

Electric Circuits Spring 2019

Practice problems (Lecture4-Lecture7)

Exercise 9.2

(a)

$$\frac{1\mu F \cdot 2\mu F}{1\mu F + 2\mu F} = \frac{2}{3} \mu F$$

(b)

$$\frac{1\mu F \cdot 10\text{pF}}{1\mu F + 10\text{pF}} = 9.9 \text{ pF}$$

(c)

$$\frac{40\text{pF} \cdot 1\mu F}{40\text{pF} + 1\mu F} = 38.5 \text{ pF}$$

(d)

$$1\text{mH} + 2\text{mH} = 3 \text{ mH}$$

(e)

$$2\text{mH} + 1\mu\text{H} = 2.001 \text{ mH}$$

(f)

$$\frac{2\text{mH} \cdot 1\text{mH}}{2\text{mH} + 1\text{mH}} + 1\mu\text{H} \cong \frac{2}{3} \text{ mH}$$

Exercise 9.4

$$v_c(t) = \frac{1}{C} \int_{-\infty}^t i_c(t) dt$$

Assume initial capacitor voltage = $v_{initial}$ when $t = 0$.

$$0 < t < \frac{T}{2}, \quad v_c(t) = \frac{1}{C} \int_{-\infty}^t I_0 dt = \frac{1}{C} \cdot I_0 \cdot t + v_{initial}$$

$$t = \frac{T}{2}, \quad v_c(t) = \left(\frac{1}{C} \cdot I_0 \cdot \frac{T}{2} \right) + \frac{Q_0}{C}$$

$$\frac{T}{2} < t < T, \quad v_c(t) = \left(v_{initial} + \frac{1}{C} \cdot I_0 \cdot \frac{T}{2} + \frac{Q_0}{C} \right) + \left(\frac{1}{C} \cdot I_0 \cdot \left(t - \frac{T}{2} \right) \right)$$

Capacitor voltage is given as V_0 at $t = T$,

$$\Rightarrow \text{At } t = T, \quad V_0 = \left(v_{initial} + \frac{1}{C} \cdot I_0 \cdot \frac{T}{2} + \frac{Q_0}{C} \right) + \left(\frac{1}{C} \cdot I_0 \cdot \left(T - \frac{T}{2} \right) \right)$$

$$\Rightarrow v_{initial} = V_0 - \left(\frac{1}{C} \cdot I_0 \cdot \frac{T}{2} + \frac{Q_0}{C} \right) - \left(\frac{1}{C} \cdot I_0 \cdot \frac{T}{2} \right) = V_0 - \left(\frac{1}{C} \cdot I_0 \cdot T + \frac{Q_0}{C} \right), \text{ at } t = 0.$$

Exercise 9.5

$$i_L(t) = \frac{1}{L} \int_{-\infty}^t v_L(t) dt$$

For voltage expression,

$$0 < t < \frac{T}{2}, \quad V_L(t) = \frac{2}{T}(-V_0)t + V_0$$

$$\frac{T}{2} < t < T, \quad V_L(t) = \frac{2}{T}(V_0)t - V_0$$

Assume initial inductor current = $i_{initial}$ when $t = 0$.

$$0 < t < \frac{T}{2}, \quad i_L(t) = \frac{1}{L} \int_{-\infty}^t \left(\frac{2}{T}(-V_0)t + V_0 \right) dt = \frac{1}{L} \cdot \frac{(-V_0)}{T} \cdot t^2 + V_0 t + i_{initial}$$

$$t = \frac{T}{2}, \quad i_L(t) = 0 + \frac{A_0}{L}$$

$$\frac{T}{2} < t < T,$$

$$i_L(t) = (i_{initial} + \left(\frac{1}{L} \cdot \frac{2}{T} \cdot (-V_0) \cdot \frac{1}{2} \cdot \left(\frac{T}{2} \right)^2 + V_0 \cdot \frac{T}{2} \right) + \frac{A_0}{L}) + \left(\frac{1}{L} \cdot \frac{2}{T} \cdot V_0 \cdot \frac{1}{2} \cdot \left(t - \frac{T}{2} \right)^2 - V_0 \cdot \left(t - \frac{T}{2} \right) \right)$$

$$\Rightarrow i_L(t) = \frac{V_0}{T} (t - T)^2 + \frac{A_0}{L} + i_{initial}$$

Inductor current is given as I_0 at $t = T$,

$$\Rightarrow \text{At } t = T, \quad I_0 = \frac{V_0}{T} (T - T)^2 + \frac{A_0}{L} + i_{initial}$$

$$i_{initial} = I_0 - \frac{A_0}{L}, \text{ at } t = 0.$$

Problem 9.1

$$V_{c1}(t) = 4u(t)$$

$$i(t) = C \frac{dv(t)}{dt}$$

$$4uC \delta(t) = C_1 \frac{dv(t)}{dt} = C_1 \frac{4 \cdot du(t)}{dt} = C_1 \cdot 4\delta(t)$$

$$\Rightarrow C_1 = uC$$

$$q(t) = C_1 V_{c1} = C_2 V_{c2}$$

$$\Rightarrow C_2 = 4uC$$

Problem 9.6

For ideal linear inductor:

$$V_t(t) = \frac{d(L(t) \cdot i(t))}{dt}$$

$$V_t(t) = \frac{d((L_0 + L_1 \sin(\omega t)) \cdot I)}{dt}$$

$$\Rightarrow V_t(t) = IL_1 \cos(\omega t)$$

Exercise 10.1

The inductor first acts as an open circuit and eventually becomes a wire:

$$t \geq 0 \begin{cases} \text{initially: } i_t(t) = 0 \text{ (open circuit)} \\ \text{finally: } i_1(t) = \frac{V_s(t)}{3k} + i_s(t) = \frac{4}{3} \text{ mA} \end{cases}$$

Assume $i_s(t)$ source points down.

$$i_i(t) = (\text{Final value}) + (\text{Initial value} - \text{Final value})e^{-t/\tau}$$

$$i_i(t) = 4/3 \left(1 - e^{-\frac{t}{\tau}} \right) \text{ [mA]}$$

$$\tau = \frac{L}{R} = \frac{1}{3} \text{ ms}$$

$$\text{Ans. } i_i(t) = 4/3 \left(1 - e^{-\frac{t}{\tau}} \right) \text{ mA for } t \geq 0; \tau = \frac{1}{3} \text{ ms}$$

Exercise 10.4

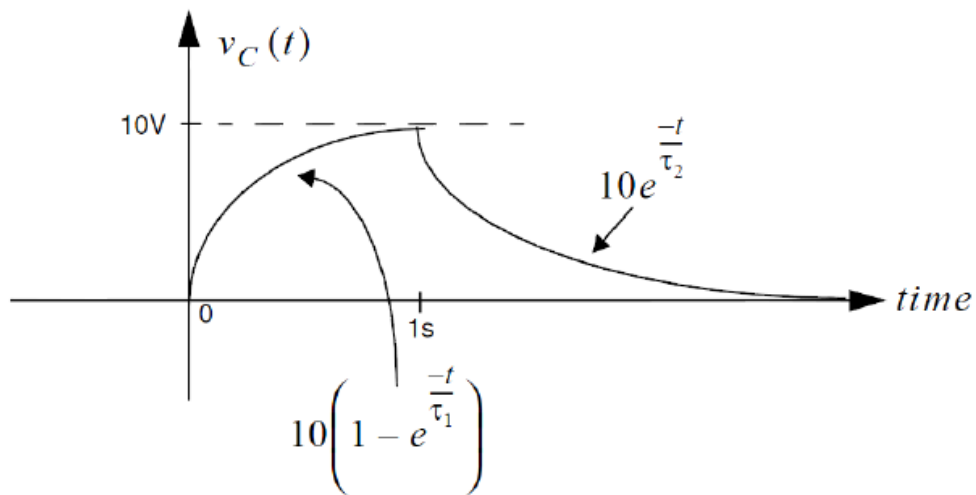
Assume $v_c = 0$ for $t < 0$. When the switch is closed at $t = 0$, v_c rises from 0 to

$$11 \cdot \frac{10k}{10k+1k} = 10V \text{ with } \tau_1 = [1k||10k] \cdot C$$

$$\tau_1 = 9.09 \text{ ms}$$

When the switch is opened,

v_c falls exponentially back to zero with $\tau_2 = 10k \cdot C = 1 \text{ second}$.

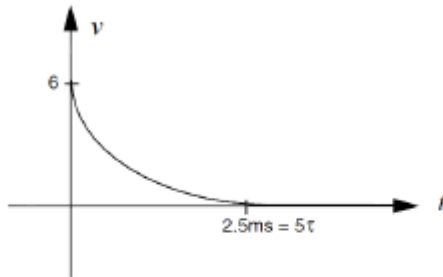


Exercise 10.5

(a)

$$\tau = [1k||1k] \cdot C = 500\mu s$$

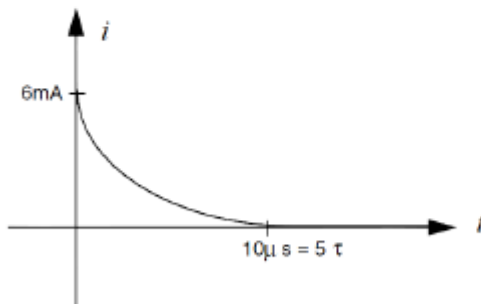
$$i = 6e^{-t/\tau}$$



(b)

$$\tau = [1k||1k] \cdot L = 2\mu s$$

$$i = (6 \cdot 10^{-3})e^{-t/\tau}$$

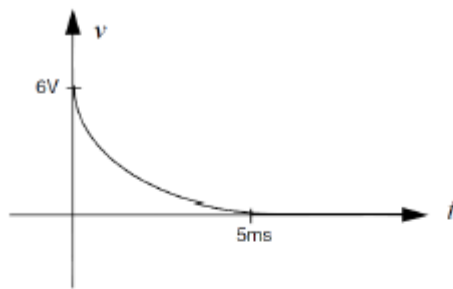


(c)

$$v(0) = 6$$

$$\tau = R \cdot C = (1k\Omega)(1\mu F) = 1ms$$

$$v = 6e^{-t/\tau}$$

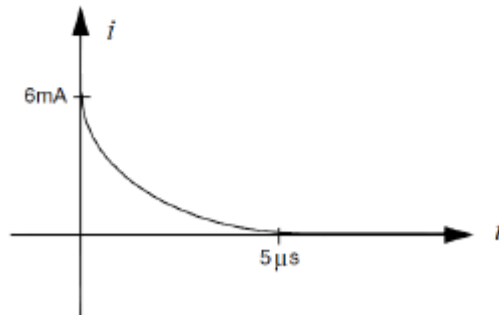


(d)

$$i(0) = \frac{6V - 0}{1000\Omega} = 6 \text{ mA}$$

$$\tau = L/1k = 1\mu s$$

$$i = 0.006e^{-t/\tau}$$



Problem 10.24

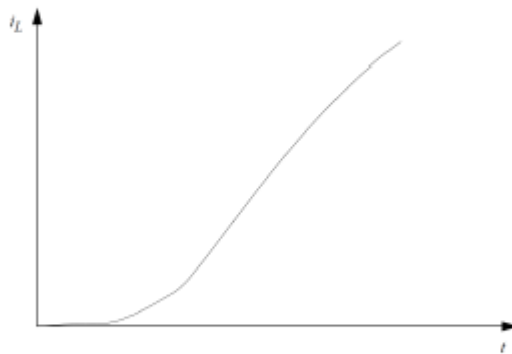
(a)

$$\tau = L/R$$

$$i_L(0^+) = 0$$

$$i_L(t) = \int \frac{K_1}{R} \left(1 - e^{-\frac{t}{\tau}}\right) dt = \frac{K_1 t}{R} + \frac{K_1 \tau}{R} e^{-\frac{t}{\tau}} - \frac{K_1 \tau}{R}$$

$$i_L(t) = \frac{K_1 t}{R} - \frac{K_1 \tau}{R} \left(1 - e^{-\frac{t}{\tau}}\right)$$



(b)

$$i_L = K_2 t$$

$$v_L = L \frac{di_L}{dt} = LK_2$$

$$v_R = Ri_L = RK_2 t$$

$$v_I = v_L + v_R = LK_2 + RK_2 t$$