2019 EE2210 Electric Circuit

Solution of the practice problems for Lecture1-3

Exercise 1.3 In the circuit in Figure 1.1, R is a linear resistor and $v = V_{DC}$ a constant (DC) voltage. What is the power dissipated in the resistor, in terms of R and V_{DC}?

Solution:

$Power = i \cdot v$

But $i = v/R$ (Ohm's Law), so

$$Power = \frac{v}{R} \cdot v = \frac{{V_{DC}}^2}{R}$$

ANS:: $\frac{V_{DC}^2}{R}$

Exercise 1.4 In the circuit of the previous exercise (Figure 1.1), $v = V_{AC}\ cos\omega t$, a sinusoidal (AC) voltage with peak amplitude V_{AC} and frequency ω, in radians/sec.

a) What is the average power dissipated in R?

b) What is the relationship between V_{DC} and V_{AC} in Figure 1.1 when the average power in R is the same for both waveforms?

Solution:

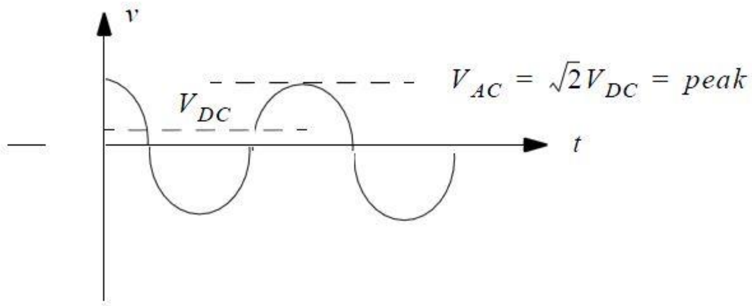

Figure 1.2:

a) If peak voltage is V_{AC}, then

$$V_{AC} = \sqrt{2}\, V_{DC}$$

where V_{DC} is the average amplitude of the voltage signal.

$$Average\ Power = \frac{(V_{average})^2}{R} = \frac{{V_{DC}}^2}{R} = \frac{(V_{AC}/\sqrt{2})^2}{R} = \frac{{V_{AC}}^2}{2R}$$

b) If peak voltage is V_{AC}, then

$$V_{AC} = \sqrt{2}\, V_{DC}$$

where V_{DC} is the average amplitude of the voltage signal.

ANS:: (a) $V_{AC}^2/2R$ (b) $V_{AC} = \sqrt{2}\, V_{DC}$

Exercise 2.5 Find the equivalent resistance at the indicated terminal pair for each of the networks shown in Figure 2.8.

Solution:

a)

$$R_{EQ} = R_1 + R_2 + R_3$$

b)

$$R_{EQ} = R_1||R_2 + R_3 = \frac{R_1 R_2 + R_3(R_1 + R_2)}{R_1 + R_2}$$

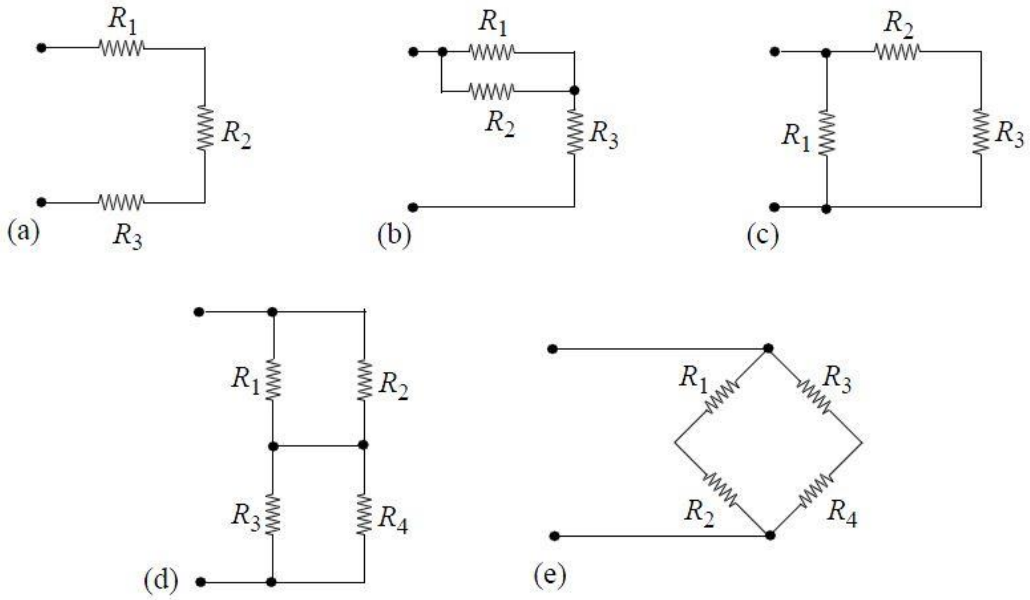

Figure 2.8:

c)

$$R_{EQ} = R_1||R_2 + R_3 = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}$$

d)

$$R_{EQ} = R_1||R_2 + R_3||R_4 = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4}$$

e)

$$R_{EQ} = (R_1 + R_2)||(R_3 + R_4) = \frac{(R_1 + R_2)(R_3 + R_4)}{R_1 + R_2 + R_3 + R_4}$$

ANS:: (a) $R_1 + R_2 + R_3$, (b) $\frac{R_1 R_2 + R_3(R_1 + R_2)}{R_1 + R_2}$ (c) $\frac{R_1(R_2+R_3)}{R_1+R_2+R_3}$ (d) $\frac{R_1 R_2}{R_1+R_2} + \frac{R_3 R_4}{R_3+R_4}$ (e) $\frac{(R_1+R_2)(R_3+R_4)}{R_1+R_2+R_3+R_4}$

Problem 2.4 For the circuit in Figure 2.26, find values of R_1 to satisfy each of the following conditions:

a) v = 3 V

b) v = 0 V

c) i = 3 A

d) The power dissipated in R_1 is 12 watts.

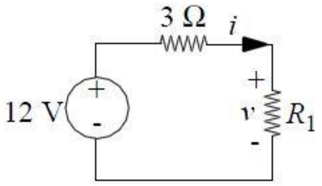

Figure 2.26:

Solution:

a) Voltage divider. Solve $12V * \frac{R_1}{3+R_1} = 3V$

$R_1 = 1\Omega$

b) $v = i * R_1$. Since the current is not 0, the resistance must be zero.

$R_1 = 0$

c) Solve $i = 3A = \frac{12V}{R_{eq}} = \frac{12V}{3\Omega + R_1}$

$R_1 = 1\Omega$

d) Power dissipated in $R_1 = 12W = i * v$ where $v = 12V * \frac{R_1}{3+R_1}$ and $i = \frac{12V}{3+R_1}$.

$R_1 = 3\Omega$

ANS:: (a) $R_1 = 1\Omega$ (b) $R_1 = 0$ (c) $R_1 = 1\Omega$ (d) $R_1 = 3\Omega$

Problem 2.9 Calculate the power dissipated in the resistor R in Figure 2.31.

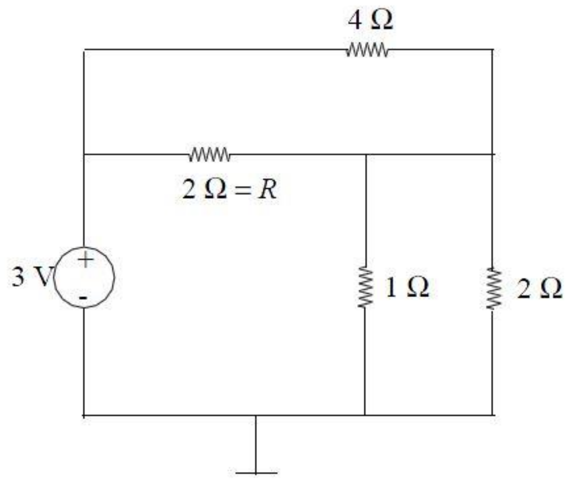

Figure 2.31:

Solution:

The equivalent resistance is 2Ω, so $\frac{3}{2}A$ of current is split between the 2Ω and 4Ω resistors. Therefore, $1A$ current goes through R.

Power $= 2W$

ANS:: Power $= 2W$

Exercise 3.2 Find the Norton equivalent at the indicated terminals for each network in Figure 3.3.

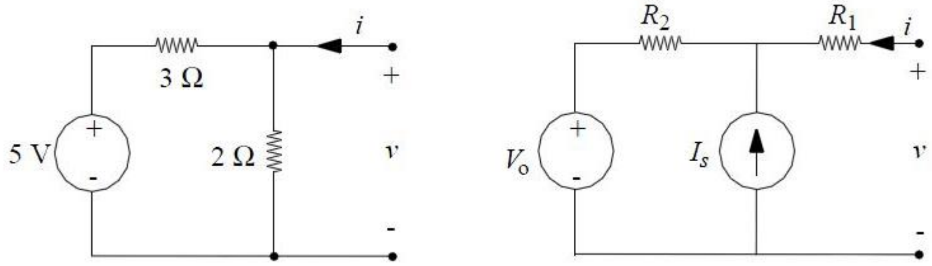

Figure 3.3:

Solution:

Left network:

$R_T = 3||2 = 1.2\ \Omega$ when $5\ V$ source is made a short circuit.

$I = 5/3\ A$ when the indicated terminals are connected with a wire ("shorted") since then no current flows through the 2 Ω resistor.

Right network:

$R_T = R_1 + R_2$, when the V_0 source is shorted and the I_S source is made an open circuit.

$$I = \frac{R_2}{R_1 + \underbrace{R_2}_{\substack{\text{current}\\ \text{divider}\\ \text{for}\\ V_0=0}}} \cdot I_S + \frac{V_0}{\underbrace{R_1 + R_2}_{\substack{\text{contribution}\\ \text{from } V_0\\ \text{when } I_S=0}}} \quad \text{by superposition}$$

ANS:: Left: 1.2Ω, $5/3A$, Right: $R_1 + R_2$, $\frac{R_2}{R_1+R_2} I_S + \frac{V_0}{R_1+R_2}$

Exercise 3.3 Find the Thévenin Equivalent for each network in Figure 3.4.

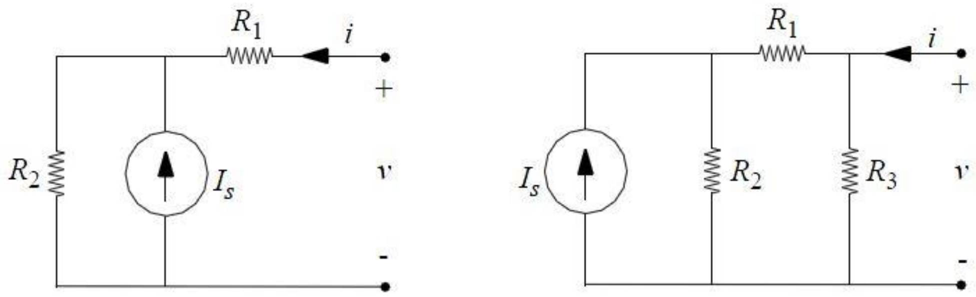

Figure 3.4:

Solution:

Left network:

$R_T = R_1 + R_2$ when I_S is made an open circuit.

$V_{OC} = I_S R_2$ since no current flows through R_1 in the open circuit case.

$R_T = R_3 || (R_1 + R_2)$ when I_S current source is made an open circuit.

Since $V_{OC} = R_3 \cdot$ (current through R_3) by Ohm's Law,

$$V_{OC} = \underbrace{\frac{I_S \cdot R_2}{R_1 + R_2 + R_3}}_{\text{current divider relation for fraction of } I_S \text{ that will flow through } R_1 \text{ and } R_3} \cdot R_3$$

ANS:: Left: $V_{OC} = I_S R_2, R_T = R_1 + R_2$, Right: $V_{OC} = \frac{I_S R_2 R_3}{R_1+R_2+R_3}, R_T = R_3||(R_1 + R_2)$

Exercise 3.4 Find v_0 in (a) and (b) by superposition in Figure 3.5.

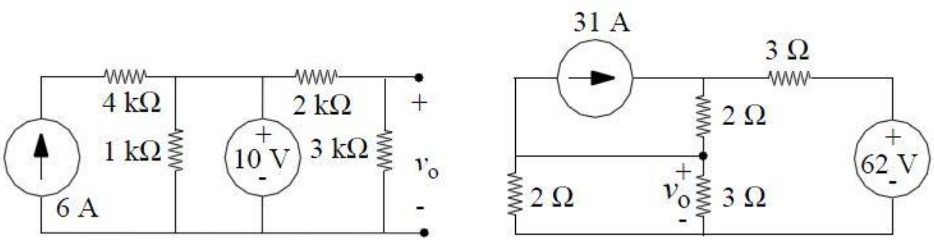

Figure 3.5:

Solution:

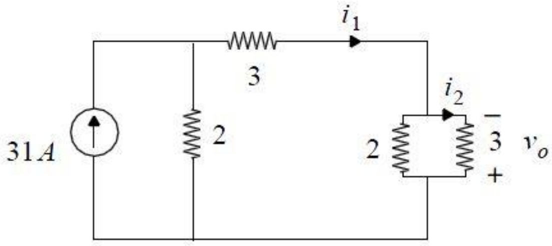

Figure 3.6:

(a):

1. Set voltage source to zero (short circuit):

$$v_0 = 0$$

2. Set current source to zero (open circuit):

$$v_0 = 10\ V \cdot \underbrace{\frac{3000}{3000+2000}}_{\text{voltage divider}}$$

$$v_0 = 6\ Volts$$

$$v_0 = 0 + 6V\ \text{[superposition]}$$

$$v_0 = 6\ Volts$$

(b):

1. Set current source to zero (open circuit):

$$v_0 = \left[\underbrace{\frac{2\ ||\ 3}{2\ ||\ 3+2+3}}_{\text{voltage divider}}\right] \cdot 62\ V = 12\ Volts \text{ since } 2\ ||\ 3 = 1.2$$

2. Set voltage source to zero (short circuit):

$$i_1 = 31\ A\left[\underbrace{\frac{2}{3+2\ ||\ 3+2}}_{\text{current divider}}\right] = 10\ A$$

$$v_0 = 3 \cdot (-i_2) = -12\ Volts \qquad i_2 = \left[\frac{2}{3+2}\right] \cdot i_1 = 4\ A$$

$$v_0 = 12 + (-12) \quad \text{[superposition]}$$

$$v_0 = 0$$

ANS:: (a) 6V (b) 0V

Exercise 3.9 The resistive network shown in Figure 3.11 is excited by two voltage sources $v_1(t)$ and $v_2(t)$.

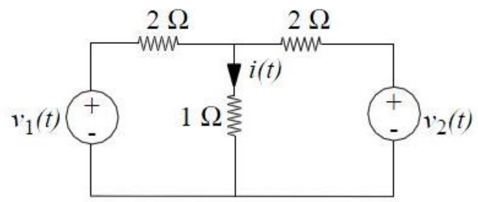

Figure 3.11:

a) Express the current $i(t)$ through the 1Ω resistor as a function of $v_1(t)$ and $v_2(t)$.

b) Determine the total energy dissipated in the 1Ω resistor due to both $v_1(t)$ and $v_2(t)$ from time T_1 to time T_2.

c) Derive the constraint between $v_1(t)$ and $v_2(t)$ such that the value for b) can be computed by adding the energies dissipated when each source acts alone (i.e. by superposition).

Solution:

a)

$$i(t) = \left[\frac{1 \,||\, 2}{1 \,||\, 2 + 2}\right](v_1(t) + v_2(t)) = \frac{1}{4}(v_1(t) + v_2(t))$$

b)

$$\text{Energy} = \frac{1}{16}\int_{T_1}^{T_2}(v_1(t) + v_2(t))^2 dt$$

c)

$$\text{For superposition to apply, } \int_{T_1}^{T_2} v_1 \cdot v_2 \cdot dt \equiv 0 \quad \text{[orthogonal]}$$

ANS:: (a) $i(t) = \frac{1}{4}(v_1(t) + v_2(t))$ (b) Energy $= \frac{1}{16}\int_{T_1}^{T_2}(v_1(t) + v_2(t))^2 dt$ (c) $\int_{T_1}^{T_2} v_1 \cdot v_2 \cdot dt \equiv 0$

Exercise 3.23

a) Find the Norton equivalent of the circuit in Figure 3.36.

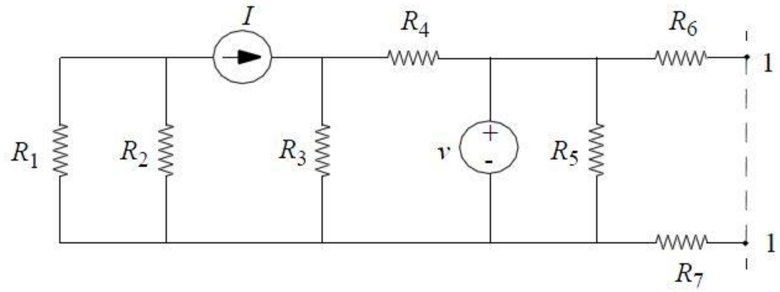

Figure 3.36:

b) Find the Thévenin equivalent of the circuit in Figure 3.37.

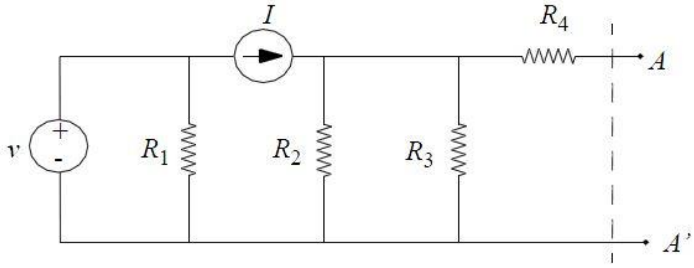

Figure 3.37:

Solution:

a) $R_T = R_6 + R_7$
$I_{sc} = V/(R_6 + R_7)$

b) $R_T = (R_2 \mid\mid R_3) + R_4$
$V_{OC} = I\ (R_2 \mid\mid R_3)$

ANS:: (a) $R_T = R_6 + R_7$, $I_{sc} = V/(R_6 + R_7)$, (b) $R_T = (R_2 \mid\mid R_3) + R_4$, $V_{OC} = I\ (R_2 \mid\mid R_3)$

Exercise 3.25 Find the node potential E in Figure 3.39.

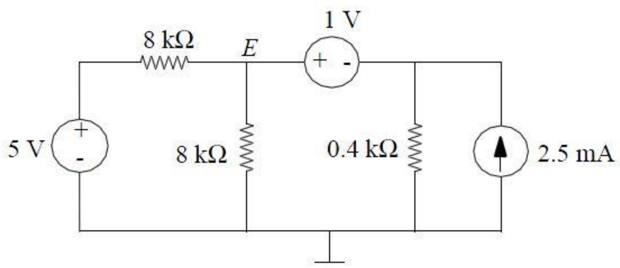

Figure 3.39:

Solution:

$E = 0.8V + 0.8V + 0.8V = 2.4V$, by superposition.

ANS:: 2.4 Volts