Electric Circuits
Lecture 3 Circuit Analysis Methods and Theorems

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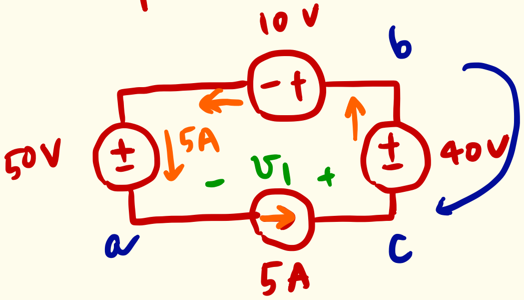
Lecture Outline

- Review
  - KCL, KVL (method 1)
  - Element combination (method 2)
- Chapter 4 in the textbook
  - Node analysis (method 3)
  - Mesh analysis (method 4)
- Chapter 5 in the textbook
  - Superposition (method 5)
  - Thévenin and Norton (method 6 and 7)

Example 1.

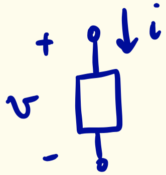


Is the circuit valid? Yes

What is v_1? KVL

Power of each source?

$$v_1 = 50 + 10 - 40 = 20 \text{ V}$$



$p = v \cdot i > 0$

$$P_{50V} = 50 \times 5 = 250 \text{ W}$$

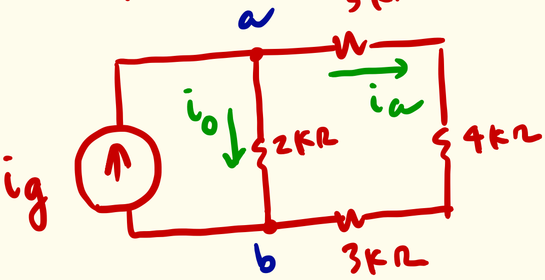
$$P_{10V} = 10 \times 5 = 50 \text{ W}$$

$$P_{40V} = 40 \times (-5) = -200 \text{ W}$$

$$P_{5A} = 20 \times (-5) = -100 \text{ W}$$

Example 2.

Assume $i_a = 2mA$. Find i_o and i_g.



KCL: $i_g - i_o - i_a = 0$ (1)

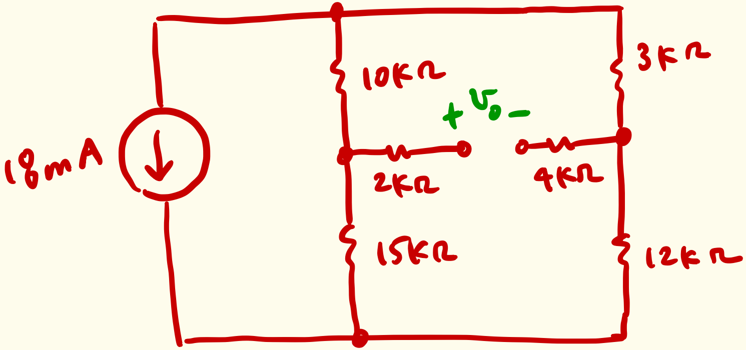
KVL: $2k \cdot i_o - 5k \cdot i_a - 4k \cdot i_a - 3k \cdot i_a = 0$ (2)

From (2), $i_o = 6 i_a = 12 \text{ mA}$

(1), $i_g = i_a + i_o = 14 \text{ mA}$

Example 3.

Find v_o



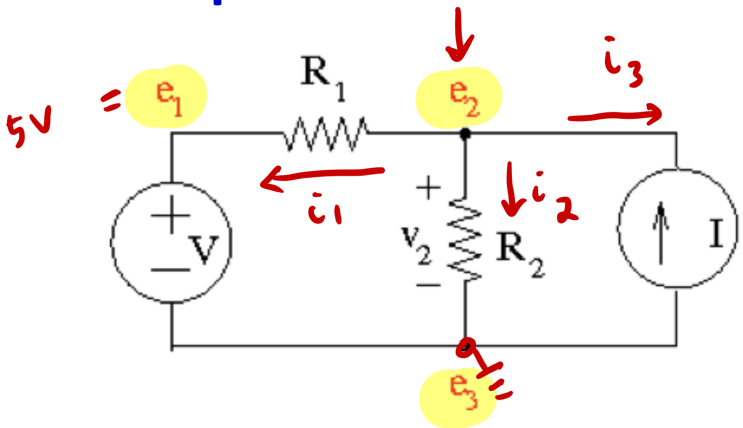
Chapter 4 Methods of Analysis of Resistive Circuits

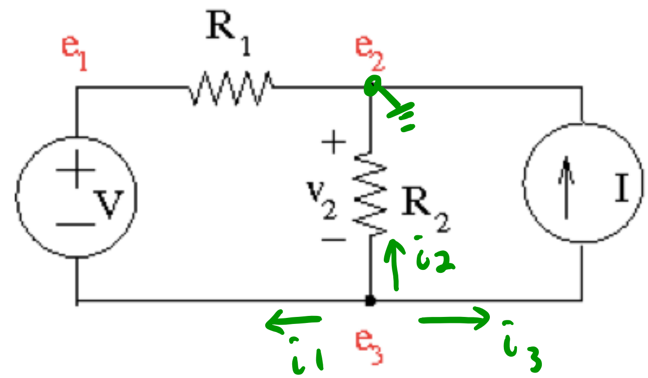
Node Voltage

- Node voltage: the potential difference between the given node and a reference node, which is usually ground (zero V).
  - Current flows from the node with higher potential to the node with lower potential.
  - It is common to choose the node that has the maximum number of elements connected to it as the reference node.

Example – Reference Node

ground (0 V)





- Given V = 5 V, I = 3 A, R_1 = 3 Ω, R_2 = 5 Ω.

Case 1. Set e_3 as reference node.

KCL @ e_2 : $i_1 + i_2 + i_3 = 0$

$$\Rightarrow \frac{e_2 - e_1}{R_1} + \frac{e_2}{R_2} - I = 0$$

$\Rightarrow e_2 = 8.75$ V

Case 2. Set e_2 as reference node.

KCL @ e_3 : $i_1 + i_2 + i_3 = 0$

$$\Rightarrow \frac{e_3 + V - e_2}{R_1} + \frac{e_3}{R_2} + I = 0$$

$e_1 = e_3 + V$

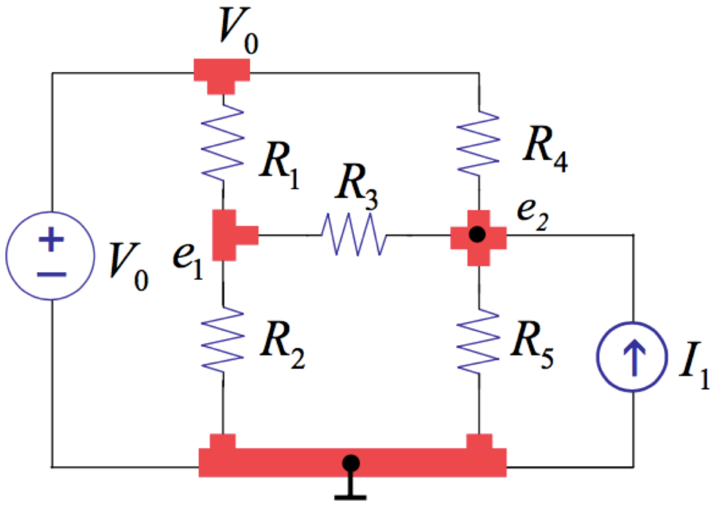
$\Rightarrow e_3 = -8.75$ V

Method 3: Node Analysis

- The most efficient utilization of KCL and circuit elements' laws to solve for the voltages and currents of all elements of a given circuit.
  1. Select a node as as reference (ground) node.
  2. Label node voltages to the remaining nodes with respect to ground.
     - These are the primary unknowns.
  3. Apply KCL to all but the ground node.
     Use device laws to express the branch currents in terms of node voltages.
  4. Solve the $n-1$ equations for node voltages.
  5. Calculate all branch voltages and currents based on elements' laws.

Example – Using Node Method

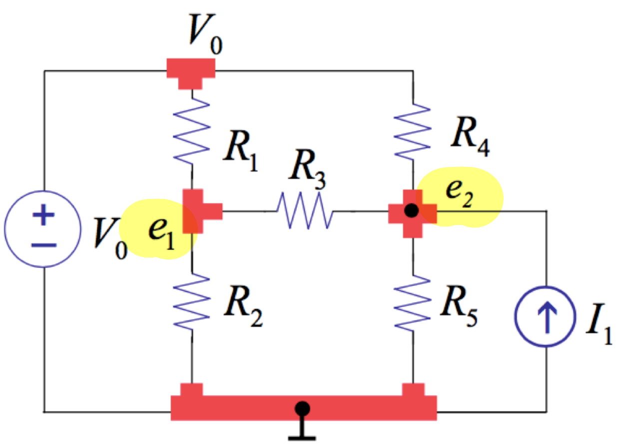
1. Select a node as as reference (ground) node.
2. Label node voltages to the remaining nodes with respect to ground.



Example – Using Node Method

3. Apply KCL to all but the ground node. Use device laws to express the branch currents in terms of node voltages.

- For convenience, write $G_i = \frac{1}{R_i}$
- To avoid mistakes use convention: e.g., always sum currents leaving a node



KCL @ e_1: $\frac{e_1 - V}{R_1} + \frac{e_1 - e_2}{R_3} + \frac{e_1}{R_2} = 0 \quad (1)$

KCL @ e_2: $\frac{e_2 - V}{R_4} + \frac{e_2 - e_1}{R_3} + \frac{e_2}{R_5} - I = 0 \quad (2)$

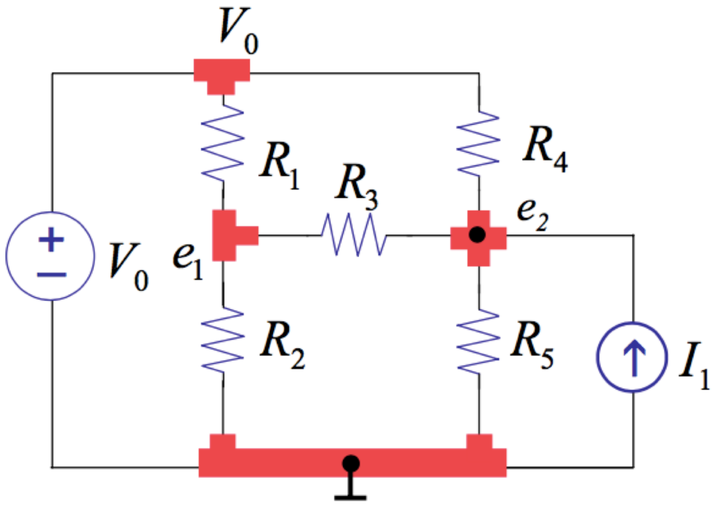
From (1) and (2), express e_1 and e_2 as functions of $V, I, R_1, R_2, R_3, R_4, R_5$ (known variables)

Example – Using Node Method

4. Solve the n – 1 equations for node voltages

KCL at e_1 $\quad (e_1 - V_0)G_1 + (e_1 - e_2)G_3 + (e_1 - 0)G_2 = 0$

KCL at e_2 $\quad (e_2 - e_1)G_3 + (e_2 - V_0)G_4 + (e_2 - 0)G_5 - I_1 = 0$



Example – Using Node Method

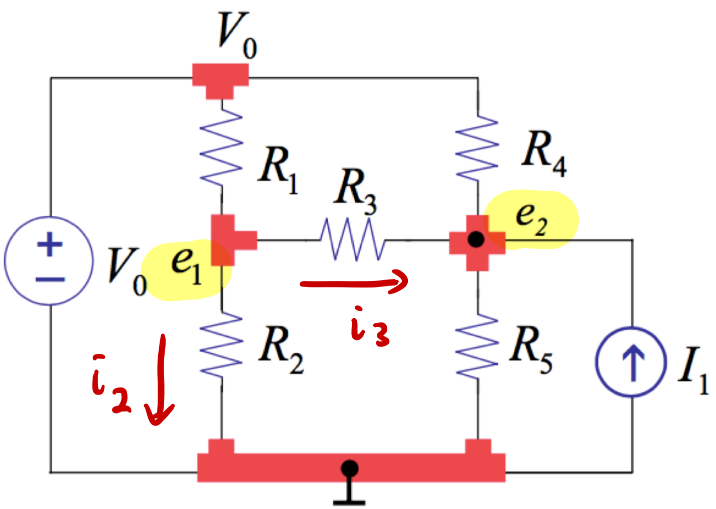
5. Calculate all branch voltages and currents based on elements' laws.

For example

$$i_2 = \frac{e_1 - 0}{R_2}$$

$$i_3 = \frac{e_1 - e_2}{R_3}$$

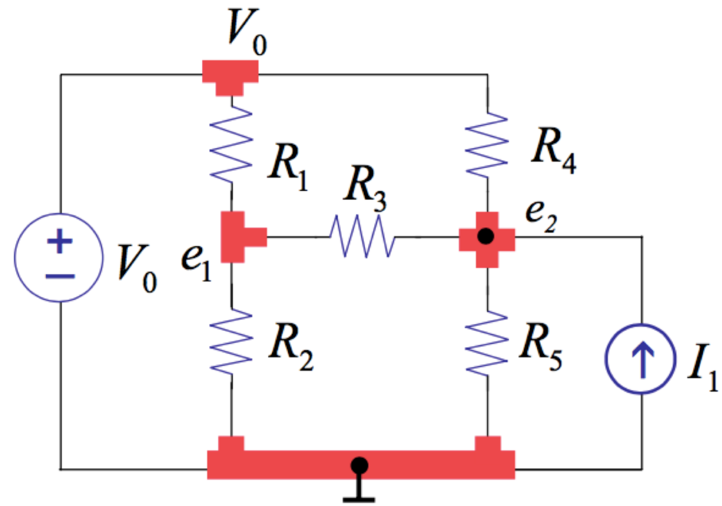
⋮



Revisit Step 4

$$e_1(G_1+G_2+G_3)+e_2(-G_3)=V_0(G_1)$$

$$e_1(-G_3)+e_2(G_3+G_4+G_5)=V_0(G_4)+I_1$$



$$\begin{bmatrix} G_1+G_2+G_3 & -G_3 \\ -G_3 & G_3+G_4+G_5 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}$$

conductivity matrix

unknown node voltages

sources

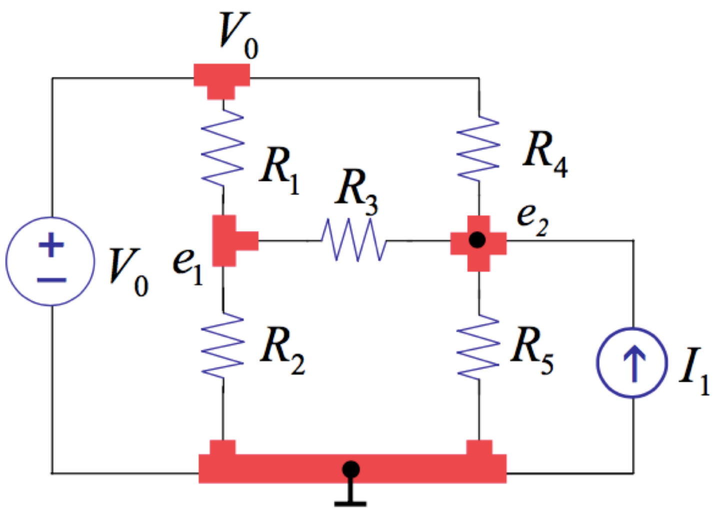
Revisit Step 4

$$\begin{bmatrix} G_1+G_2+G_3 & -G_3 \\ -G_3 & G_3+G_4+G_5 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}$$

$$e_1 = \frac{(G_3+G_4+G_5)(G_1V_0)+(G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3G_4+G_3G_5}$$

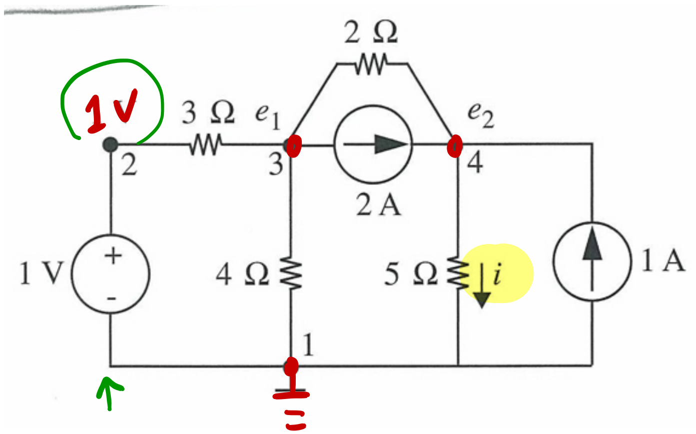
$$e_2 = \frac{(G_3)(G_1V_0)+(G_1+G_2+G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3G_4+G_3G_5}$$

- The same denominator for e_1 and e_2.
- No negative terms in denominator.
- Linear in V_0 and I_1.



More Example – Node Method

- Determine the current i.



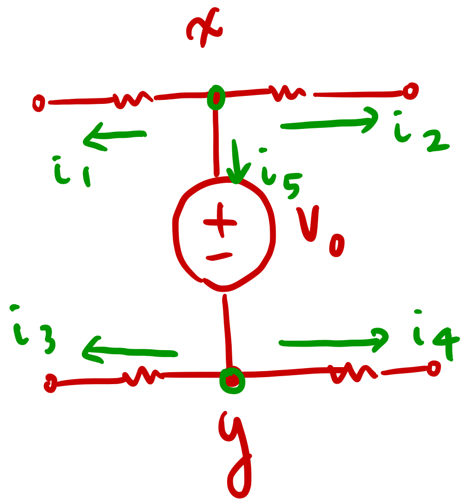
1. Choose node 1 as ref. node
2. Label all other unknowns (e_1, e_2)
3. KCL @ e_1: $\frac{e_1 - 1}{3} + \frac{e_1}{4} + \frac{e_1 - e_2}{2} + 2 = 0$

   KCL @ e_2: $-2 + \frac{e_2 - e_1}{2} + \frac{e_2}{5} - 1 = 0$

$\Rightarrow e_1 = 0.65V, \quad e_2 = 4.75\ V$

$i = \frac{e_2 - 0}{5} = 0.95\ A$

Floating Independent Sources



KCL @ x: $i_1 + i_5 + i_2 = 0$

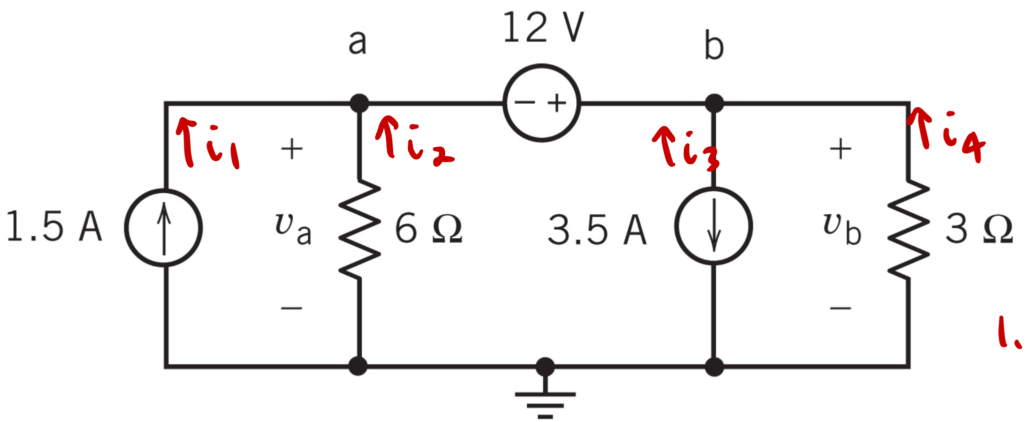
+ KCL @ y: $i_3 - i_5 + i_4 = 0$

$i_1 + i_2 + i_3 + i_4 = 0$

x, y: super node

Example 4.3-2

- Determine v_a and v_b.



a, b : supernode

$v_b = v_a + 12 \quad (1)$

KCL @ supernode : $i_1 + i_2 + i_3 + i_4 = 0$

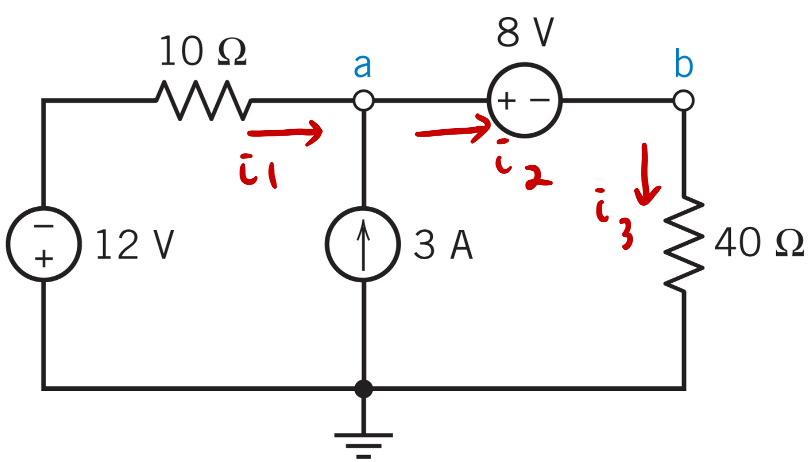
$$1.5 + \frac{-v_a}{6} - 3.5 + \frac{-v_b}{3} = 0 \quad (2)$$

From (1) and (2), $v_a = -12\text{ V}$

$v_b = 0\text{ V}$

Exercise 4.3-2

- Determine v_a and v_b.



a, b: supernode

KCL @ a: $i_1 + 3 - i_2 = 0$ (1)

KCL @ b: $i_2 - i_3 = 0$ (2)

$v_a = v_b + 8$

(1)+(2)

$\Rightarrow i_1 + 3 - i_3 = 0$

From (1), $\frac{-12 - v_a}{10} + 3 - i_2 = 0$

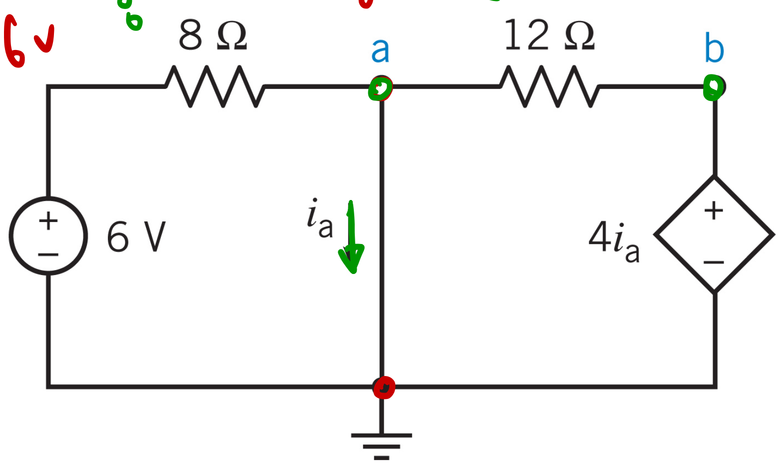
From (2), $i_2 = i_3 = \frac{v_b}{40}$

$\Rightarrow v_a = 16\,V, \; v_b = 8\,V$

Exercise 4.4-1

- Determine v_b.

$$\frac{v_b}{12} = \frac{4i_a}{12}$$



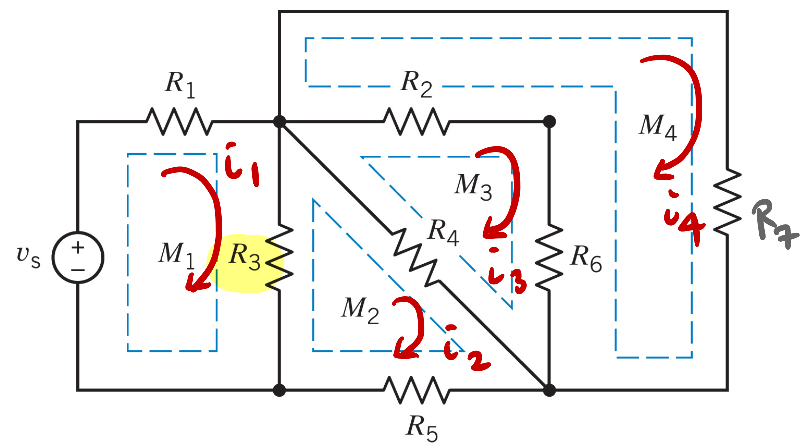
$$\text{KCL @ } a: \frac{6}{8} - i_a + \frac{4i_a}{12} = 0$$

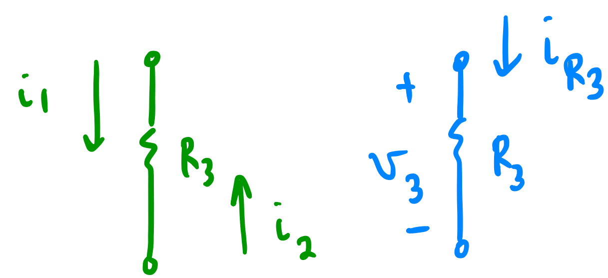
$$\Rightarrow i_a = \frac{9}{8}\text{ A}$$

$$v_b = 4i_a = \frac{9}{2}\text{ V}$$

Method 4: Mesh Analysis

- Mesh: a loop that does not contain any other loops within it.
- Mesh analysis
  - Write element voltages as f Functions of mesh currents.
  - Apply KVL to each mesh of the circuit.

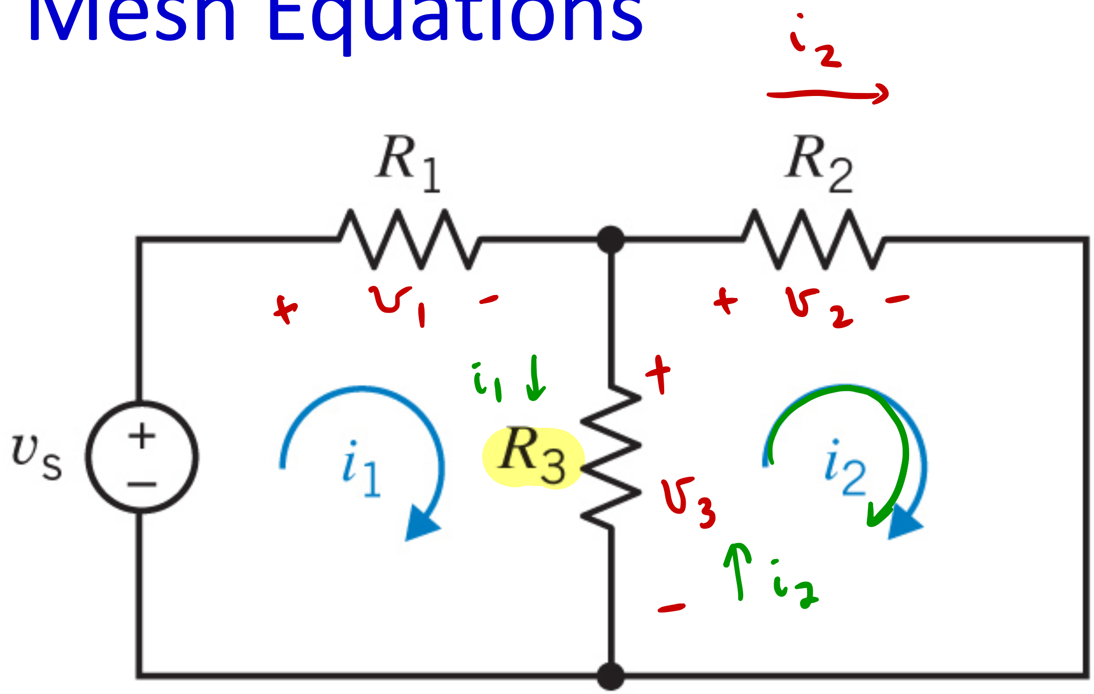




$$i_{R_3} = i_1 - i_2$$

$$v_3 = i_{R_3} \cdot R_3$$

Mesh Equations



KVL mesh 1 : $v_s - v_1 - v_3 = 0$

$v_1 = i_1 \cdot R_1$

$v_3 = (i_1 - i_2) \cdot R_3$

mesh 2 : $v_3 - v_2 = 0$

$v_2 = i_2 \cdot R_2$