EE2210 Electric Circuits

Practice problems solution for Lecture11~13 **Spring 2019**

1.

$$\frac{V_o}{V_i} = \frac{Ls}{Ls+R} = \frac{\omega L}{\sqrt{(\omega L)^2 + R^2}}\, e^{j\phi}$$

$$\phi = \tan^{-1}\left(\frac{R}{\omega L}\right)$$

2.

Solution:

$$Z = \frac{(R+\frac{1}{Cs})(R+Ls)}{2R+\frac{1}{Cs}+Ls} = \frac{R(LCs^2+(\frac{L}{R}+RC)s+1)}{LCs^2+2RCs+1}$$

In order for Z to always be purely real,

$$\left(\frac{L}{R}+RC\right) = 2RC$$

$$L = R^2C$$

Then

$$Z = R = 2000$$

independent of ω.

$$.001 = 2000^2\ C$$

$$C = 2.5 \cdot 10^{-10} Farads$$

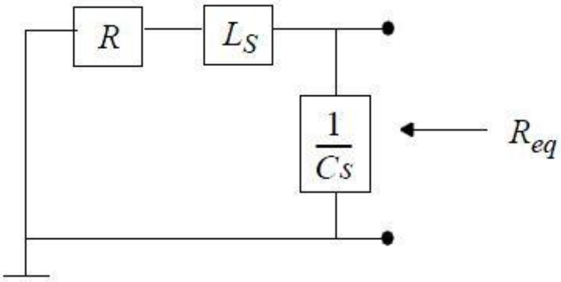
ANS:: $R = 2000$ and $C = 2.5 \cdot 10^{-10}$ Farads

3.

Solution:

a)

$$H(s) = \frac{1}{LCs^2 + RCs + 1} = \frac{1}{(1-\omega^2 LC) + j\omega RC}$$



4.

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{\frac{T}{2}} V^2 dt} = \sqrt{\frac{1}{T}\int_0^{\frac{T}{2}} \left(V_m \sin\left(\frac{2\pi}{T}t\right)\right)^2 dt} = \sqrt{\frac{{V_m}^2}{T}\int_0^{\frac{T}{2}} \sin^2\left(\frac{2\pi}{T}t\right) dt}$$

$$= \sqrt{\frac{{V_m}^2}{T}\frac{T}{2\pi}\int_0^{\frac{T}{2}} \sin^2\left(\frac{2\pi}{T}t\right) d\,\frac{2\pi}{T}t}$$

$$= \sqrt{\frac{{V_m}^2}{T}\frac{T}{2\pi}\int_0^{\frac{T}{2}} \frac{1-\cos\left(2*\frac{2\pi}{T}t\right)}{2} d\,\frac{2\pi}{T}t}$$

$$= \sqrt{\frac{{V_m}^2}{2\pi}[\frac{2\pi}{T}\frac{t}{2} - \frac{\sin(2*\frac{2\pi}{T}t)}{4}]_0^{T/2}} = \sqrt{\frac{{V_m}^2}{2\pi}[\frac{\pi}{2}]} = \frac{V_m}{2}$$

Note:

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

5.

$$\omega = 50$$

$$Z = R + j\omega L + \frac{1}{j\omega C} = 4 + \mathrm{j} * 50 * 0.24 - \mathrm{j}\frac{1}{50 * 0.0025} = 5.66\angle 45^{o}\,\Omega$$

$$I_0 = \frac{V}{Z} = \frac{0.1\angle -90^{o}}{5.66\angle 45^{o}} = 17.67\angle -135^{o}\ mA$$

$$i_0 = 17.67\cos(50\mathrm{t}-135^{o})\ mA$$

6.

(a)

$$Z_{ab} = j\omega L + R\|\left(\frac{1}{j\omega C}\right) = j\omega L + \frac{-\frac{jR}{\omega C}}{R - \frac{j}{\omega C}} = j\omega L + \frac{-jR}{\omega CR - j} = j\omega L + \frac{-jR(\omega CR + j)}{(\omega CR)^2 + 1}$$

Pure resistive→ $\mathrm{Im}(Z_{ab}) = 0$

$$\therefore \omega L - \frac{\omega CR^2}{(\omega CR)^2 + 1} = 0 \rightarrow \omega^2 = \frac{\left(\frac{CR^2}{L}\right) - 1}{(CR)^2} = 900 * 10^8$$

$$\omega = 300\ krad/s$$

(b)

$$Z_{ab}(300 * 10^3) = j48 + \frac{100(-j133.33)}{100 - j133.33} = 64\ \Omega$$

7.

a) $\mathrm{H(jw)} = \frac{v_o}{v_i} = \frac{1/sC\|R_L}{R+1/sC\|R_L} = \frac{R_L}{sRCR_L+(R+R_L)}$

b) $|\mathrm{H(jw)}| = \frac{R_L}{\sqrt{wRCR_L{}^2+(R+R_L)^2}}$ |H(jw)| is maximum at w=0

c) $|H(jw)|_{max} = \frac{R_L}{R+R_L}$

d) $|H(jw)| = \frac{R_L}{\sqrt{2}(R+R_L)} = \frac{1/RC}{\sqrt{w_c{}^2+[(R+R_L)/RR_LC]^2}}$ $w_c = \frac{1}{RC}(1 + \left(\frac{R}{R_L}\right))$