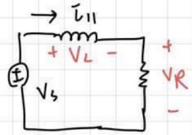
EE2210 Electric circuits HW2 Ans.

HW2 - 1

By superposition : $V_s = L \cdot \frac{di_{11}}{dt} + i_{11}R$



① particular solution : $i_{11} = \frac{V_s}{R}$

② homogeneous solution :

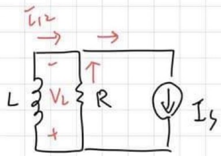
$$L\frac{di_{11}}{dt} + i_{11}R = 0 \rightarrow \text{Assume } i_{11} = Ae^{st}$$

$$sLAe^{st} + RAe^{st} = 0 \rightarrow (sL+R) = 0 \rightarrow s = \frac{-R}{L}$$

$$\Rightarrow i_{11} = \frac{V_s}{R} + Ae^{-\frac{R}{L}t}$$

initial condition : $i_{11}(0) = \frac{V_s}{R} + A = 0 \rightarrow A = -\frac{V_s}{R}$

$$\Rightarrow i_{11}(t) = \frac{V_s}{R} - \frac{V_s}{R}e^{-\frac{R}{L}t}$$



$$I_s = i_R + i_{12} = \frac{V_L}{R} + \frac{1}{L}\int_{-\infty}^{t} V_L\,dt$$

$$I_s = \frac{V_L}{R} + i_{12}(0) + \frac{1}{L}\int_{0}^{t} V_L\,dt$$

$$\xRightarrow{\frac{d}{dt}} \frac{1}{R}\frac{dV_L}{dt} + \frac{1}{L}V_L = 0 \Rightarrow V_L = Ae^{-\frac{R}{L}t}$$

initial condition : $i_{12}(0) = 0 = I_s - \frac{A \cdot 1}{R} \rightarrow A = I_s \cdot R$

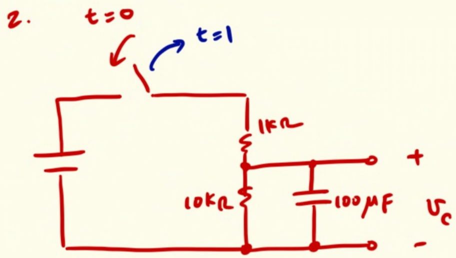
$$\Rightarrow i_{12}(t) = I_s - I_s e^{-\frac{R}{L}t}$$

$$\Rightarrow i_1(t) = i_{11}(t) + i_{12}(t)$$

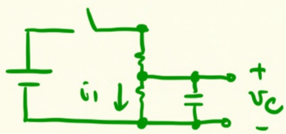
$$= \frac{1}{3000}\left[1 - e^{-3000t}\right] + \frac{1}{1000}\left[1 - e^{-3000t}\right]$$

#

2.



1) For $t<0$,

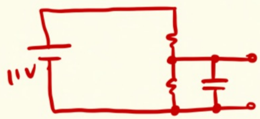


$i_1 = 0$

$v_c = 0$

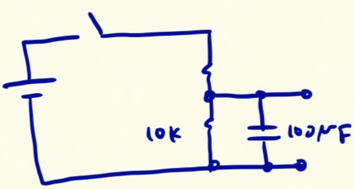
2) $0<t<1$,

$$\frac{11 - v_c}{1k} = \frac{v_c}{10k} + 100\mu \cdot \frac{dv_c}{dt} \Rightarrow v_c = 10 - 10 \cdot e^{-t/(9.09m)} \text{ V}$$

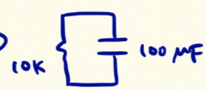


when $t \to \infty$, $v_c \to 10V$

3) $t>1$,



$\Rightarrow$



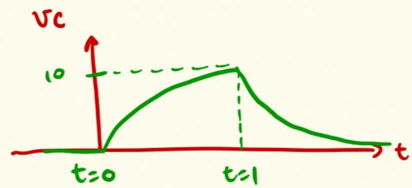
with initial condition

$v_c(t=1)$

$= 10 - 10 \cdot e^{-1/(9.09m)}$

≈ 10 V

$v_c = 10 \cdot e^{-t}$ V



$v_c = 0$

$v_c = 10 - 10 \cdot e^{-t/9.09m}$

$v_c = 10 \cdot e^{-t}$

3、

$$i(t) = C\frac{dV_C}{dt} \quad , \quad V_C = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$$

$$V(t) = i(t)\times 8 + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$$

$$= 8i(t) + \frac{1}{C}\int_{-\infty}^{0} i(\tau)\,d\tau + \frac{1}{C}\int_{0}^{t} i(\tau)\,d\tau$$

$$= 8\cdot 3e^{-25t} + (-2) + \frac{1}{0.1}\int_{0}^{t} 3e^{-25\tau}\,d\tau$$

$$= 24e^{-25t} - 2 + 3\times\frac{1}{0.1}\times\left(\frac{1}{-25}\right)e^{-25\tau}\,d\tau\Big|_{0}^{t}$$

$$= 24e^{-25t} - 2 - \frac{6}{5}\left(e^{-25t} - 1\right)$$

$$= 22.8e^{-25t} - 0.8 \ \#$$

$$V_C = V(t) - i(t)\times 8 = 22.8e^{-25t} - 0.8 - 8\times 3e^{-25t}$$

$$= -1.2e^{-25t} - 0.8 \ \#$$

HW2-4

For $0 < t < 1\mu$,

$$i_L(t) = \frac{1}{L}\int_0^t V_s(t)\,dt + I_0$$

$$= \frac{1}{5m}\int_0^t 4m\,dt + (-2\mu) = 0.8t - 2\mu \text{ A} \ \#$$

For $1\mu < t < 3\mu$,

$$i_L(t) = \frac{1}{5m}\int_{1\mu}^t (-1m)\,dt + i(1\mu)$$

$$= -0.2(t - 1\mu) + (-1.2\mu) = -0.2t - 1\mu \text{ A} \ \#$$

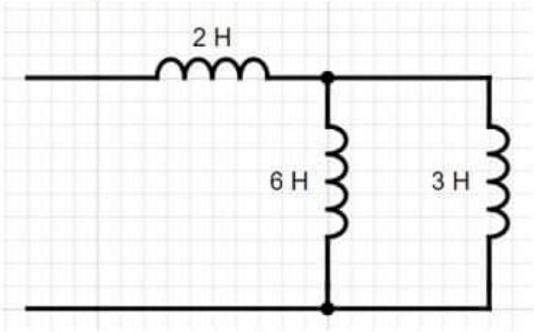
For $t > 3\mu$,

$$i_L(t) = \frac{1}{5m}\int_{3\mu}^t 0\cdot dt + i(3\mu)$$

$$= -0.2 \times 3\mu - 1\mu = -1.6\mu \text{A} \ \#$$

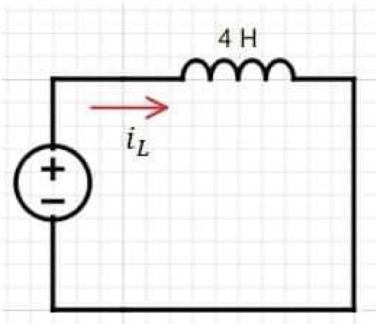
5、

Use equivalent circuit



$$L_{eq} = 2 + \frac{1}{\frac{1}{6}+\frac{1}{3}} = 4\ H$$

Thus, ordinary circuit can be re-draw as below



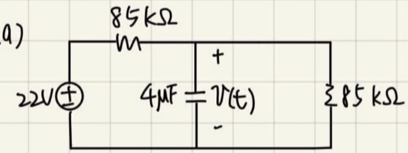
Then,

$$V = L\frac{di_L}{dt} \rightarrow i = \frac{1}{L}\int_0^t V\,d\bar{t} = \frac{1}{4}\int_0^t 10 * cos(10\bar{t})\,d\bar{t} = 0.25 * \sin(10t)\,A$$

HW2

6、

(a)

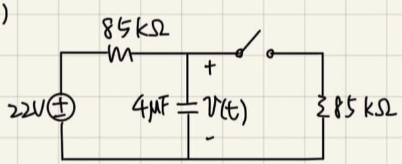


at steady state:

$v(t) = 11\ V$

$E = \frac{1}{2} C v^2 = \frac{1}{2} \cdot 4\mu \cdot 11^2 = 242\ \mu J$ ※

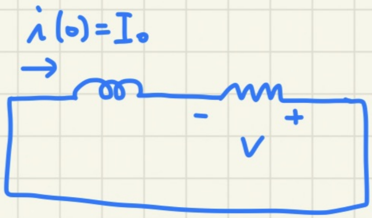
(b)



at steady state:

$v(t) = 22\ V$

$E = \frac{1}{2} C v^2 = \frac{1}{2} \cdot 4\mu \cdot 22^2 = 968\ \mu J$ ※

7. 

$$R = R_1 /\!/ R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$v(t) = -\frac{R_1 R_2}{R_1 + R_2} I_0 \, e^{-\frac{R_1 R_2}{R_1 + R_2} \cdot \frac{t}{L}}$$

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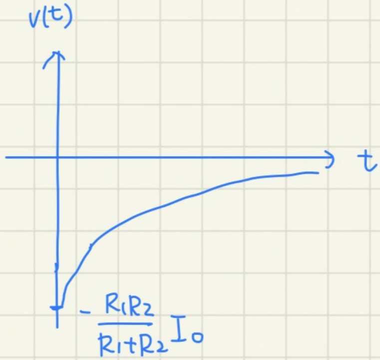

8.

$$V = L\frac{di}{dt}$$

$$= \cos(100t)\cdot L\cdot -\sin 100t \cdot 100$$

$$P = IV$$

$$= I\cdot L\frac{di}{dt} = \begin{cases} -500\sin(100t)\cos(100t), & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$= \frac{d}{dt}\left(\frac{1}{2}LI^2\right)$$

$$E = \frac{1}{2}LI^2 = \begin{cases} \frac{5}{2}(\cos(100t))^2, & t \geq 0 \\ 0, & t < 0 \end{cases}$$