Under-damped

$$v(t) = V_I + K_1 e^{-\alpha t} \cos \omega_d t + K_2 e^{-\alpha t} \sin \omega_d t$$

- **Furthermore,** $v(0) = 0$ **and** $i(0) = 0$

1.) $v(0) = 0 \Rightarrow V_I + K_1 = 0 \Rightarrow K_1 = -V_I$

2) $i(0) = 0 \Rightarrow i = C \frac{dv}{dt} = C \cdot K_1 \left(-\alpha e^{-\alpha t} \cos \omega_d t - e^{-\alpha t} \cdot \omega_d \cdot \sin \omega_d t\right) + C \cdot K_2 \left(-\alpha e^{-\alpha t} \sin \omega_d t + e^{-\alpha t} \cdot \omega_d \cdot \cos \omega_d t\right)$

$i(0) = C K_1 (-\alpha) + C K_2 \omega_d = 0 \Rightarrow K_2 = -\frac{\alpha}{\omega_d} \cdot V_I$

3) $v(t) = V_I - V_I \cdot e^{-\alpha t} \cos \omega_d t - \frac{V_I \cdot \alpha}{\omega_d} \cdot e^{-\alpha t} \cdot \sin \omega_d t$

Under-damped

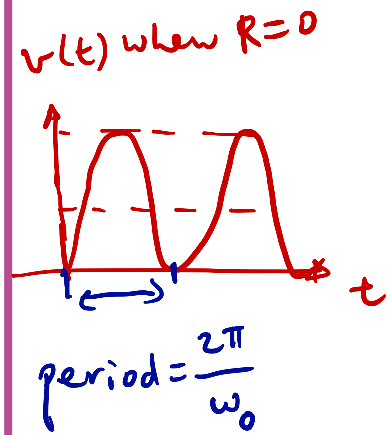
$$v(t) = V_I - V_I e^{-\alpha t}\cos\omega_d t - V_I \frac{\alpha}{\omega_d} e^{-\alpha t}\sin\omega_d t$$

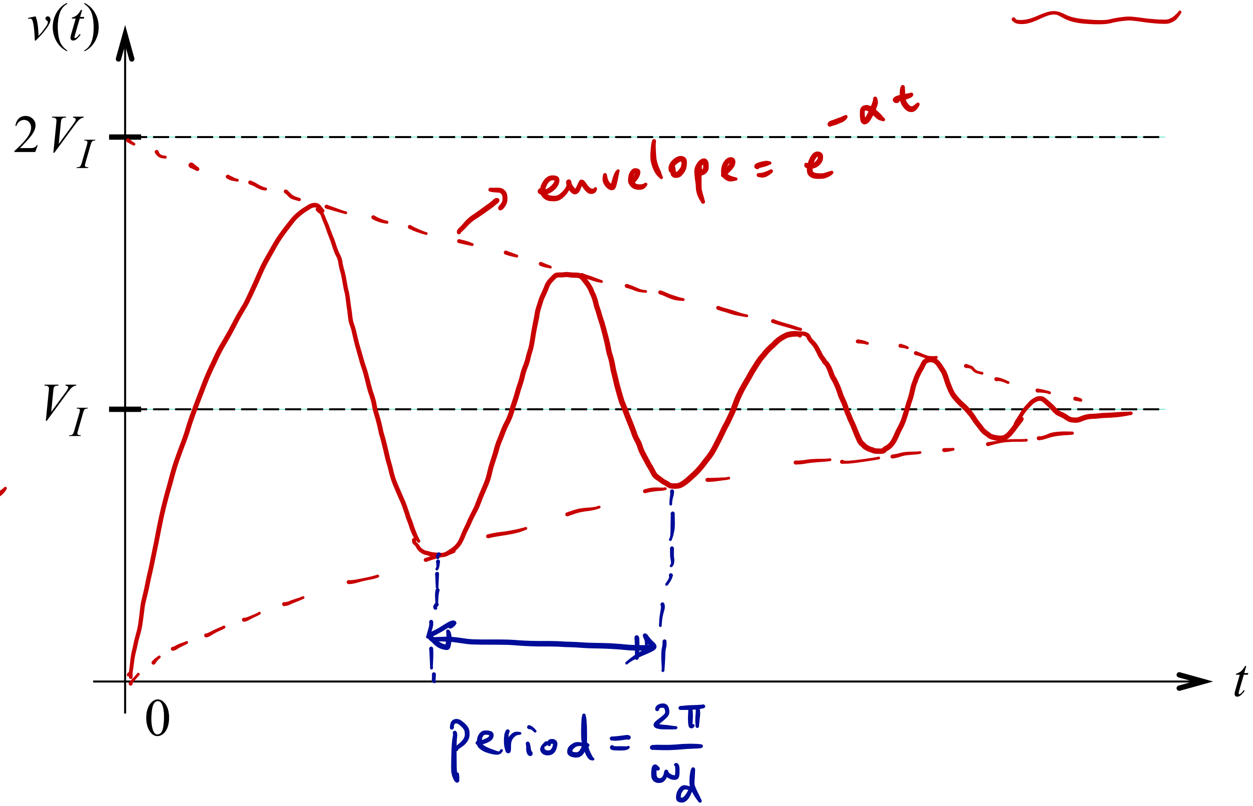
- **Scaled sum of sines (of the same frequency) are also sines (Appendix B.7)**

Use

• $A_1 \cos\omega_d t + A_2 \sin\omega_d t = \sqrt{A_1^2 + A_2^2}\cdot\cos\left(\omega_d t - \tan^{-1}\frac{A_2}{A_1}\right)$

$\Rightarrow$ Rewrite $v(t) = V_I + e^{-\alpha t}\cdot V_I\cdot\frac{\omega_0}{\omega_d}\cdot\cos\left(\omega_d t - \tan^{-1}\frac{\alpha}{\omega_d}\right)$





Critically-Damped

Section 12.2.3

$\alpha > \omega_0$ over-damped

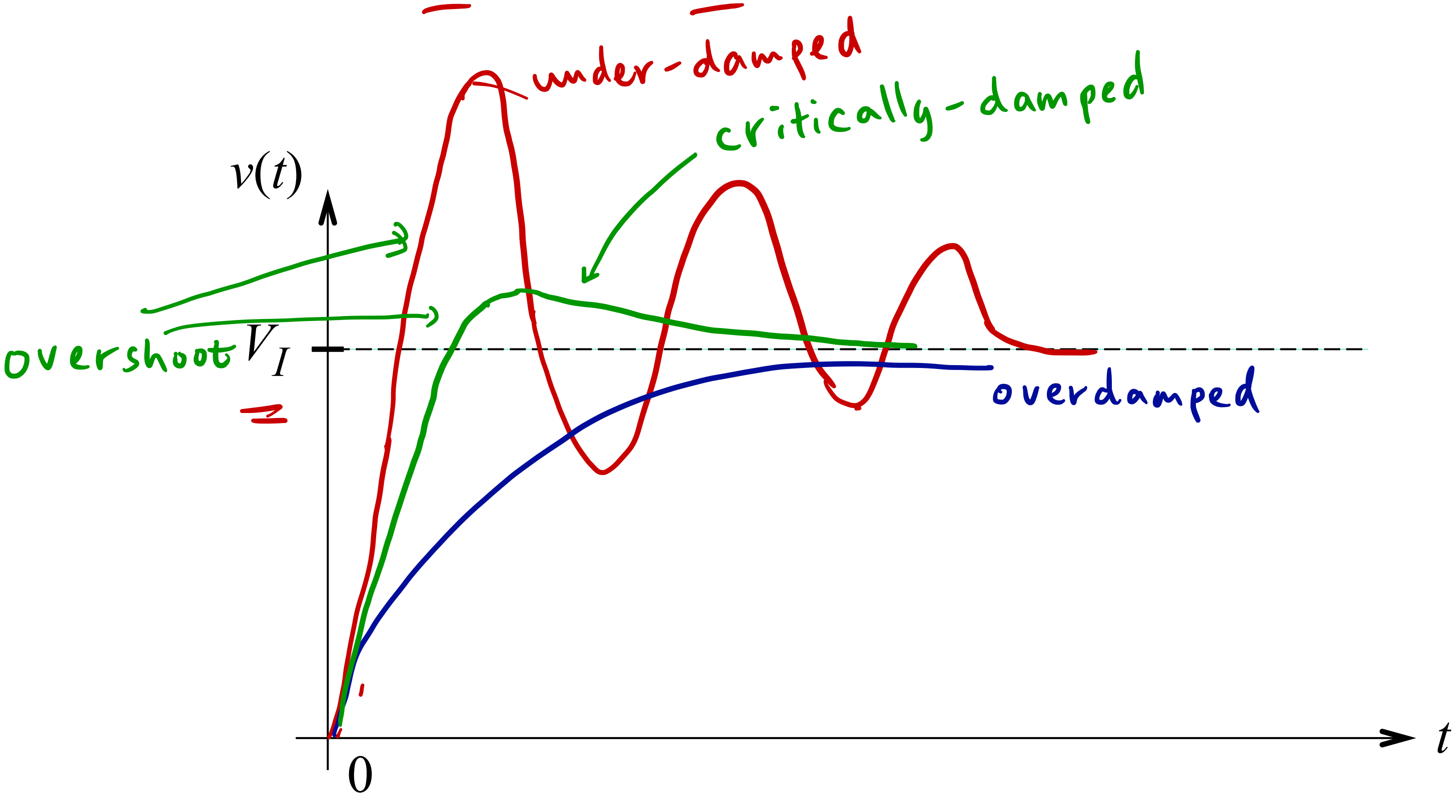
$\alpha < \omega_0$ under-damped

$\alpha = \omega_0$ critically-damped

$$s_1 = s_2 = -\alpha$$

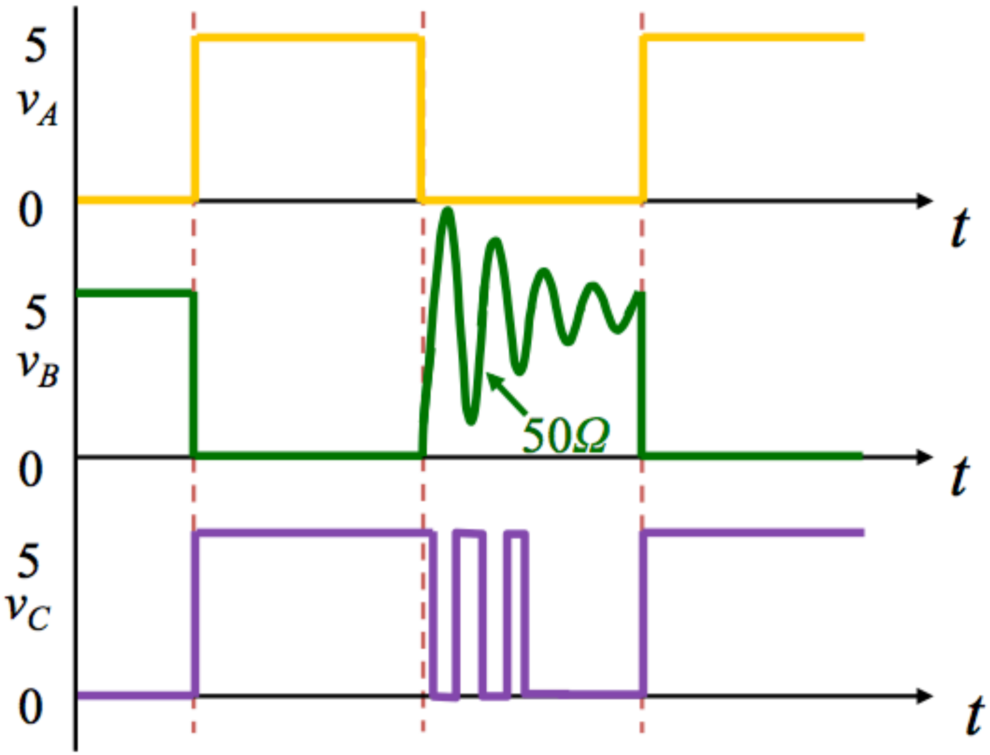
- **Total solution will be in the form of**

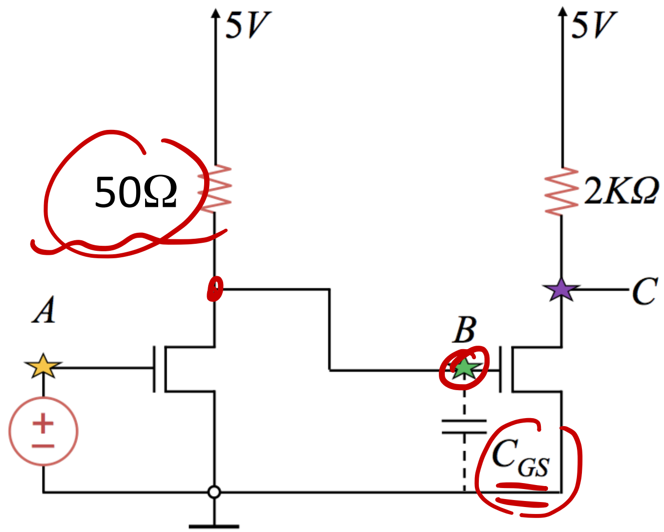
$$v(t) = V_I + A_1 e^{-\alpha t} + A_2 e^{-\alpha t}$$



Remember This?

- With 50Ω load resistor, hoping to speed up the pull up





R ↑

$\frac{1}{RC}$

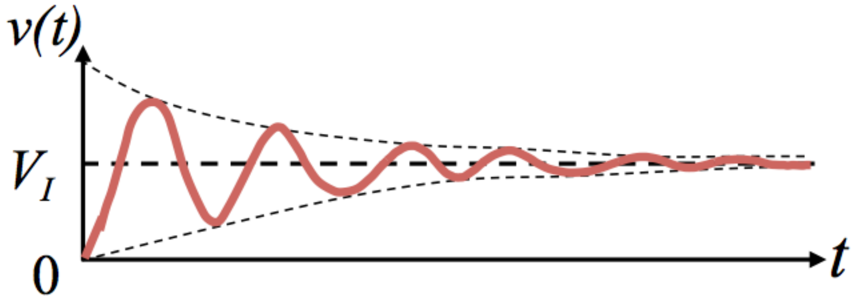
Speed ⇒ R ↓

↕

oscillation

Example 12.2.3

Easy Way: Characteristic Equation Tells the Whole Story



- **For series RLC circuit:**

$$\frac{d^2 v(t)}{dt^2} + \frac{R}{L}\frac{dv(t)}{dt} + \frac{1}{LC} = v_I(t)$$

Characteristic equation $= \underbrace{s^2 + \frac{R}{L}s + \frac{1}{LC} = 0}_{s^2 + 2\alpha s + \omega_0^2 = 0}$

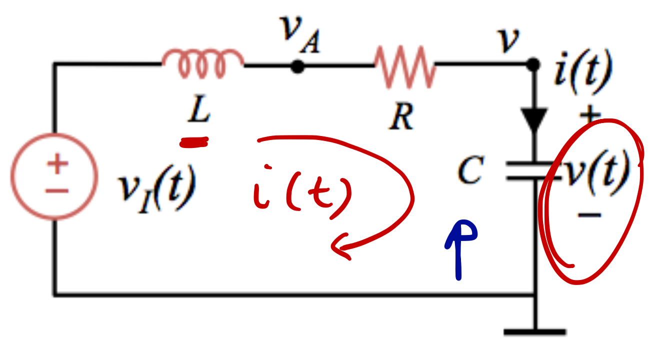
$\Rightarrow$

$\Rightarrow \underset{\sim}{\alpha},\ \underset{\sim}{\omega_0}$

Section 12.7

Intuitive Analysis (12.7)

- **What if** $v(0) > 0$ **and** $i(0) < 0$?



1. Given $v(0)$ and $i(0)$

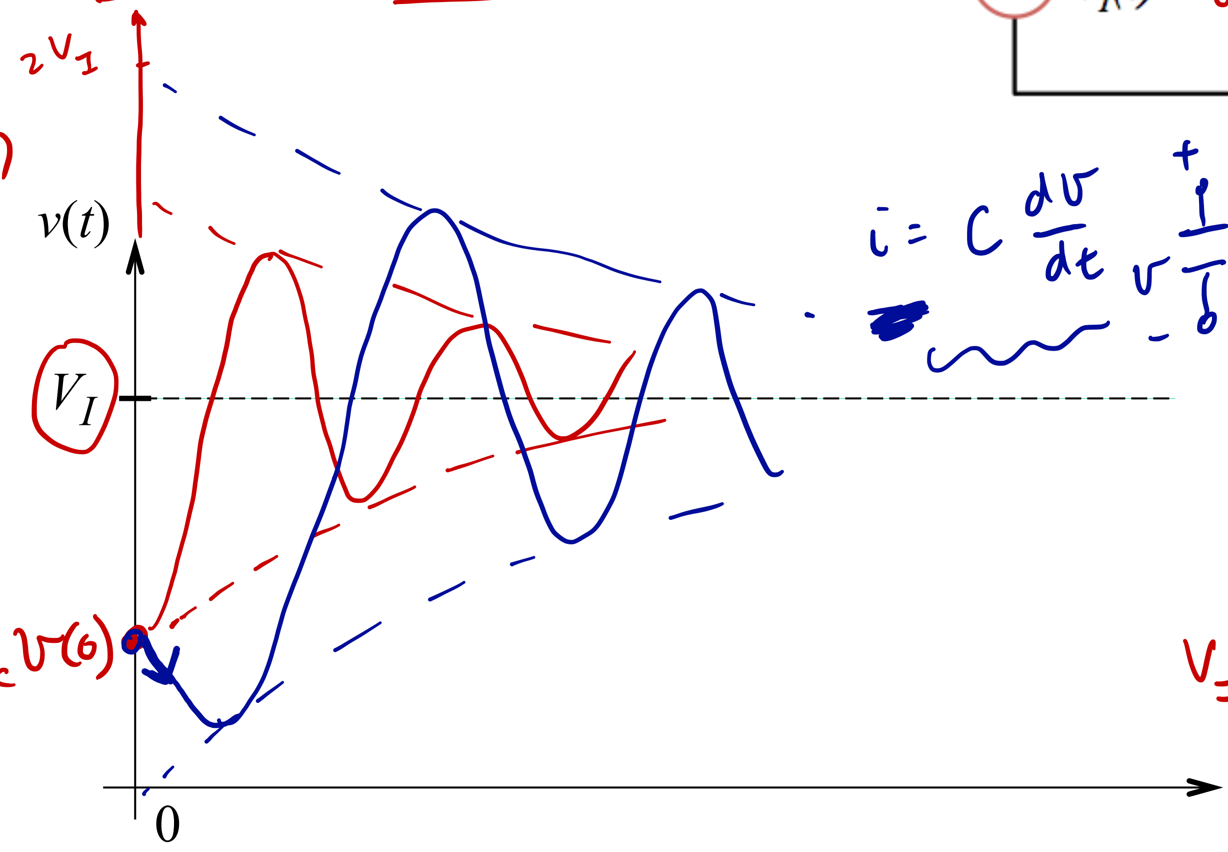
2. Find $v(t \to \infty) = V_I$

3. Characteristic equation

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$\Rightarrow \alpha, \omega_0$

判断 $\Rightarrow$ underdamped, overdamped, critically-damped

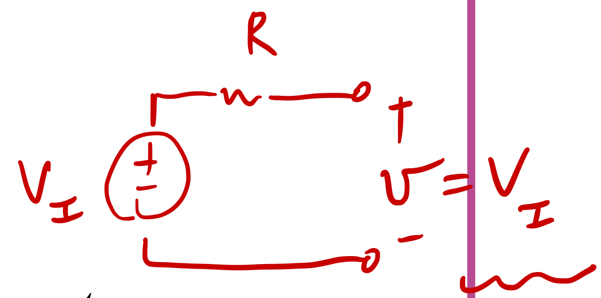


$i = C\frac{dv}{dt}$

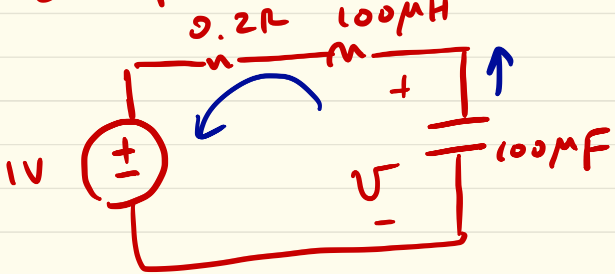
$i = C\frac{dv}{dt}$

$i_c = 0$

$= 0$



Example



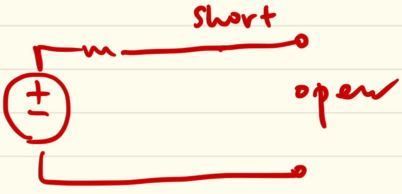
1) Given $v_c(0)=0.5V$

$i_L(0)=-0.5A$

$v_I=1V$

$i_L(t=0^+)<0$

2) Final value $v(t\to\infty)=v_I=1V$

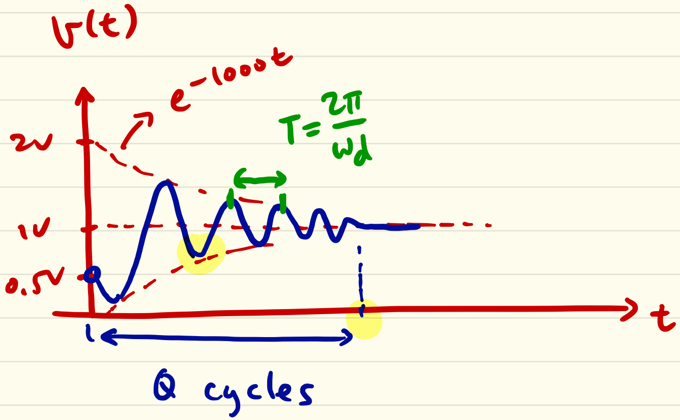


3) Characteristic equation: $s^2+\frac{R}{L}s+\frac{1}{LC}=0$ $\quad s^2+2\alpha s+\omega_0^2=0$

$\alpha=\frac{R}{2L}=10^3$ rad/s, $\omega_0=\frac{1}{\sqrt{LC}}=10^4$ rad/s $\Rightarrow$ $\alpha<\omega_0$

underdamped.

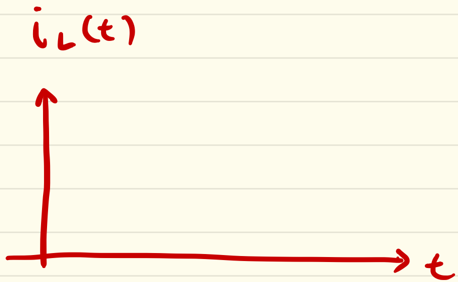
$\omega_d=\sqrt{\omega_0^2-\alpha^2}=9950$ rad/s. $Q=\frac{\omega_0}{2\alpha}=5$



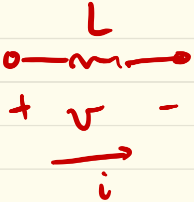
In the plot

1) initial value ✓
2) Final value ✓
3) $\underline{\underline{Q}}$ cycles
4) T

Practice

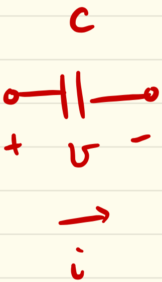


In steady state



$$v = L \cancel{\frac{di}{dt}}^{=0} = 0$$

L = short circuit



$$i = C \frac{dv}{dt} = 0$$

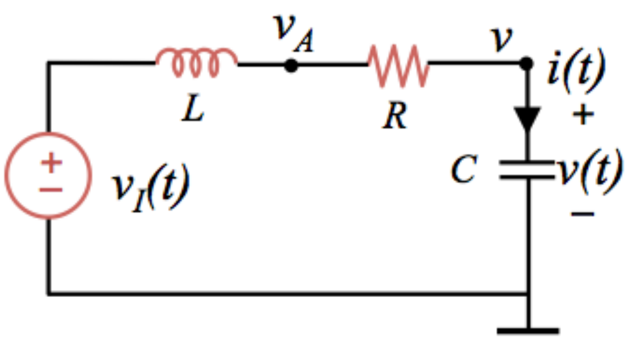
C = open circuit

What about Other Variables?

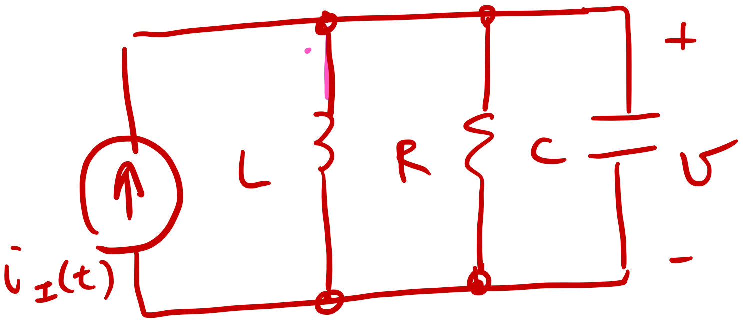
$$\underbrace{v(t)} \rightarrow \underbrace{i_C(t)} = i_L(t) = i_R(t)$$

$$i(t) = C\frac{dv}{dt} \qquad v_L(t) = L\frac{di_L(t)}{dt}$$

$$v_R(t) = R \cdot i_R(t)$$



Parallel RLC – Characteristic Equation Says It All



KCL: $C\frac{dv}{dt} + \frac{v}{R} + \frac{1}{L}\int v(t)\,dt = i_I(t)$

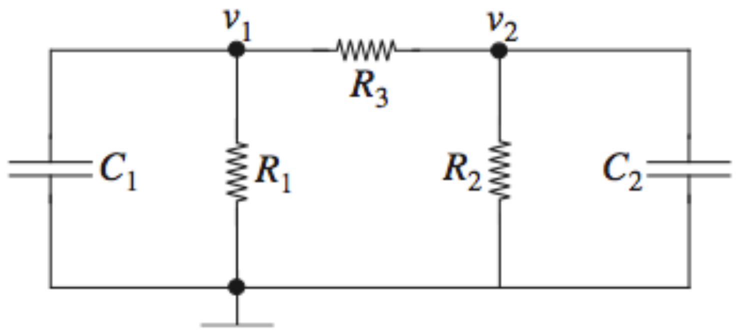
$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

$\Rightarrow$ Characteristic equation: $\underbrace{s^2 + \frac{1}{RC}s + \frac{1}{LC}}_{} = 0$

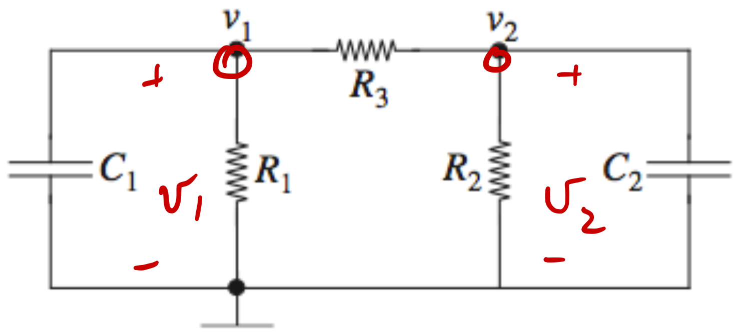
$$s^2 + 2\alpha s + \omega_0^2 = 0 \quad \Rightarrow \quad \alpha = \frac{1}{2RC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Two-Capacitor Circuits



Two-Capacitor Circuits



KCL

For Node #1 $$C_1\frac{dv_1(t)}{dt}+\frac{1}{R_1}v_1(t)+\frac{1}{R_3}(v_1(t)-v_2(t))=0$$

For Node #2 $$C_2\frac{dv_2(t)}{dt}+\frac{1}{R_2}v_2(t)+\frac{1}{R_3}(v_2(t)-v_1(t))=0$$

Express $v_2(t)$ in terms f $v_1(t)$ $$v_2(t)=R_3C_1\frac{dv_1(t)}{dt}+\left(1+\frac{R_3}{R_1}\right)v_1(t)$$

Differential equation of $v_1(t)$ $$\frac{d^2v_1(t)}{dt^2}+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_3C_1}+\frac{1}{R_3C_2}\right)\frac{dv_1(t)}{dt}+\left(\frac{1}{R_1R_2C_1C_2}+\frac{1}{R_1R_3C_1C_2}+\frac{1}{R_2R_3C_1C_2}\right)v_1(t)=0.$$

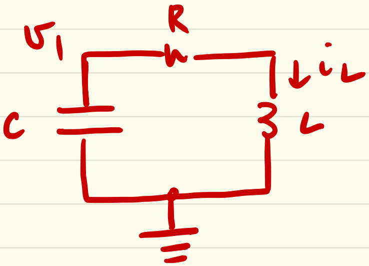
$\Rightarrow s^2+2\alpha s+\omega_0^2=0$

$\Rightarrow \alpha, \omega_0 \qquad s_{1,2}=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$

- **Circuits with only resistors and capacitors have characteristic equations with only real non-positive roots.**

Undriven RLC

Given $v_1(0)$ and $\frac{dv_1(0)}{dt}$.



$\Rightarrow v_1, \; i_L = ?$