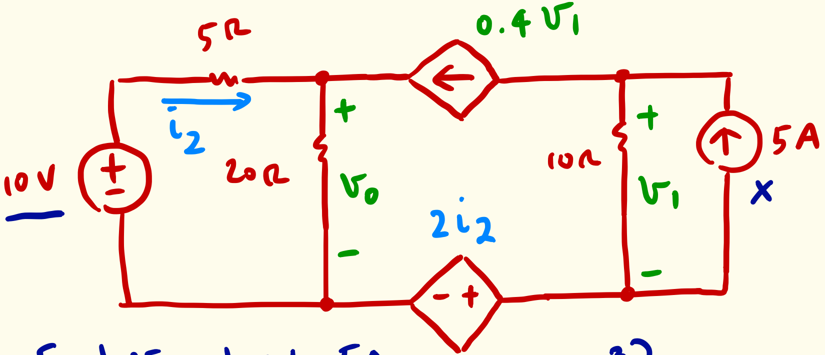
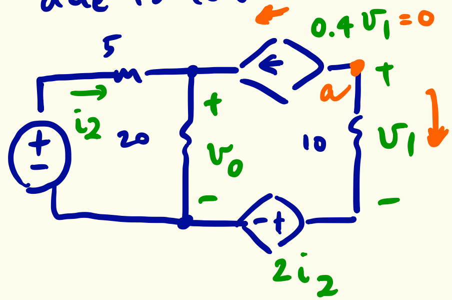
• Example. Use superposition. Find v_o.

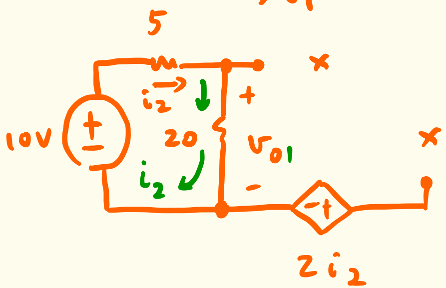


1) Find partial response v_{o1} due to 10V source.



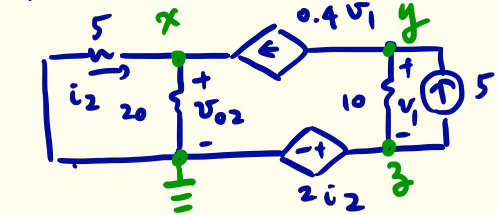
KCL @ a : $0.4v_1 + \frac{v_1}{10} = 0$

$\Rightarrow v_1 = 0$



$i_2 = \frac{10}{(5+20)} = 0.4\ A$, $v_{o1} = 0.4 \times 20 = 8\ V$

2) Find v_{o2} due to 5A source.



KCL @ x : $\frac{v_x}{5} + \frac{v_x}{20} - 0.4v_1 = 0$

@ y : $0.4v_1 + \frac{v_1}{10} - 5 = 0$

$v_3 = 2i_2 = 2 \cdot \frac{-v_x}{5} \Rightarrow v_{o2} = 16\ V$

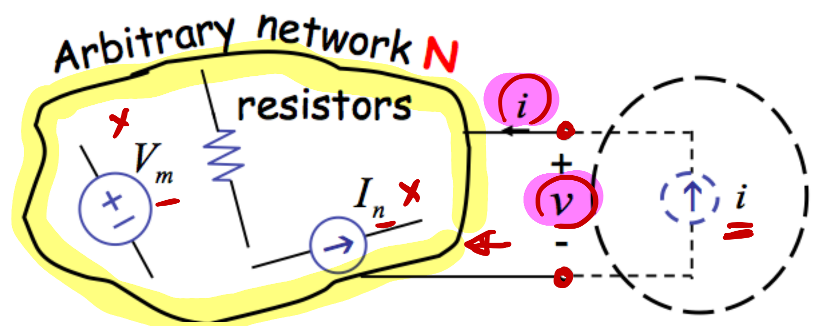
3) Total response

$v_o = v_{o1} + v_{o2}$

$= 8 + 16$

$= 24\ V$

Yet Another Method?



linear

- Arbitrary network

$$v = \sum_m \alpha_m V_m + \sum_n \beta_n I_n + Ri$$

depends only on the network N. independent of i.

where all $V_m = 0$, $I_n = 0$

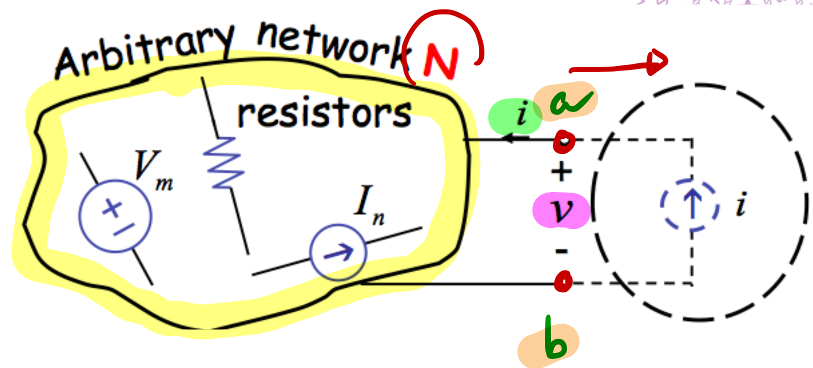
α : no unit

β : Ω

1. Independent of external excitation and behave like a voltage.
   - Let's call it 'v_{TH}'
2. Independent of external excitation and behave like a resistor.
   - Let's call it 'R_{TH}'

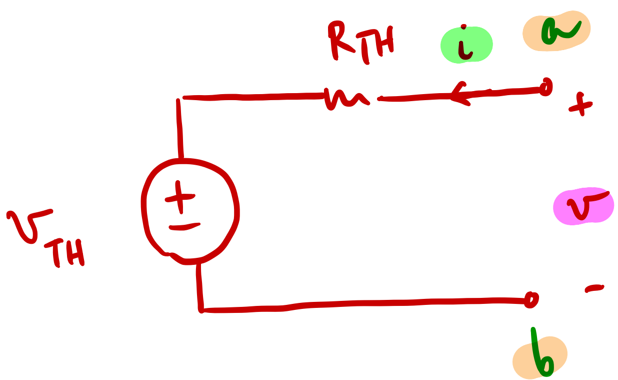
Arbitrary Network

$$v = \sum_m \alpha_m V_m + \sum_n \beta_n I_n + Ri$$



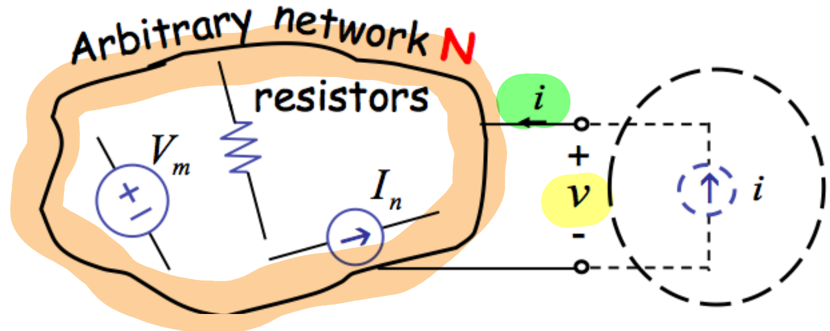
$$v = v_{TH} + R_{TH} \cdot i$$

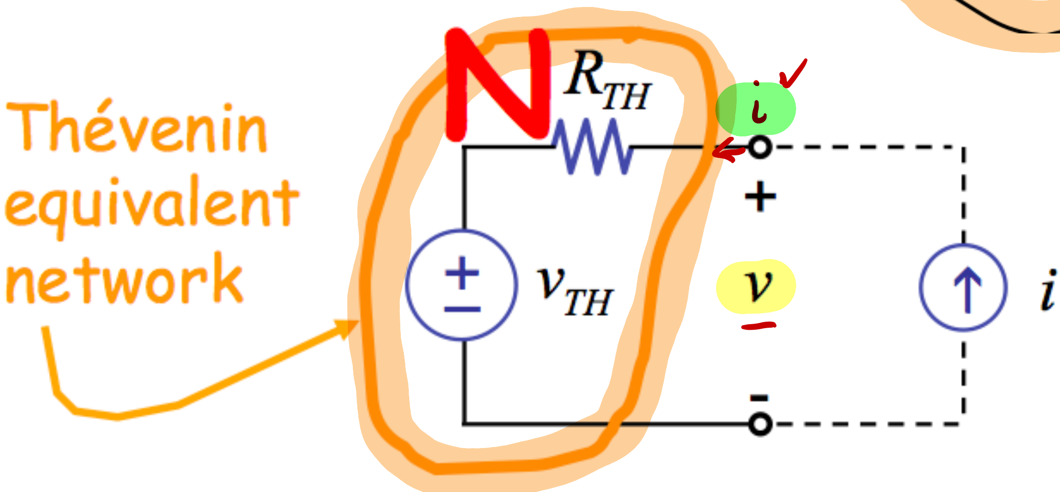
- In other words, as far as the external world is concerned (for the purpose of the *i*-*v* relation), 'arbitrary network N' is indistinguishable from:



Arbitrary Network

$$v = v_{\mathrm{TH}} + R_{\mathrm{TH}} \cdot i$$



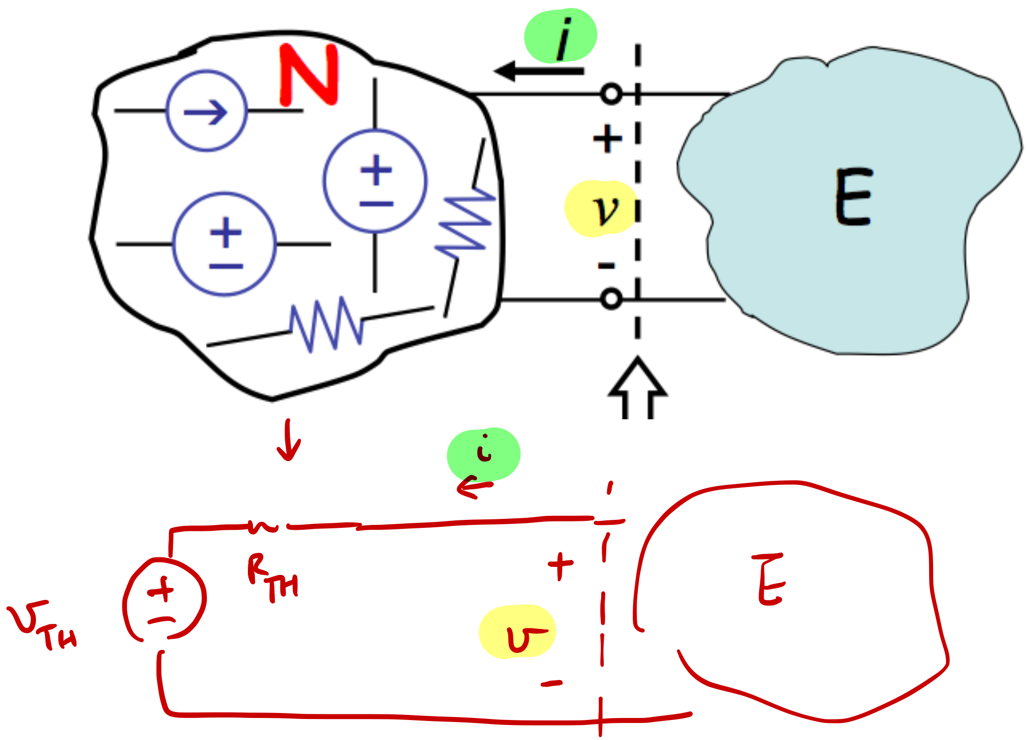


$V_{TH} = V_{open},\ i=0$

- How to derive v_{TH} and R_{TH}?
- v_{TH} → Open circuit voltage seen at terminal pair (aka port).
- R_{TH} → Resistance of network seen from port (with V_m's and I_n's set to 0).

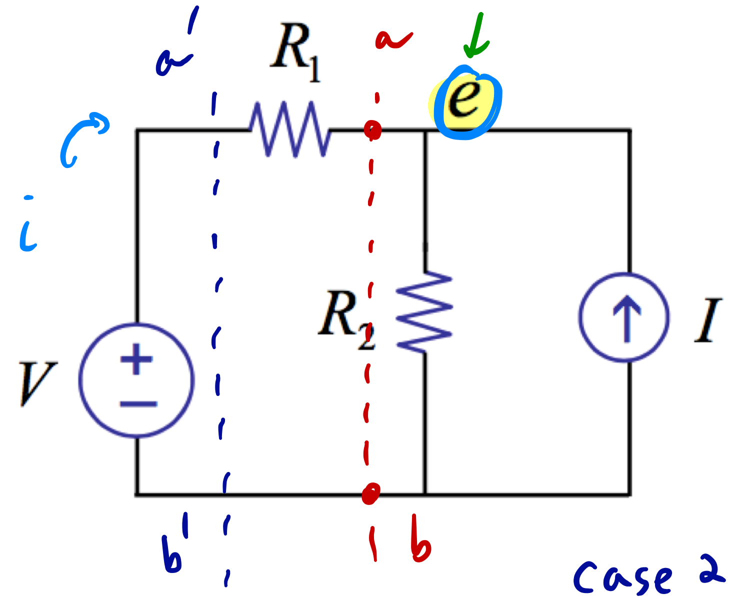
$R_{TH} = \dfrac{v}{i}$ when independent sources $= 0$

Method 6: The Thévenin Method

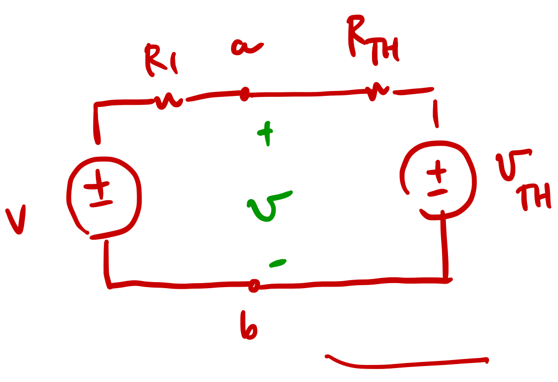


- Replace network N with its Thévenin equivalent
- Solve with external network E

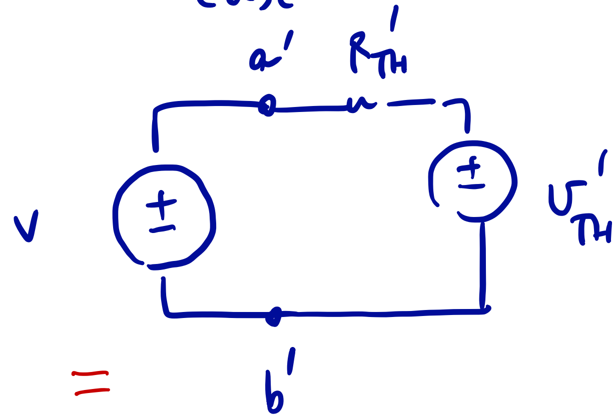
Example – Using Thévenin Method



Case 1.



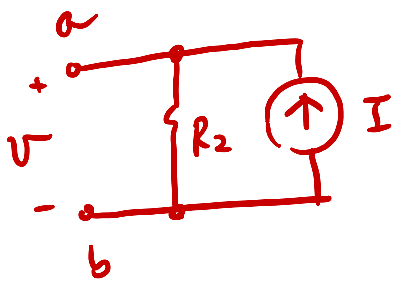
Case 2



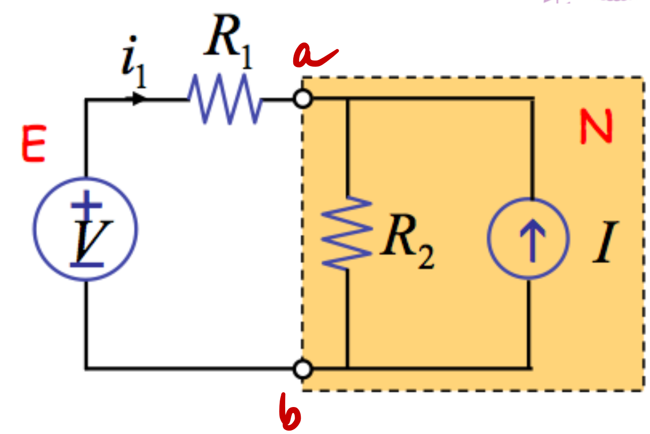
Example – Using Thévenin Method

- Case 1

1) v_{TH}

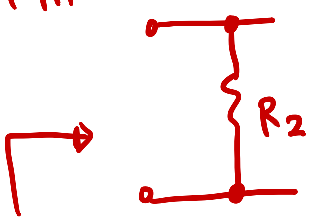


$v_{TH} = I \cdot R_2$

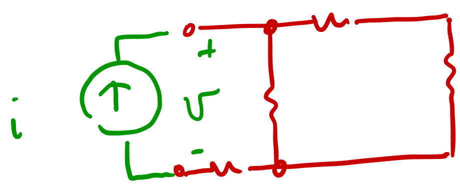


$$V_{TH} = IR_2$$
$$R_{TH} = R_2$$

2) R_{TH}



$R_{TH} = R_2$

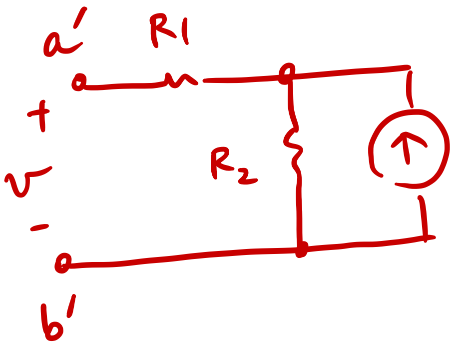


$\frac{v}{i} = R_{TH}$

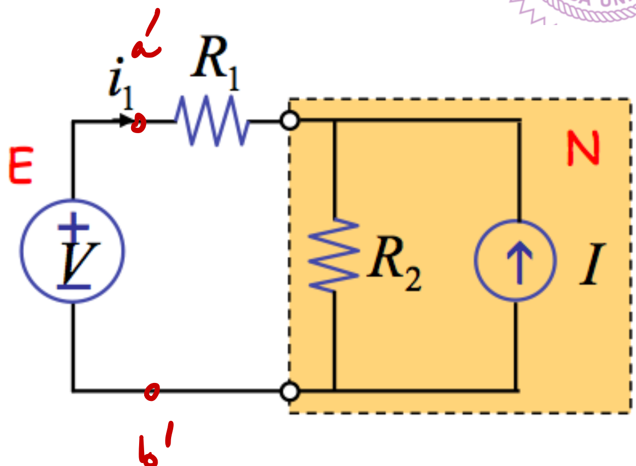
Example – Using Thévenin Method

- Case 2

1)



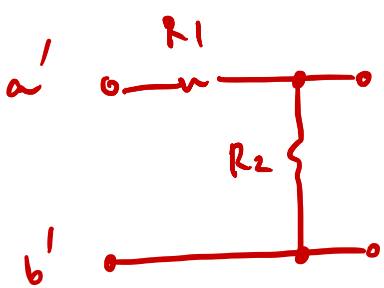
$v'_{TH} = R_2 \cdot I$



~~$V_{TH} = IR_2$~~

~~$R_{TH} = R_2$~~

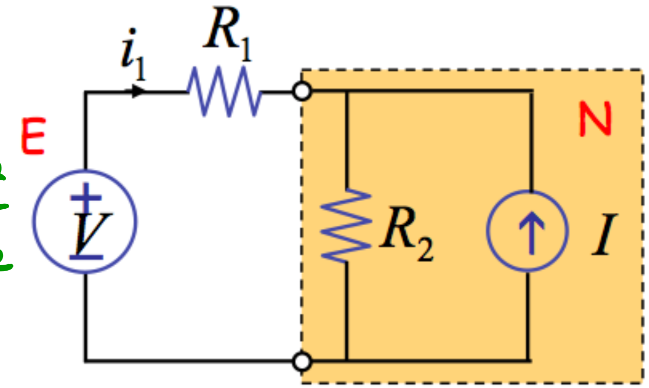
2) $R_{TH} = R_1 + R_2$



$I = 0$

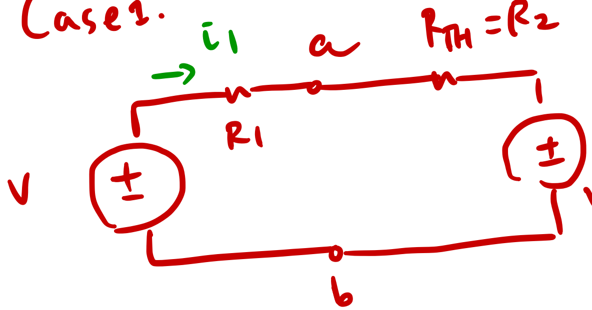
Example – Using Thévenin Method

- Solve with external network E



$$V_{TH} = IR_2$$
$$R_{TH} = R_2$$

Case 1.

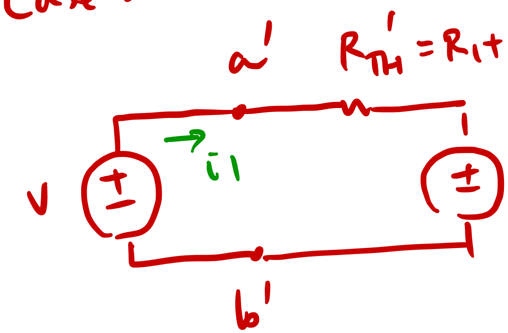


$V_{TH} = I \cdot R_2$

$$i_1 = \frac{V - V_{TH}}{R_1 + R_{TH}} = \frac{V - I \cdot R_2}{R_1 + R_2}$$

$$V_a = V - i_1 \cdot R_1$$

Case 2.



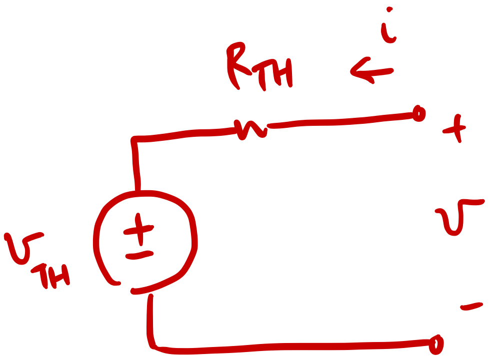
$V'_{TH} = I \cdot R_2$

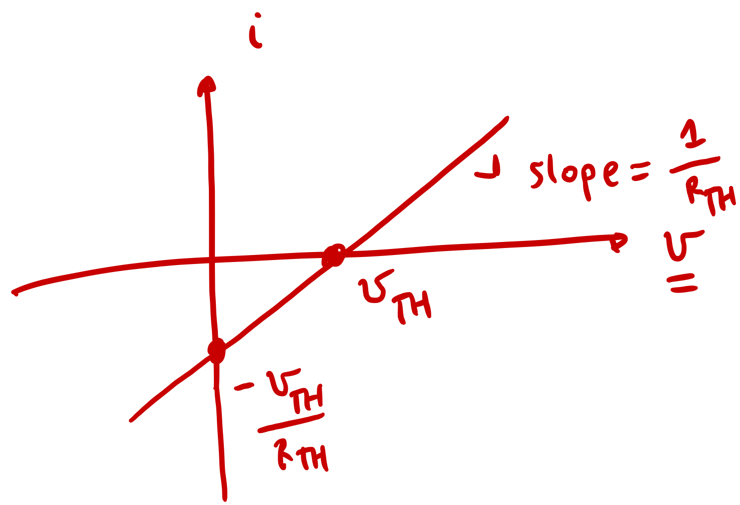
$$i_1 = \frac{V - V'_{TH}}{R'_{TH}} = \frac{V - I \cdot R_2}{R_1 + R_2}$$

Example – Using Thévenin Method

- Graphically…

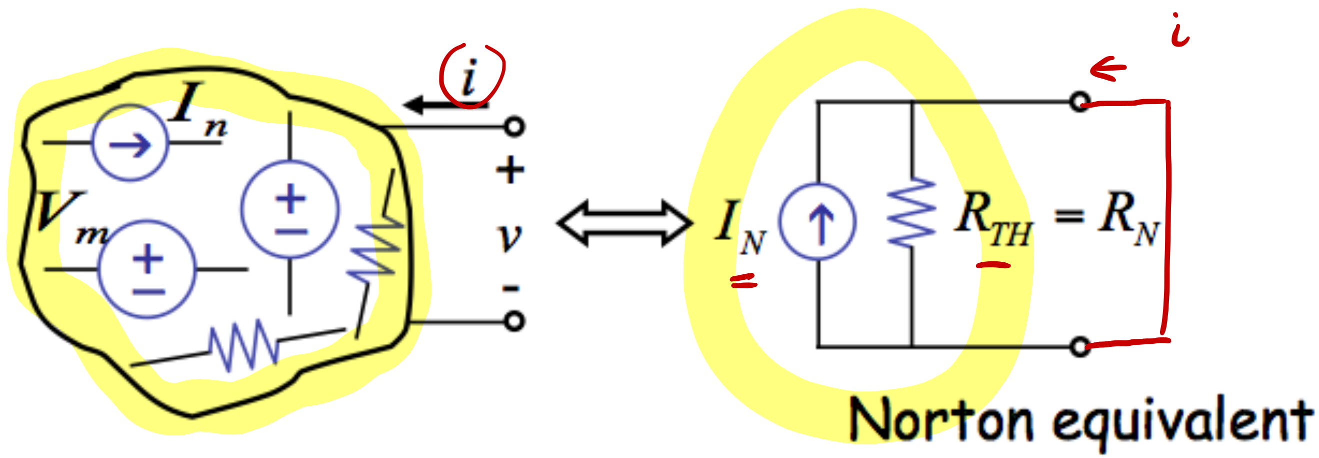
$v = v_{TH} + R_{TH} \cdot i$





i-v characteristic

Method 7: The Norton Method

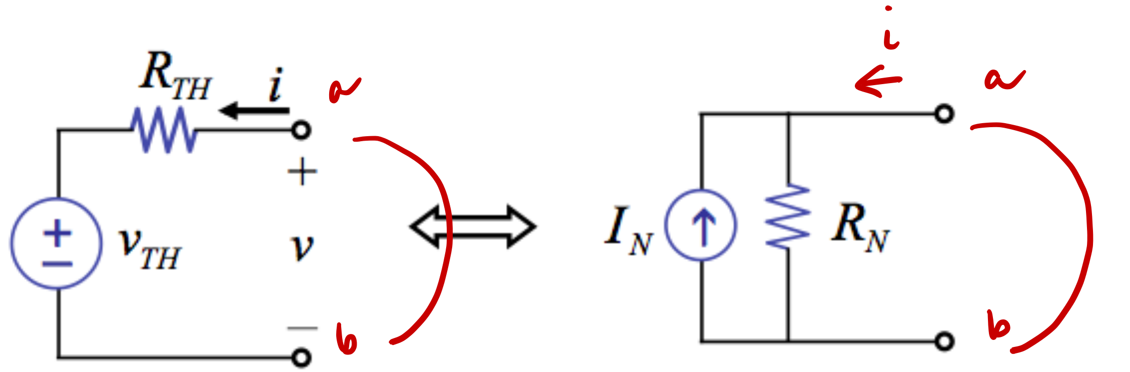


- I_N: short circuit current, $I_N = -i$

- $R_N = R_{TH}$

  resistance of network seen from the port when all independent sources $= 0$.

Thévenin and Norton



Thevenin equivalent　　　　Norton equivalent

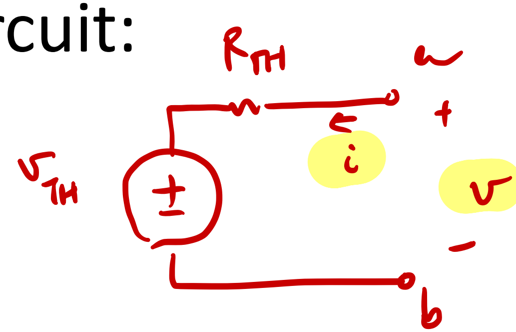
Equivalent resistance　　$R_{TH} \quad = \quad R_N$

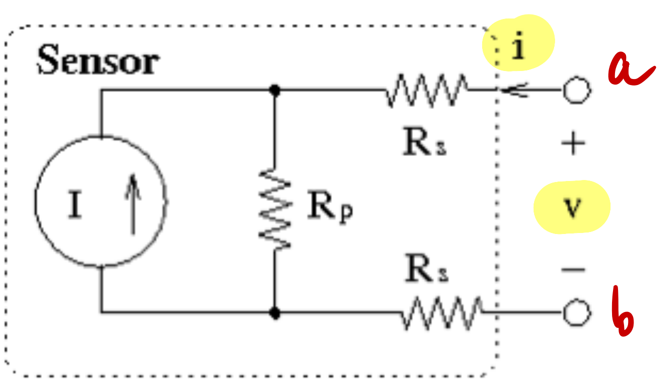
open-circuit voltage, v_{oc}　　$v_{TH} \quad = \quad I_N \cdot R_N$

Short-circuit current, i_{sc}　　$-\dfrac{v_{TH}}{R_{TH}} \quad = -I_N$

Example – Thévenin and Norton

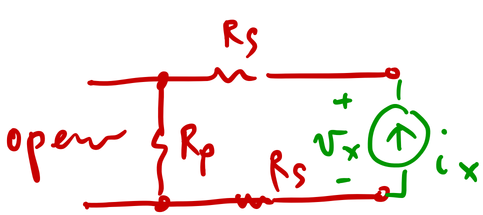
- A light sensor is modeled as a current source that produces a current proportional to the intensity of light.
  - Leakage through the sensor is modeled as R_P.
  - Resistance in the contacts (wires) is modeled as R_S.
- Thévenin equivalent circuit:





$$V_{TH} = I \cdot R_p$$

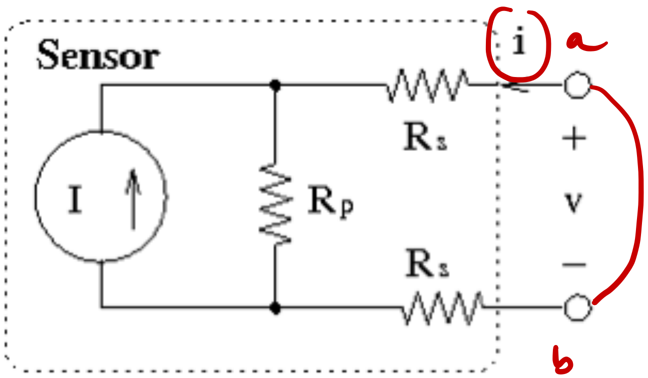
$$R_{TH} = 2R_s + R_p$$

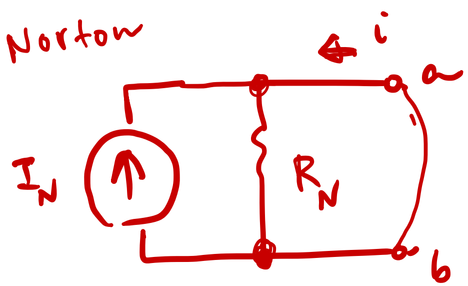


$$V_x = i_x (R_s + R_p + R_s) \Rightarrow R_{TH} = \frac{V_x}{i_x}$$

Example – Thévenin and Norton

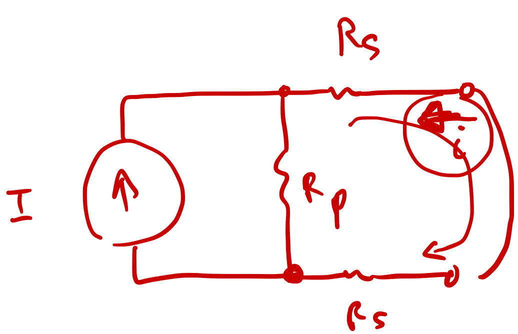
- Norton equivalent circuit:





$$I_N = -i = -I \cdot \frac{R_p}{R_p + 2R_s}$$

$$R_N = 2R_s + R_p$$



Summary

- Basic circuit analysis methods
  - KCL, KVL
  - Element combination rules
  - Node method
  - Mesh method
  - Superposition
  - Thévenin
  - Norton

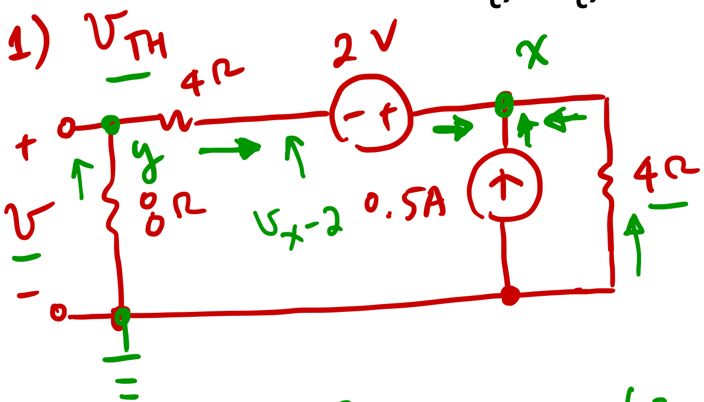
For all circuits

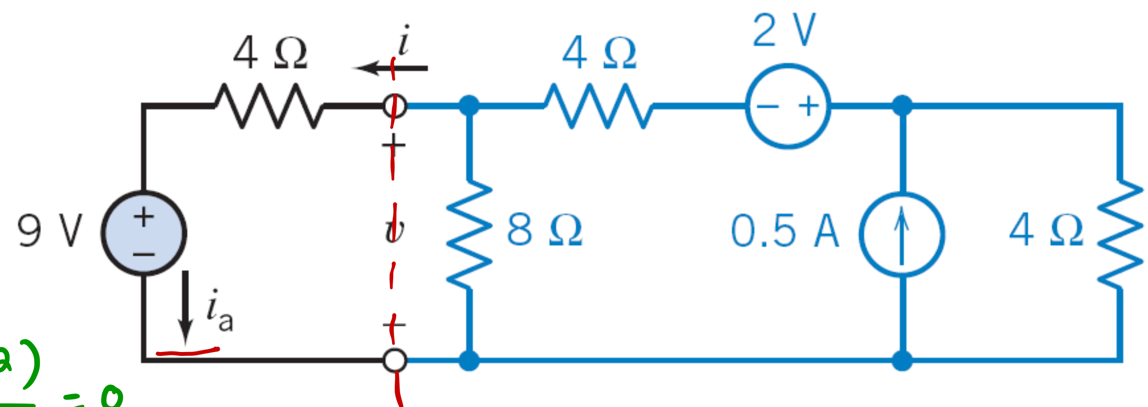
For linear circuits only

Problem 5.2-1

- Determine R_t, v_t, and i_a.

1) v_{TH}



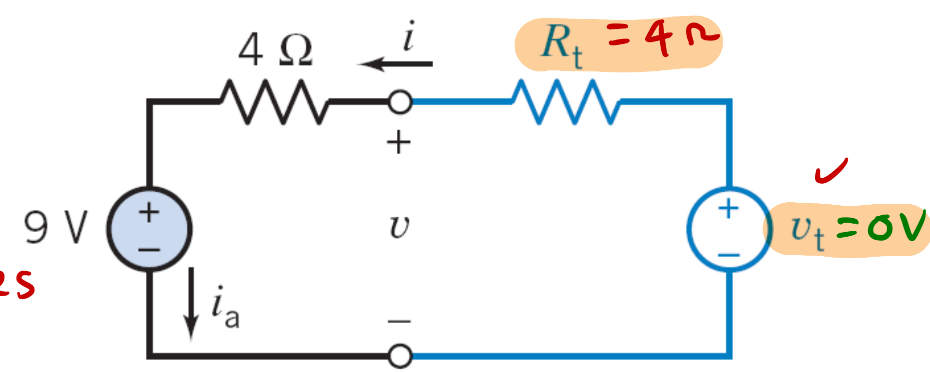


(a)

KCL @ x: $\frac{-v_x}{4} + 0.5 + \frac{v_{TH} - (v_x - 2)}{4} = 0$

@ y: $\frac{v_{TH}}{8} + \frac{v_{TH} - (v_x - 2)}{4} = 0$

$\Rightarrow$ $v_x = 2$ V, $v_y = v_{TH} = 0$ V



(b)

2) R_{TH} : turn OFF all independent sources

$R_{TH} = (4+4) \parallel 8$

$= 4\,\Omega$

$i_a = \frac{-9}{8}$ A