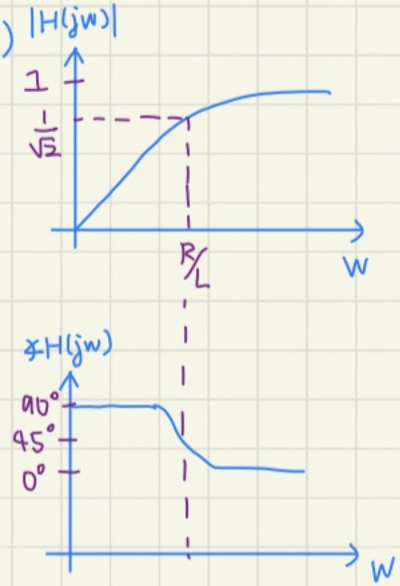
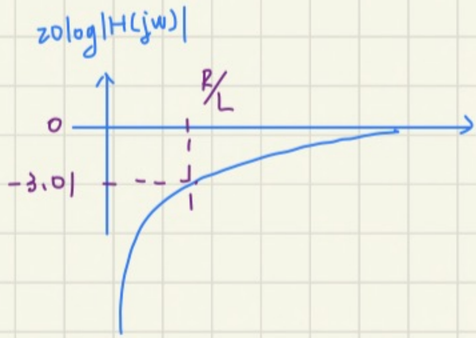
EE2210 Electric circuits HW5 Ans.

1. (a) $\dfrac{V_o}{V_i} = \dfrac{jwL}{R + jwL}$, $|H(jw)| = \dfrac{wL}{\sqrt{R^2 + w^2L^2}}$

$$\measuredangle H(jw) = 90^\circ - \tan^{-1}\left(\frac{wL}{R}\right) = \tan^{-1}\left(\frac{R}{wL}\right)$$

(b)

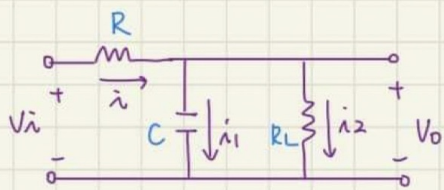




2. $V_{in}(t) = V_1 \cos(120\pi t)$, where $\omega = 120\pi$

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1/sC}{R + sL + 1/sC} = \frac{1/jw\cdot 80\cdot 10^{-6}}{50 + jw0.15 + 1/jw\cdot 80\cdot 10^{-6}} = -0.193 - j0.062 = 0.203\angle 197.8^\circ$$ #

3.



(a) Step 1 : set up DE

$$i = i_1 + i_2 \Rightarrow \frac{V_i - V_o}{R} = C\frac{dV_o}{dt} + \frac{V_o}{R_L} \Rightarrow RC\cdot\frac{dV_o}{dt} + \left(1+\frac{R}{R_L}\right)V_o = V_i$$

Step 2 : Find Particular Solution

$$\because V_i = 10\cos(100t+45°) \Rightarrow \tilde{V_i} = 10\cdot e^{j100t}\cdot e^{j45°}$$

Let $\tilde{V_{op}} = Ae^{st}$ where $s = j\cdot 100$, $\tilde{V_{op}} = Ae^{j100t}$ 代入 DE

$$RC\cdot A\cdot j100\cdot \cancel{e^{j100t}} + A\cancel{e^{j100t}} + \frac{R}{R_L}A\cancel{e^{j100t}} = 10\cancel{e^{j100t}}\cdot e^{j45°}$$

$$\Rightarrow \left(RC\cdot j100 + 1 + \frac{R}{R_L}\right)\cdot A = 10e^{j45°} \quad \therefore A = \frac{10}{\left(1+\frac{R}{R_L}\right)+j(100RC)}e^{j45°}$$

$$= \frac{10}{\sqrt{\left(1+\frac{R}{R_L}\right)^2+(100RC)^2}}\cdot e^{j\cdot\tan^{-1}\left(-\frac{100RC}{1+R/R_L}\right)}\cdot e^{j45°}$$

代回 $\tilde{V_{op}}$

$$V_{op} = Re\left[\tilde{V_{op}}\right] = \frac{10}{\sqrt{\left(1+\frac{R}{R_L}\right)^2+(100RC)^2}}\cos\left[100t+45° - \tan^{-1}\left(\frac{100RC}{1+R/R_L}\right)\right] \text{ (V)}$$

Steady-state response #

Step 3 : Find Homogeneous Solution ($V_i = 0$)

Let $V_{oh} = Be^{\alpha t}$ 代入 DE $\Rightarrow RC\cdot\alpha\cdot B\cancel{e^{\alpha t}} + \left(1+\frac{R}{R_L}\right)\cdot B\cancel{e^{\alpha t}} = 0$

$$\therefore \alpha = \frac{-(1+R/R_L)}{RC} \Rightarrow V_{oh} = B\cdot e^{\frac{-(1+R/R_L)}{RC}t}$$

Step 4: Total Solution $\left(V_o(0) = V_{op}(0) + V_{oh}(0) = 0\right)$

$$V_{op}(0) + B = V_1 \Rightarrow B = -V_{op}(0) + V_1 = \frac{-10}{\sqrt{\left(1+\frac{R}{R_L}\right)^2 + (100RC)^2}} \cos\left[45^\circ - \tan^{-1}\left(\frac{100RC}{1+R/R_L}\right)\right] + V_1$$

$$\therefore V_{oh} = \left\{\frac{-10}{\sqrt{\left(1+\frac{R}{R_L}\right)^2 + (100RC)^2}} \cos\left[45^\circ - \tan^{-1}\left(\frac{100RC}{1+R/R_L}\right)\right] + V_1\right\} e^{\frac{-\left(1+\frac{R}{R_L}\right)}{RC} t} \ (V) \ \#$$

transient response

$$\Rightarrow V_o(t) = V_{op}(t) + V_{oh}(t)$$

$$= \frac{10}{\sqrt{\left(1+\frac{R}{R_L}\right)^2 + (100RC)^2}} \cos\left[45^\circ - \tan^{-1}\left(\frac{100RC}{1+R/R_L}\right)\right]\left[1 - e^{\frac{-\left(1+\frac{R}{R_L}\right)}{RC} t}\right] + V_1 e^{-\frac{\left(1+\frac{R}{R_L}\right)}{RC} t} \ (V) \ \#$$

(b) $H(jw) = \dfrac{\left(\frac{1}{sC} \parallel R_L\right)}{R + \left(\frac{1}{sC} \parallel R_L\right)}$ where $\dfrac{1}{sC} \parallel R_L = \dfrac{R_L/sC}{\frac{1}{sC} + R_L} = \dfrac{R_L}{1+sCR_L}$

$$= \frac{\frac{R_L}{1+sCR_L}}{R + \frac{R_L}{1+sCR_L}} = \frac{R_L}{R_L + R + sCR\cdot R_L} \text{ where } s = jw$$

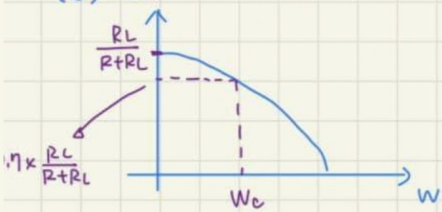
$$= \frac{R_L}{R + R_L + jwR\cdot R_L\cdot C}$$

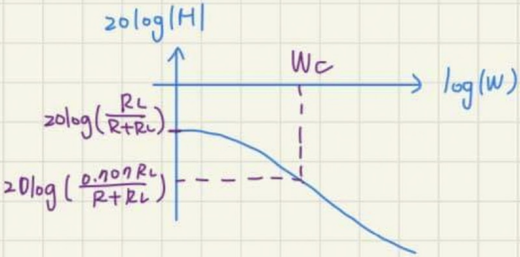
(c) $$|H(jw)| = \frac{R_L}{\sqrt{(R+R_L)^2 + (wRR_LC)^2}}$$

$\therefore$ when $w \to 0$, $|H(jw)|$ will be maximum.

(d) $w=0$ 代入, $|H(jw)| = \dfrac{R_L}{R+R_L}$

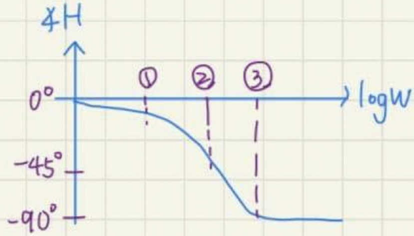
(e)





when $W_c = \dfrac{1}{C\cdot\dfrac{R\cdot R_L}{R+R_L}} = \dfrac{R+R_L}{R\cdot R_L\cdot C}$

$|H(jw)| = \dfrac{1}{\sqrt{2}}\cdot\dfrac{R_L}{R+R_L}$



① : $\log(0.1W_c)$

② : $\log(W_c)$

③ : $\log(10W_c)$

4.

$$Z(s) = (R+sL) \parallel (R + 1/sC) = \frac{R^2 + \frac{R}{sC} + sLR + \frac{L}{C}}{2R + sL + \frac{1}{sC}}$$

$$= \frac{sCR^2 + R + s^2RLC + sL}{2sCR + s^2LC + 1} = \frac{(R - w^2RLC) + jw(L + CR^2)}{(1 - w^2LC) + jw(2RC)}$$

$$= \frac{[(1-w^2LC) - jw(2RC)][(R - w^2RLC) + jw(L + CR^2)]}{(1-w^2LC)^2 + w^2 4R^2C^2}$$

① $Im(Z) = 0$, for all w

$$-2RC(R - w^2RLC) + (1 - w^2LC)(L + R^2C) = 0$$

$$-R^2C + w^2R^2C^2L + L - w^2L^2C = 0$$

$$(L - R^2C) + w^2LC(R^2C - L) = 0$$

$$\Rightarrow L = R^2C = 0.001 \Rightarrow C = \frac{10^{-3}}{R^2}$$

② $Re(Z) = 2000$, for all w

$$(1-w^2LC)(R - w^2RLC) + w^2 2RC(L + R^2C)$$

$$= R - w^2RLC \cdot 2 + w^4RC^2L^2 + w^2 4RCL$$

$$= R + 2w^2RLC + w^4RC^2L^2$$

$$= R - 2w^2L\left(\frac{10^{-3}}{R}\right) + w^4L^2\left(\frac{10^{-6}}{R^3}\right)$$

$$(1-w^2LC)^2 + w^2 4R^2C^2 = w^2 4LC + (1 + w^4L^2C^2 - 2w^2LC)$$

$$= w^4L^2C^2 + 2w^2LC + 1$$

$$2 \times 10^3 [w^4L^2C^2 + 2w^2LC + 1] = R[w^4L^2C^2 + 2w^2LC + 1]$$

$$\Rightarrow R = 2000\,\Omega, \quad C = \frac{10^{-3}}{4 \times 10^6} = 250\,pF \;\#$$