Exercise 12.1

a) Is the zero input response of the circuit shown in Figure 12.1 underdamped, over-damped, or critically damped?

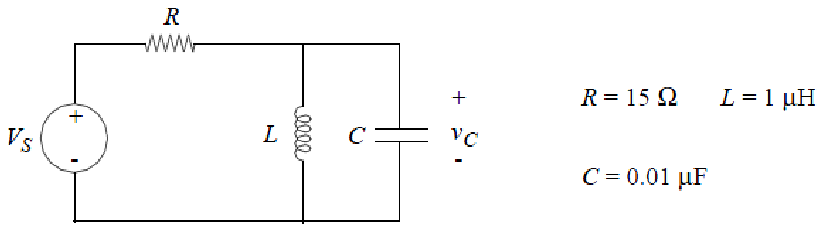


Figure 12.1:

b) What is the form of the zero input response (v_C) for the same circuit? Make a rough sketch.

c) Compare the envelope of the zero input response with the rate of delay of the zero input response of the RC circuit in Figure 12.2:

How do they differ?

Solution:

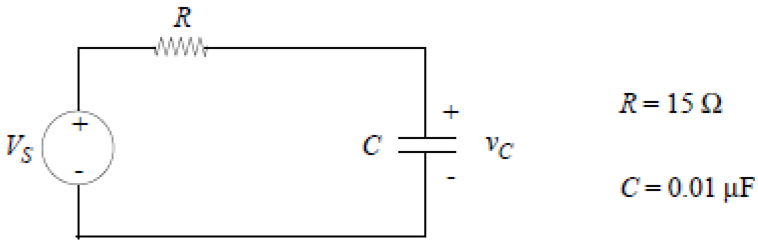


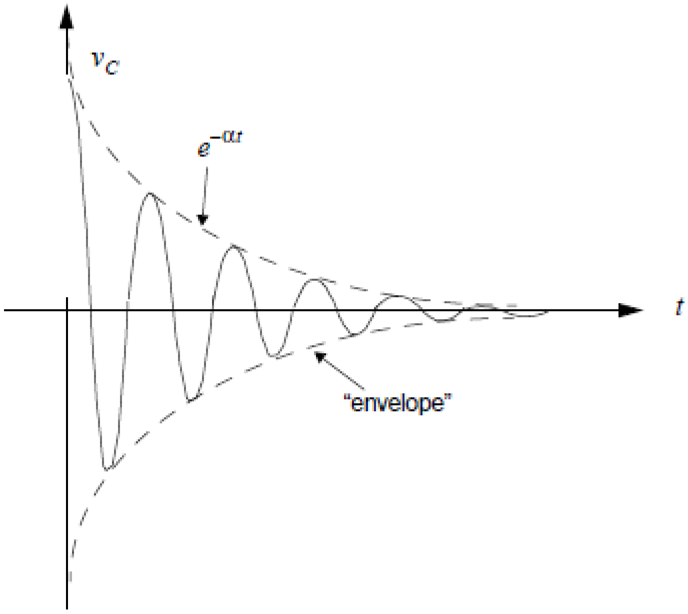
Figure 12.2:

b) $v_C = K\ e^{-\alpha t} \cdot \cos(\omega_d \cdot t + \phi)$

$\omega_d = \sqrt{\omega_o{}^2 - \alpha^2}$

$\omega_o = 10 \times 10^6$

$\alpha = 3.33 \times 10^6$



c) (1) v_C in RC circuit in zero-input case decays as $e^{-t/\tau} = e^{-t/RC}$.
(2) v_C above in the RLC circuit decays with "envelope" as $e^{-\alpha\tau} = e^{-t/2RC}$.
Therefore, the RC circuit zero-input response decays twice as fast as the RLC response;
i.e. $\tau_{RLC} = 2 \cdot \tau_{RC}$;
RLC takes twice as long to decay.

Exercise 12.2 For each of the circuits in Figure 12.4, find and sketch the indicated zero-input response corresponding to the indicated initial conditions.

a) In Figure 12.4, find v_2, assuming $v_1(0) = 1V, v_2(0) = 0$.

b) In Figure 12.5, find v, assuming $i(0) = 0, v(0) = 1V$

c) Repeat (b), but with the resistor changed to 5Ω.

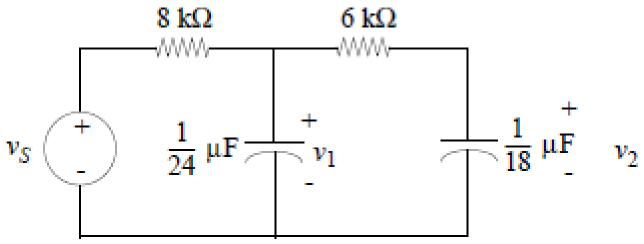


Figure 12.4:

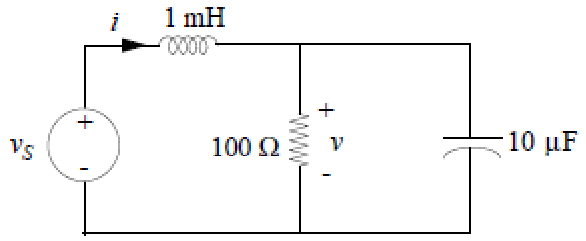


Figure 12.5:

Solution:

a) (1) $\frac{v_1}{8k} + \frac{v_1 - v_2}{6k} + \left(\frac{1}{24}\mu F\right)\frac{dv_1}{dt} = 0$

(2) $v_1 - \left(\frac{1}{18}\mu F\right)\frac{dv_2}{dt}(6000) - v_2 = 0 \quad \rightarrow \quad v_1 = v_2 + \frac{1}{3000}\frac{dv_2}{dt}$

Plug (2) into (1), find

$$v_2 = A\,e^{-1000t} + B\,e^{-9000t}$$

Initial conditions allow us to find constants A and B:

$$A + B = 0 \quad \rightarrow \quad \text{from } v_2(0) = 0$$

$$A + B - \frac{1}{3}A - 3B = 1 \quad \rightarrow \quad \text{from } v_1(0) = 1\text{Volt}$$

$$A = \frac{3}{8}$$

$$B = -\frac{3}{8}$$

$$v_2 = \frac{3}{8}(e^{-1000t} - e^{-9000t})\ ;\ t \text{ in seconds} \qquad \text{(a)}$$

b)

$$\frac{v-0}{100} + C\,\frac{dv}{dt} - i = 0$$

$$-v = L\,\frac{di}{dt} \quad \rightarrow \quad \frac{di}{dt} = -\frac{v}{L}$$

$$(s^2 + 1000s + 100 \cdot 10^6)v = 0$$

$$s_{1,2} = \underbrace{-500}_{\alpha} \pm \underbrace{9,990j}_{\omega_d} \quad \rightarrow \quad {\omega_o}^2 = {\omega_d}^2 + \alpha^2 \quad \omega_o = 10,000$$

Thus,

$$v = 1.001\;e^{-500t}\;(0.999\;\;\cos\omega_d t - 0.05\sin\omega_d t) \qquad \text{(b)}$$

c)

$$(s^2 + 20,000s + 100 \times 10^6)v$$

$$s_{1,2} = -10,000$$

$$v = A\;e^{-10,000t} + Bte^{-10,000t}$$

Initial condition: $v(0) = 1V \quad \rightarrow \quad A = 1$

$$i = -1000 \cdot \int v = -1000 \left[\frac{-A}{10^4}\,e^{-10^4 t} + B \underbrace{\int te^{-10^4 t}dt}_{\text{integrate by parts}} \right]$$

$$i(t=0) = 0 = 10^4 \cdot A + B \quad \rightarrow \quad B = -10^4 \;\; \text{since } A = 1$$

$$v = (1 - 10^4)e^{-10^4 t} \qquad \text{(c)}$$

Exercise 13.1 Find the magnitude and phase of each of the following expressions

a) $(8+j7)(5e^{j30^\circ})(e^{-j39^\circ})(0.3-j0.1)$

b) $\frac{(8.5+j34)(20e^{-j25^\circ})(60)(\cos 10^\circ + j\sin 10^\circ)}{(25e^{j20^\circ})(37e^{j23^\circ})}$

c) $(25e^{j30^\circ})(10e^{j27^\circ})(14-j13)/(1-j2)$

d) $(13e^{j(15^\circ+j1.5)})(6e^{(1-j30^\circ)})$

Solution:

a) $(8+j7) = 10.63\ e^{41.18^\circ \cdot j}$
$(0.3-j0.1) = 0.316e^{-18.43Z^\circ \cdot j}$
$MAG = 16.8$
$PHASE = 13.75\,\text{deg}$

b) $MAG = \frac{35 \cdot 20 \cdot 60 \cdot 1}{25 \cdot 37} = 45.47$
$PHASE = 76^\circ - 25^\circ + 10^\circ - 20^\circ - 23^\circ = 18^\circ$

c) $MAG = \frac{25 \cdot 10 \cdot 19 \cdot 1}{2.236} = 2136$
$PHASE = 30^\circ + 27^\circ - 42^\circ + 63^\circ = 78^\circ$

d) $13e^{j(15+1.5j)} \cdot 6e^{(1-30j)} = \frac{13 \cdot e^{j15} e^{-1.5} \cdot 6 \cdot e^{1}}{e^{j30}}$
$MAG = 47.3$
$PHASE = -15^\circ$

Exercise 13.2 Find the real and imaginary parts of the following expressions

a) $(3+j5)(4e^{j50^\circ})(7e^{-j20^\circ})$

b) $(10e^{j50^\circ})(e^{j20^\circ})$

c) $(10e^{j50^\circ})(e^{j\omega t})$

d) $Ee^{j\omega t}$where $E = |E|e^{j\Theta}$

Solution:

a) $5.83e^{j59^\circ} \cdot 4e^{j50^\circ} \cdot 7e^{-j20^\circ} = 163.26e^{89j} \ \rightarrow \ 2.84 + j163$

b) $10e^{j70^\circ} \ \rightarrow \ 3.42 + j9.4$

c) $10e^{j(\omega t+50)} \ \rightarrow \ 10(\cos(\omega t + 50^\circ) + j\sin(\omega t + 50^\circ))$

d) $|E|\ e^{j(\omega t+\theta)} \ \rightarrow \ |E|(\cos(\omega t+\theta) + j\sin(\omega t+\theta))$

ANS:: (a) $2.84 + j163$, (b) $3.42 + j9.4$, (c) $10(\cos(\omega t + 50^\circ) + j\sin(\omega t + 50^\circ))$, (d) $|E|(\cos(\omega t+\theta) + j\sin(\omega t+\theta))$

5.

a) $i_R(0) = \frac{15}{200} = 75\,\text{mA}$

$i_L(0) = -45\,\text{mA}$

$i_C(0) = -i_L(0) - i_R(0) = 45 - 75 = -30\,\text{mA}$

b) $\alpha = \frac{1}{2RC} = \frac{1}{2(200)(0.2 \times 10^{-6})} = 12{,}500$

$\omega_o^2 = \frac{1}{LC} = \frac{1}{(50 \times 10^{-3})(0.2 \times 10^{-6})} = 10^8$

$s_{1,2} = -12{,}500 \pm \sqrt{1.5625 \times 10^8 - 10^8} = -12{,}500 \pm 7500$

$s_1 = -5000 \text{ rad/s}; \qquad s_2 = -20{,}000 \text{ rad/s}$

$v = A_1 e^{-5000t} + A_2 e^{-20{,}000t}$

$v(0) = A_1 + A_2 = 15$

$\frac{dv}{dt}(0) = -5000A_1 - 20{,}000A_2 = \frac{-30 \times 10^{-3}}{0.2 \times 10^{-6}} = -15 \times 10^4 \text{V/s}$

Solving, $A_1 = 10; \quad A_2 = 5$

$v = 10e^{-5000t} + 5e^{-20{,}000t}\,\text{V}, \qquad t \geq 0$

c)
$$\begin{aligned} i_C &= C\frac{dv}{dt} \\ &= 0.2 \times 10^{-6}[-50{,}000e^{-5000t} - 100{,}000e^{-20{,}000t}] \\ &= -10e^{-5000t} - 20e^{-20{,}000t}\,\text{mA} \end{aligned}$$

$i_R = 50e^{-5000t} + 25e^{-20{,}000t}\,\text{mA}$

$i_L = -i_C - i_R = -40e^{-5000t} - 5e^{-20{,}000t}\,\text{mA}, \quad t \geq 0$

6.

a) The first step to finding $i(t)$ is to calculate the roots of the characteristic equation. For the given element values,

$$\omega_0^2 = \frac{1}{LC}$$

$$= \frac{(10^3)(10^6)}{(100)(0.1)} = 10^8,$$

$$\alpha = \frac{R}{2L}$$

$$= \frac{560}{2(100)} \times 10^3$$

$$= 2800 \text{ rad/s.}$$

Next, we compare ω_0^2 to α^2 and note that $\omega_0^2 > \alpha^2$, because

$$\alpha^2 = 7.84 \times 10^6$$

$$= 0.0784 \times 10^8.$$

At this point, we know that the response is underdamped and that the solution for $i(t)$ is of the form

$$i(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t,$$

where $\alpha = 2800$ rad/s and $\omega_d = 9600$ rad/s. The numerical values of B_1 and B_2 come from the initial conditions. The inductor current is zero before the switch has been closed, and hence it is zero immediately after. Therefore

$$i(0) = 0 = B_1.$$

To find B_2, we evaluate $di(0^+)/dt$. From the circuit, we note that, because $i(0) = 0$ immediately after the switch has been closed, there will be no voltage drop across the resistor. Thus the initial voltage on the capacitor appears across the terminals of the inductor, which leads to the expression,

$$L\frac{di(0^+)}{dt} = V_0,$$

or

$$\frac{di(0^+)}{dt} = \frac{V_0}{L} = \frac{100}{100} \times 10^3$$
$$= 1000 \text{ A/s}.$$

Because $B_1 = 0$,

$$\frac{di}{dt} = 400B_2e^{-2800t}(24 \cos 9600t - 7 \sin 9600t).$$

Thus

$$\frac{di(0^+)}{dt} = 9600B_2,$$

$$B_2 = \frac{1000}{9600} \approx 0.1042 \text{ A}.$$

The solution for $i(t)$ is

$$i(t) = 0.1042e^{-2800t} \sin 9600t \text{ A}, \quad t \geq 0.$$

b) To find $v_C(t)$, we can use either of the following relationships:

$$v_C = -\frac{1}{C}\int_0^t i\,d\tau + 100 \text{ or}$$

$$v_C = iR + L\frac{di}{dt}.$$

Whichever expression is used (the second is recommended), the result is

$$v_C(t) = (100 \cos 9600t + 29.17 \sin 9600t)e^{-2800t} \text{ V}, \quad t \geq 0.$$