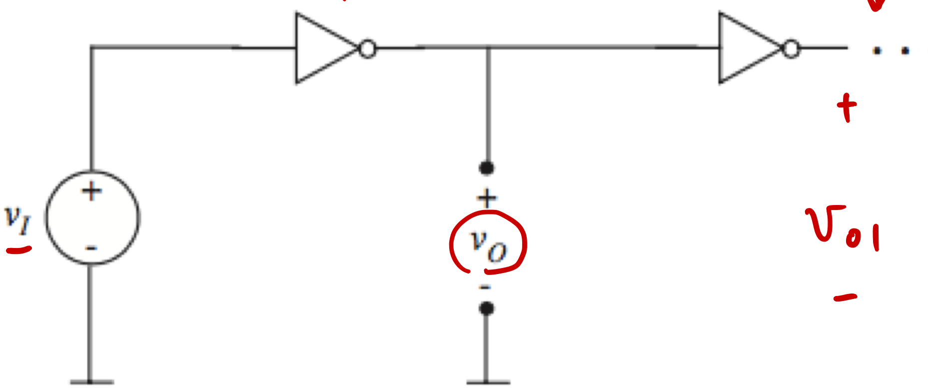
Chapter 7 Energy Storage Elements - Capacitors, Inductors

Chapter 8 The Complete Response of RL and RC Circuits

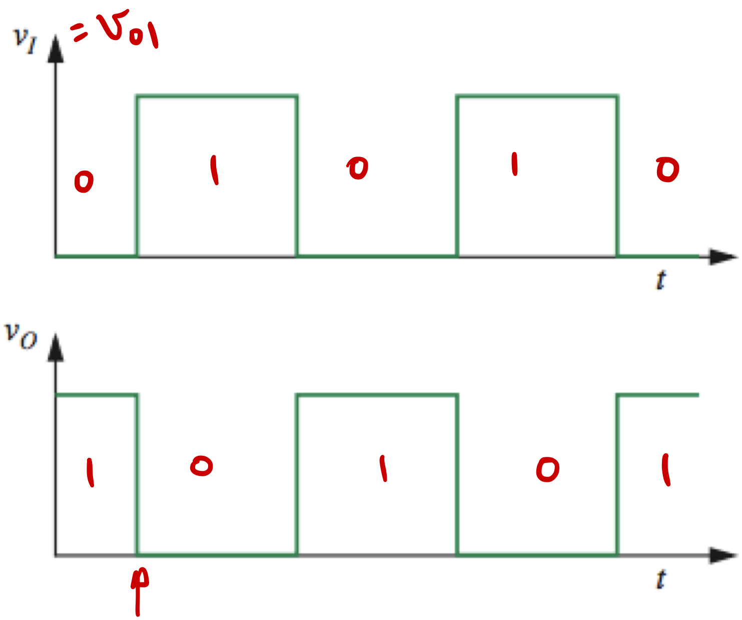
First-order circuits

Motivation

inverter

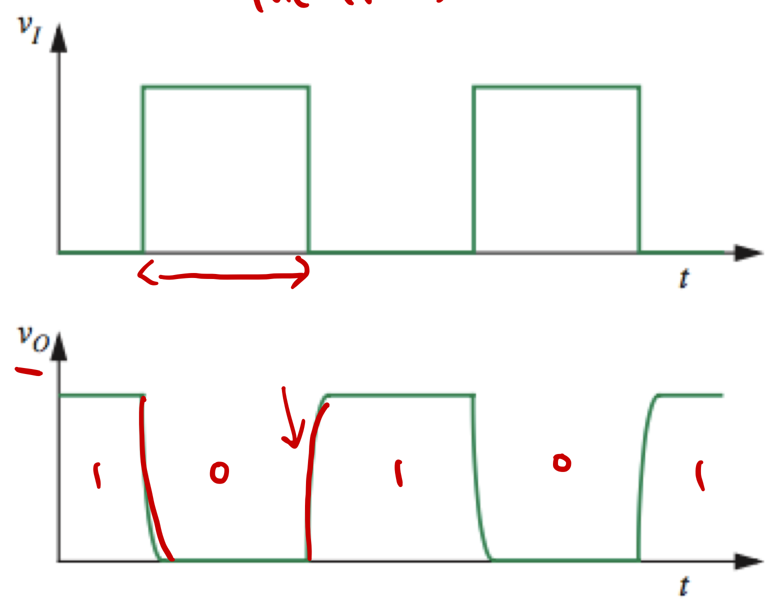


ideal



寄生

Actual: considering parasitic capacitance in the transistors

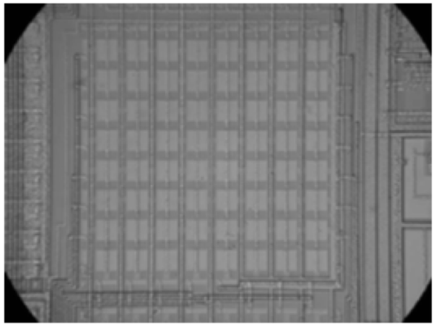


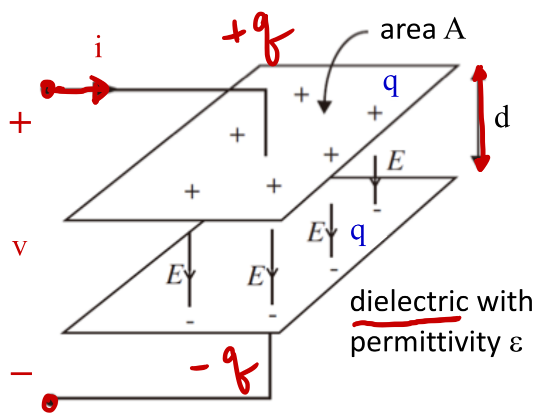
Ideal Linear Capacitor

- A capacitor stores energy in an electric field. (CMOS)
- A pair of parallel plates (metal plates)
  sandwich structure

Discrete capacitors ✓

Integrated capacitor





- Total charge $q = C \cdot v$ (coulomb, C)

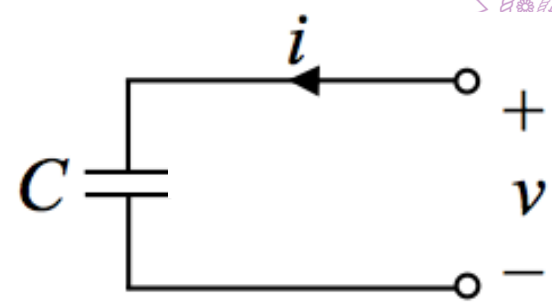
- No net charge in the element.
  - Total charge on capacitor $(+q)+(-q)=0$

Ideal Linear Capacitor

- The element law of a capacitor

$\frac{dq(t)}{dt} = i$, $q = C \cdot v \Rightarrow i = \frac{d(C \cdot v)}{dt} = C \cdot \frac{dv}{dt}$

C: constant



$$\boxed{q = C \cdot v}$$

- The branch voltage of a capacitor

$$v = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\, d\tau$$

$v = i \cdot R$

$v(t) = i(t) \cdot R$

R: memoryless

  - Depends on the entire past history of its branch current which is the essence of memory.

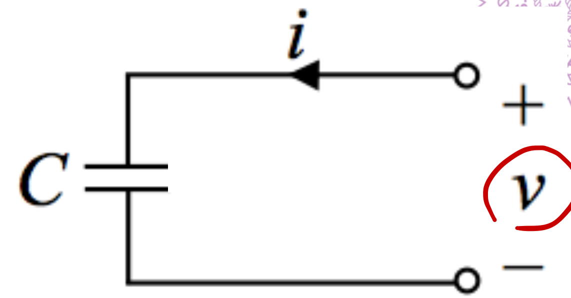
- Capacitors are linear devices.
  - Verify for yourselves that capacitors satisfy both the properties of homogeneity and superposition.

Ideal Linear Capacitor

$$P = v \cdot i = v \cdot C\frac{dv}{dt} = \frac{d}{dt}\left(\frac{1}{2}Cv^2\right) \quad (\text{watts}, W)$$

$$E = \frac{1}{2}Cv^2 \quad (\text{Joule}, J)$$

=



$$q = C \cdot v$$

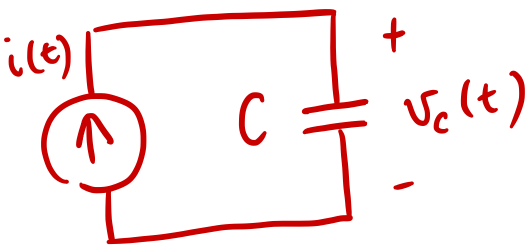
$$i = C \cdot \frac{dv}{dt}$$

- Associated with the ability to exhibit memory is the property of energy storage, in the form of electric field.
- Unlike a resistor, a capacitor stores energy not dissipates it.

Capacitor and a Current Source

- Given $i(t)$ and $v_C(t = t_0) = V_0$. Find $v_C(t)$ for $t \geq t_0$.

initial condition

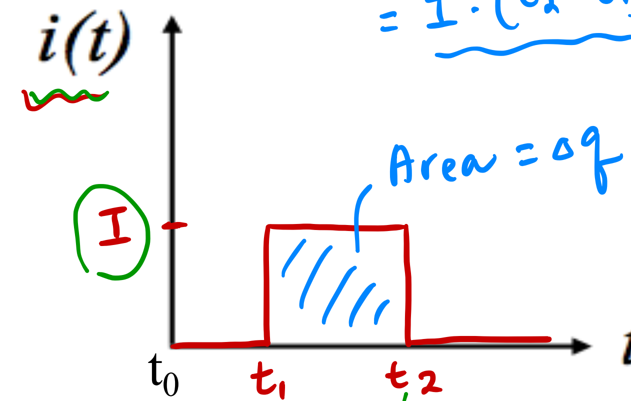


$\Delta v = \frac{I}{C}(t_2 - t_1)$

$\Delta q = C \cdot \Delta v$

$= I \cdot (t_2 - t_1)$

Given



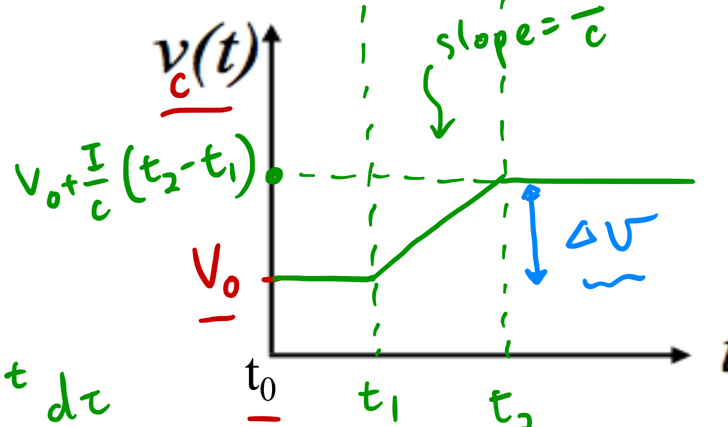
$t < t_0$

$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau = \frac{1}{C}\int_{-\infty}^{t_0} i(\tau)\,d\tau +$$

$t \geq t_0$

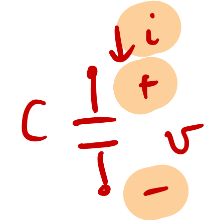
$$\frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau$$

$$= V_0 + \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau$$

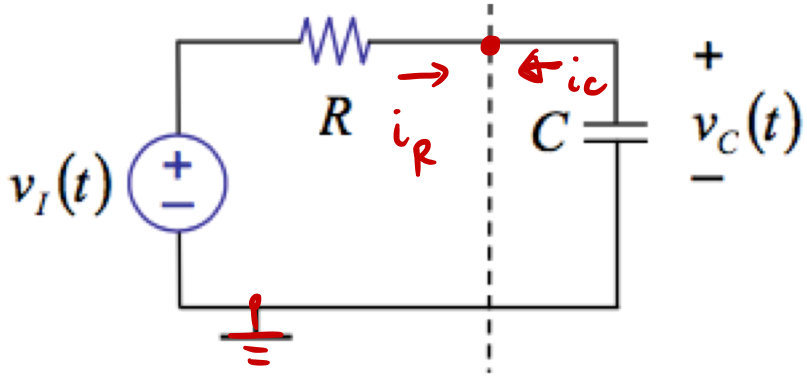


$t_1 < t < t_2$, $\quad v_c = V_0 + \frac{1}{C}\int_{t_1}^{t} I \cdot d\tau = V_0 + \frac{I}{C}\int_{t_1}^{t} d\tau$

Analyzing an RC Circuit



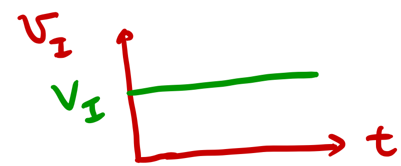
$i = C \cdot \frac{dv}{dt}$



- Circuits with one energy storage element are called first-order circuits.

- Given $v_C(0) = V_0$ initial condition of capacitor

  $v_I(t) = V_I$ for $t \geq 0$



$t \geq 0$, one unknown: v_C

KCL @ v_C: $i_R + i_C = 0$ $\Rightarrow$ $\frac{v_I - v_C}{R} - C\frac{dv_C}{dt} = 0$

$\Rightarrow RC\frac{dv_C}{dt} + v_C = v_I$

- The next step is just math!

How to Solve the Differential Equation

- Method of homogeneous and particular solutions

$v_C(0) = V_0$ given

$v_I(t) = V_I$ for $t \geq 0$

$$RC\frac{\mathrm{d}v_C}{\mathrm{d}t} + v_C = V_I$$

1. Find the particular solution.
2. Find the homogeneous solution.
3. The total solution is the sum of the particular and homogeneous solutions.
4. Use the initial conditions to solve the remaining constraints.

Step 1: Particular Solution

- v_{CP}: any solution that satisfies the original equation.
- In general, use trial and error (guesswork).

$v_C(0) = V_0$ given

$v_I(t) = V_I$ for $t \geq t_0$

✓ $$RC\frac{\mathrm{d}v_C}{\mathrm{d}t} + v_C = V_I$$

Guess $v_{cp} = V_I$

Plug into $RC \cdot \frac{dv_{cp}}{dt} + v_c = V_I$

$RC \cdot 0 + V_I = V_I$ valid

$\Rightarrow v_{cp} = V_I$