EE2210 Electric circuits HW4 Ans.

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HW4 - 1

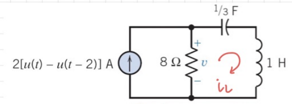


Figure 1

superposition : $I_{in} = 2u(t)$

$$\begin{cases} (I_{in} - i_L)\cdot R = V_C + L\cdot \dfrac{di_L}{dt} \\ i_C = i_L = C\cdot \dfrac{dV_C}{dt} \end{cases}$$

$$\Rightarrow \frac{d^2V_C}{dt} + \frac{R}{L}\frac{dV_C}{dt} + \frac{V_C}{LC} = \frac{I_{in}\cdot R}{LC}$$

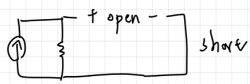
$$V_{C1.p} = I_{in}\cdot R = 16u(t)$$

$$V_{C1.h} = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

charavsitic equ : $s^2 + 8s + 3 = 0$

$$\rightarrow s = -4 \pm \sqrt{13}$$

initial condition :



$i_L(0) = 0$

$V_C(0) = 0$

$$\begin{cases} V_C = 16 + A_1 + A_2 = 0 \\ i_L = C(A_1 s_1 + A_2 s_2) = 0 \end{cases}$$

$$\Rightarrow A_1 = \frac{16s_2}{s_1 - s_2} = \frac{-32}{13}\sqrt{13} - 8 \approx -16.875$$

$$A_2 = \frac{16s_1}{s_2 - s_1} = \frac{32}{13}\sqrt{13} - 8 \approx 0.875$$

$$V_{C1} = 16 - 16.875e^{(-4+\sqrt{13})t} + 0.8e^{(-4-\sqrt{13})t}$$

② $I_{in} = -2u(t-2)$

→ replace $t \rightarrow t-2$ and multiply negative sign. from above.

$$\Rightarrow V_{C2} = -16 + 16.875e^{(-4+\sqrt{13})(t-2)} - 0.8e^{(-4-\sqrt{13})(t-2)}$$

$V_C = V_{C1} + V_{C2}$

$$= \left[16 - 16.875e^{(-4+\sqrt{13})t} + 0.8e^{(-4-\sqrt{13})t}\right]u(t)$$

$$+ \left[-16 + 16.875e^{(-4+\sqrt{13})(t-2)} - 0.8e^{(-4-\sqrt{13})(t-2)}\right]u(t-2).$$

(V) #

2. (a) $i_R = \frac{V_0}{R} = \frac{15}{200} = 75\ mA$

$i_C = -i_L - i_R = 45 - 75 = -30\ mA$ #

(b) $i_R + i_L + i_C = 0$

$\frac{V}{R} + \frac{1}{L}\int_0^t v\,d\tau + i_L(0) + C\frac{dv}{dt} = 0 \Rightarrow C\frac{d^2V}{dt^2} + \frac{1}{R}\frac{dV}{dt} + \frac{1}{L}V = 0$

$\Rightarrow \frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{1}{LC}V = 0 \Rightarrow s^2 + 25000s + 10^8 = 0 \Rightarrow s = -20000 / -5000$

$\therefore v(t) = A_1e^{-20000t} + A_2e^{-5000t}$, $v(0) = A_1 + A_2 = 15$ —①

$-20000A_1 - 5000A_2 = -375000 + 225000 = -150000 \Rightarrow A_1 + 0.25A_2 = 7.5$ —②

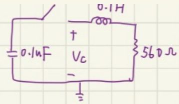
solve ①, ② $\Rightarrow A_1 = 5,\ A_2 = 10,\ v(t) = 5e^{-20000t} + 10e^{-5000t}$

(c) $i_L(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i_L(0) = 20\int_0^t (5e^{-20000t} + 10e^{-5000t})d\tau - 0.045$

$= -0.005e^{-20000t} - 0.04e^{-5000t}$ #

3.

(a) By KVL, $V_c(t) + L\frac{di(t)}{dt} + i(t)R = 0$



$i(t) = C\frac{dv(t)}{dt} \Rightarrow V_c + LC\frac{d^2V_c}{dt} + RC\frac{dV_c}{dt} = 0$

$\Rightarrow \frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{1}{LC}V_c = 0 \Rightarrow \frac{d^2V_c}{dt^2} + 5600\frac{dV_c}{dt} + 10^8 V_c = 0$

$\because \begin{cases} W_0 = \frac{1}{\sqrt{LC}} = 10000 \\ \alpha = \frac{R}{2L} = 2800 \end{cases} \Rightarrow W_0 > \alpha \Rightarrow \begin{cases} S_1, S_2 \text{ complex conjugate} \\ \text{under-damped} \end{cases}$

$\therefore V_c(t) = A_1 e^{-\alpha t} e^{(\sqrt{\alpha^2 - W_0^2})t} + A_2 e^{-\alpha t} e^{-(\sqrt{\alpha^2 - W_0^2})t}$

$= A_1 e^{-2800t} e^{9600t} + A_2 e^{-2800t} e^{-9600t}$

$= e^{-2800t}(K_1 \cos 9600t + K_2 \sin 9600t)$

$\because V_c(0^+) = 100V = K_1$

$\therefore i(t) = C\frac{dV_c}{dt} = C \times [-2800 e^{-2800t}(100\cos 9600t + K_2 \sin 9600t) + e^{-2800t} \times (-9.6\times 10^5 \sin 9600t + 9600 K_2 \cos 9600t)]$

$i(0) = 0 = C \times [-2800 \times 100 + 9600 K_2] \Rightarrow K_2 = 29.1\overline{6}$

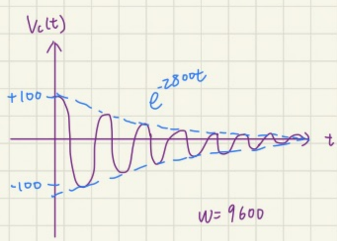
$\therefore V_c(t) = e^{-2800t}(100\cos 9600t + 29.1\overline{6}\sin 9600t)$.

$i(t) = C\frac{dV_c}{dt} = (10^{-7}) \times e^{-2800t} \times [(-2.8\times 10^5 + 9600 \times 29.1\overline{6})\cos 9600t + (-2800 \times 29.1\overline{6} - 9.6\times 10^5)\sin 9600t]$

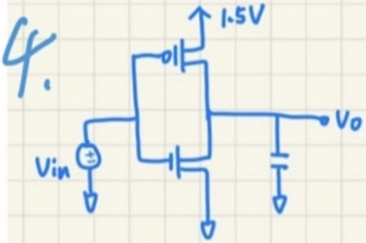
$= -\frac{5}{48}e^{-2800t}\sin 9600t$ #

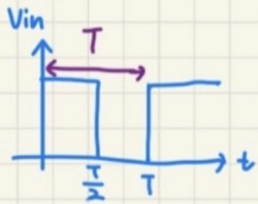
(b) $\begin{cases} \alpha = \frac{R}{2L} = 2800 \\ w_0 = \frac{1}{\sqrt{LC}} = 10000 \end{cases} \Rightarrow W_0 > \alpha \Rightarrow V_c$ is under damping

$V_c(t) = e^{-2800t}(100\cos 9600t + 29.1\overline{6}\sin 9600t)$

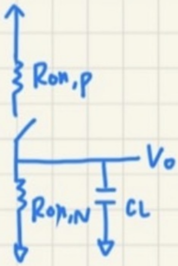


4.





$\boxed{0 < t < \frac{T}{2}}$ $V_{in} = V_{high}$ < Nmos on, Pmos off — discharging C_L



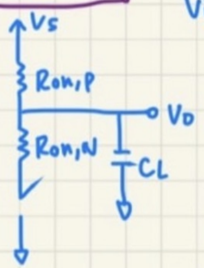
$V_{o,\,initial} = V_s, \quad V_{o,\,final} = 0$

$V_o(t) = 0 + (0 - V_s)\, e^{-t/R_{on}C_L}$

$E_1 = \int_0^{T/2} \frac{V_o^2}{R_{on}}\, dt = \frac{V_s^2 C_L}{2}\left(1 - e^{\frac{-2(T/2)}{R_{on}C_L}}\right)$

$\approx \frac{V_s^2 C_L}{2} \quad (\because T/2 \gg R_{on}C_L)$

$\boxed{T/2 < t < T}$ $V_{in} = V_{low}$ < Nmos off, Pmos on — charging C_L



$V_{o,\,initial} = 0, \quad V_{o,\,final} = V_s$

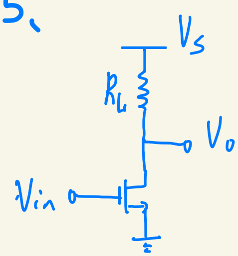
$V_o(t) = V_s + (V_s - 0)\, e^{\frac{-t}{R_{on}C_L}}$

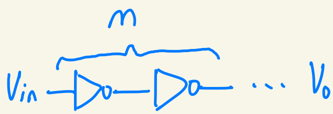
$E_2 = \int_{T/2}^{T} \frac{(V_s - V_o)^2}{R_{on}}\, dt \approx \frac{V_s^2 C_L}{2} \quad (\because T/2 \gg R_{on}C_L)$

$E_{total} = E_1 + E_2 = V_s^2 C_L, \quad \overline{P} = \frac{E_{total}}{T} = E_{total} \cdot f = V_s^2 C_L f$

Static power $= 0\,W$ Dynamic power $= 1.5^2 \cdot 10 \cdot 10^{-15} \cdot 10^7 = 0.225\ \mu W$ ※

5.





If V_{in} is high, nmos turns on, then the static power consumption is

$$V_s \cdot \left(\frac{V_s}{R_L + R_{on}} \right) = \frac{V_s^2}{R_L + R_{on}}$$

Over the inverter chain, only half of inverters will turn on because of the inverting property, the total static power cons. is

$$\frac{m}{2} \times \frac{V_s^2}{R_L + R_{on}} \quad \#$$