Over-damped $\alpha > \omega_0$

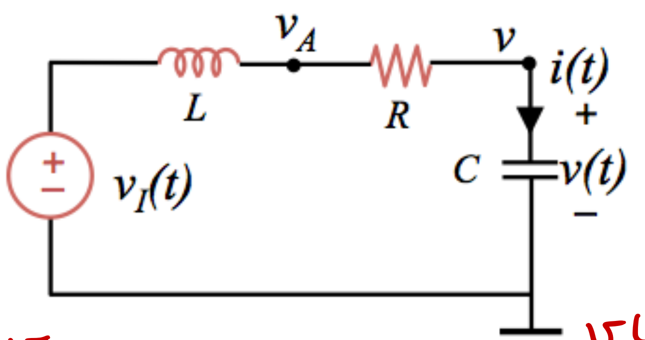
$$v(t) = V_I + A_1 e^{-\alpha t} e^{\left(\sqrt{\alpha^2-\omega_0^2}\right)t} + A_2 e^{-\alpha t} e^{\left(-\sqrt{\alpha^2-\omega_0^2}\right)t}$$

$\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

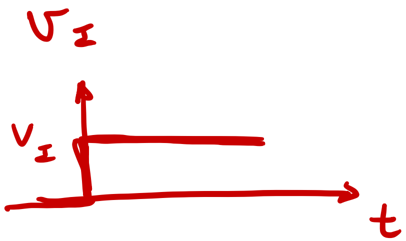
$\alpha > \omega_0$ over-damped

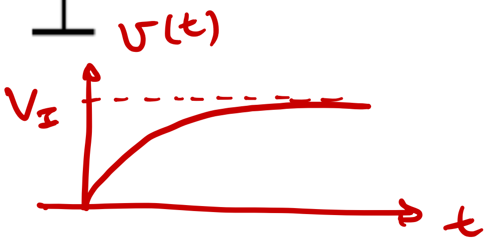
$\alpha < \omega_0$ under-damped

$\alpha = \omega_0$ critically-damped



$$v(t) = V_I + A_1 e^{-\alpha_1 t} + \underbrace{A_2 e^{-\alpha_2 t}}_{\text{decay faster}}$$





when $\alpha \gg \omega_0$, $v(t) \cong V_I + A_1 e^{-\alpha_1 t}$ (use Taylor expansion)

$$-\alpha_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\alpha + \alpha\left(1 - \frac{\omega_0^2}{\alpha^2}\right)^{\frac{1}{2}} \approx -\alpha + \alpha\left(1 - \frac{1}{2}\frac{\omega_0^2}{\alpha^2}\right) = \frac{-1}{2}\frac{\omega_0^2}{\alpha}$$

$$\Rightarrow v(t) = V_I + A_1 e^{\frac{-1}{2}\frac{\omega_0^2}{\alpha}t} = V_I + A_1 e^{-\frac{1}{RC}t}$$

Under-damped $\alpha < \omega_0$

$$v(t) = V_I + A_1 e^{-\alpha t} e^{\left(\sqrt{\alpha^2-\omega_0^2}\right)t} + A_2 e^{-\alpha t} e^{\left(-\sqrt{\alpha^2-\omega_0^2}\right)t}$$

$\alpha > \omega_0$ over-damped
$\alpha < \omega_0$ under-damped
$\alpha = \omega_0$ critically-damped

$$v(t) = V_I + A_1 e^{-\alpha t} \cdot e^{j\sqrt{\omega_0^2-\alpha^2}\,t} + A_2 e^{-\alpha t} e^{-j\sqrt{\omega_0^2-\alpha^2}\,t}$$

$$= V_I + A_1 e^{-\alpha t} e^{j\omega_d t} + A_2 e^{-\alpha t} e^{-j\omega_d t}, \quad \omega_d = \sqrt{\omega_0^2-\alpha^2} \text{ : damped natural frequency}$$

$$= V_I + \underbrace{e^{-\alpha t}\left(K_1 \cos\omega_d t + K_2 \sin\omega_d t\right)}_{\text{decaying sinusoids}}, \quad K_1 = A_1 + A_2, \quad K_2 = jA_1 - jA_2.$$

Define $Q = \dfrac{\omega_0}{2\alpha} = \dfrac{1}{R} \cdot \sqrt{\dfrac{L}{C}}$

Q : quality factor