EE2210 Electric circuits

QUIZ3 Ans.

1.

Quiz 3

1.

$0 < t < \frac{T}{2}$: $V_o(t) = 0 + (0 - V_S)\, e^{-\frac{t}{R_{on} C_L}}$

$$E_1 = \int_0^{\frac{T}{2}} \frac{V_o^2}{R_{on}}\, dt \approx \frac{V_S^2 C_L}{2}$$

$\frac{T}{2} < t < T$: $V_o(t) = V_S + (V_S - 0)\, e^{-\frac{t}{R_{on} C_L}}$

$$E_2 = \int_{\frac{T}{2}}^{T} \frac{(V_S - V_o)^2}{R_{on}}\, dt \approx \frac{V_S^2 C_L}{2}$$

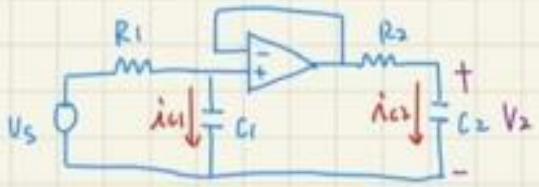
$E_{total} = E_1 + E_2 = V_S^2 C_L$

$P = \frac{E_{total}}{T} = V_S^2 C_L f$

Static power = 0 W #

Dynamic power = $V_S^2 \times 1f \times 100M = V_S^2 \times 10^{-7}$ W #

2.



$$\frac{V_S - V_1}{R_1} - i_{C1} = 0 \quad \left(i_{C1} = C_1 \frac{dV_{C1}}{dt}\right) \Rightarrow \frac{V_1}{R_1} + C_1 \frac{dV_1}{dt} = \frac{V_S}{R_1} \quad -①$$

$$\frac{V_1 - V_2}{R_2} - i_{C2} = 0 \quad \left(i_{C2} = C_2 \frac{dV_{C2}}{dt}\right) \Rightarrow \frac{V_2}{R_2} + C_2 \frac{dV_2}{dt} = \frac{V_S}{R_2}$$

$$\Rightarrow V_1(t) = R_2 C_2 \frac{dV_2}{dt} + V_2 \quad -②$$

將②代入①

$$\frac{R_2 C_2}{R_1} \cdot \frac{dV_2}{dt} + \frac{V_2}{R_1} + C_1 \cdot \frac{d}{dt}\left[R_2 C_2 \frac{dV_2}{dt} + V_2\right] = V_S/R_1$$

$$\frac{R_2 C_2}{R_1} V_2' + \frac{1}{R_1} V_2 + R_2 C_1 C_2 V_2'' + C_1 V_2' = \frac{V_S}{R_1} \quad (\text{同乘} \frac{1}{R_2 C_2 C_1})$$

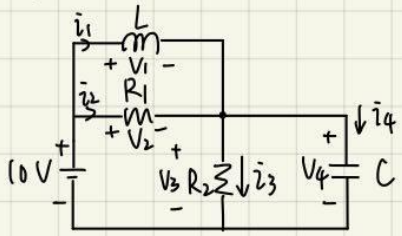
$$V_2'' + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2}\right) V_2' + \frac{1}{R_1 R_2 C_1 C_2} V_2 = \frac{1}{R_1 R_2 C_1 C_2} V_S \quad \#$$

3.

Quiz 3.

3. $V_4(0^+) = 4\ V$, $i_1(0^+) = 2\ A$ $\Rightarrow$ capacitor : voltage continuous, inductor : current continuous

$t = 0^+$



$$i_2(0^+) = \frac{10-4}{R_1} = 3\ A$$

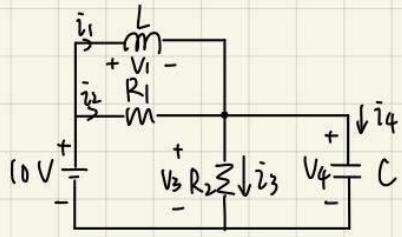
$$i_3(0^+) = \frac{4}{R_2} = 4\ A$$

$$i_4(0^+) = i_1 + i_2 - i_3 = 1\ A$$

$$V_1(0^+) = 10 - 4 = 6\ V = V_2(0^+)$$

$$V_3(0^+) = V_4(0^+) = 4\ V$$

$t \to \infty$



$$V_4(\infty) = 10\ V = V_3(\infty)$$

$$V_1(\infty) = 0\ V = V_2(\infty)$$

$$i_2(\infty) = \frac{10-10}{R_1} = 0\ A$$

$$i_3(\infty) = \frac{10}{R_2} = 10\ A$$

$$i_4(\infty) = 0\ A$$

$$i_1(\infty) = i_3 + i_4 - i_2 = 10\ A$$

4.

$$i_R + i_L + i_C = 0$$

$$\frac{V}{R} + \frac{1}{L}\int_0^t v\,d\lambda + i_L(0) + C\frac{dV}{dt} = 0 \Rightarrow C\frac{d^2V}{dt^2} + \frac{1}{R}\frac{dV}{dt} + \frac{1}{L}V = 0$$

$$\Rightarrow \frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{1}{LC}V = 0 \Rightarrow s^2 + 25000s + 10^8 = 0 \Rightarrow s = -20000, -5000$$

$$\therefore v(t) = A_1 e^{-20000t} + A_2 e^{-5000t}, \quad v(0) = A_1 + A_2 = 15 \quad -①$$

$$-20000A_1 - 5000A_2 = -375000 + 225000 = -150000 \Rightarrow A_1 + 0.25A_2 = 7.5 \quad -②$$

solve ①, ② $\Rightarrow A_1 = 5$, $A_2 = 10$, $v(t) = 5e^{-20000t} + 10e^{-5000t}$

5.

$t>0$

$$\left.\begin{aligned} i_L + 0.2\frac{dv}{dt} &= 4 \\ (v-1)\frac{di_L}{dt} - 6i_L &= 0 \end{aligned}\right\} \quad \frac{d^2v}{dt^2} + 6\frac{dv}{dt} + 5v = 120$$

$s^2 + 6s + 5 = 0 \rightarrow v(t) = A_1 e^{-t} + A_2 e^{-5t} + B$

穩態 $\rightarrow B = 24$

$$\left.\begin{aligned} v(0) &= A_1 + A_2 + 24 = 0 \\ \frac{dv(0)}{dt} &= -A_1 - 5A_2 = 20 \end{aligned}\right\} \quad A_1 = -25,\ A_2 = 1$$

$v(t) = -25e^{-t} + e^{-5t} + 24$ (V)

6.

Quiz 3

6. KVL:

$$\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_L}{dt} + \frac{V_L}{LC} = \frac{48}{LC}$$

$$\Rightarrow V_{c,p} = 48$$

$$V_{c,h} = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

characteristic equ: $s^2 + 2800s + 2.5\times10^7 = 0$

$$\rightarrow s = -1400 \pm 4800j.$$

$$V_c = 48 + e^{-1400t}\left(k_1\cos(4800t) + k_2\sin(4800t)\right)$$

$$k_1 = A_1 + A_2 \quad \text{and} \quad k_2 = jA_1 - jA_2$$

$$\begin{cases} V_c(0) = 48 + k_1 = 0 \\ i_L(0) = C\cdot(-1400k_1 + 4800k_2) = 0 \end{cases} \Rightarrow \begin{cases} k_1 = -48 \\ k_2 = -14 \end{cases}$$

$$\Rightarrow V_c(t) = 48 + e^{-1400t}\left[-48\cos(4800t) - 14\sin(4800t)\right] \ (V)$$

for $t \geq 0$

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