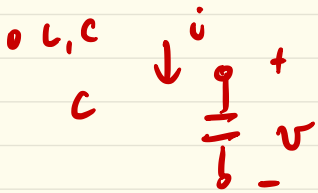


5/13/2019 (Mon)



$$i = C \frac{dv}{dt}$$

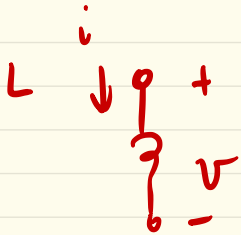
$$E = \frac{1}{2} C v^2$$

series

$$C_{eq} = \frac{1}{\sum \frac{1}{C_i}}$$

shunt

$$C_{eq} = \sum C_i$$



$$v = L \frac{di}{dt}$$

$$E = \frac{1}{2} L i^2$$

$$L_{eq} = \sum L_i \quad L_{eq} = \frac{1}{\sum \frac{1}{L_i}}$$

o RL, RC first-order

General response

$$x(t) = x_f + (x_0 - x_f) \cdot e^{-(t-t_0)/\tau}$$

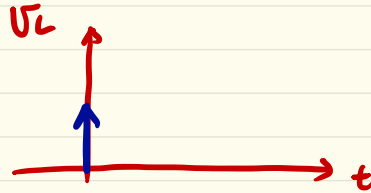
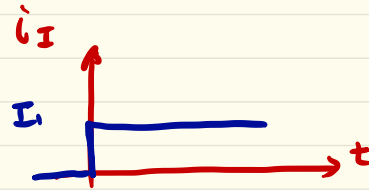
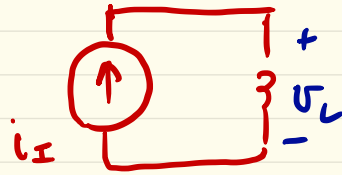
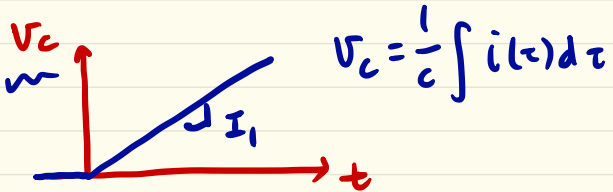
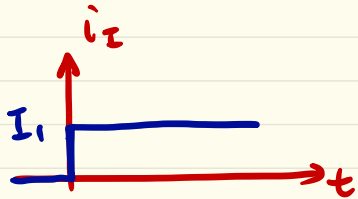
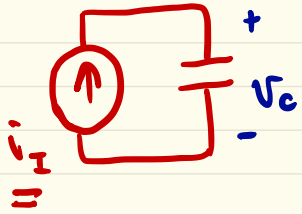
x_f : final value ($x(t)$ when $t \rightarrow \infty$), x_0 : initial value ($x(t)$ when $t = t_0$)

- 1) homogeneous sol (Ae^{st})
- 2) particular sol

Total response = 1) + 2)

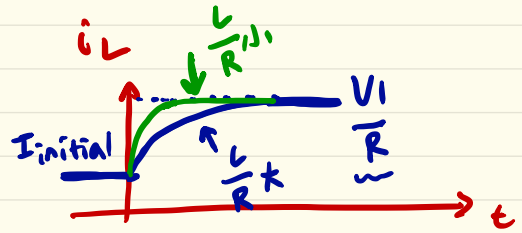
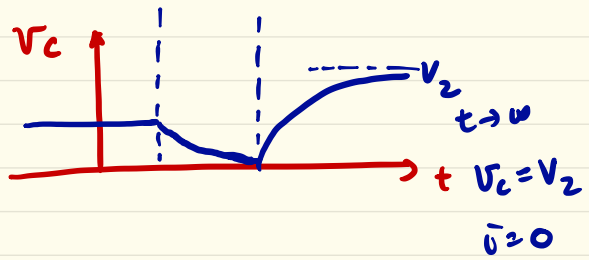
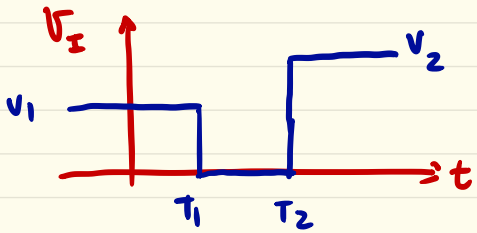
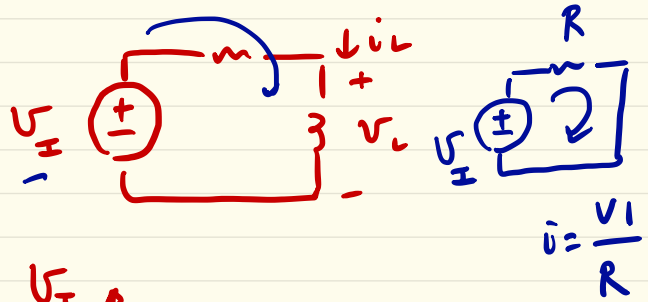
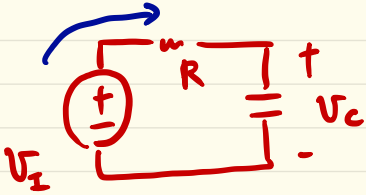
$\tau = RC$ or L/R (time constant)

• only L or C



step
↓
impulse
ramp.

o RC, RL

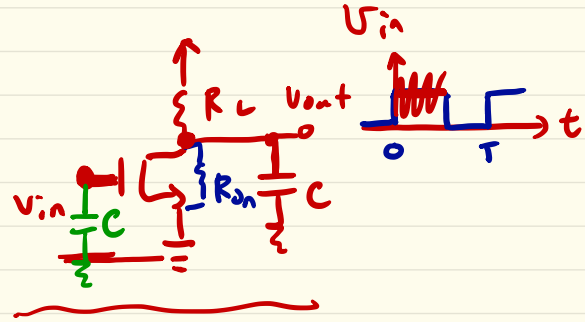


$$i_L = \frac{V_I}{R} (1 - e^{-R/L \cdot t})$$

$$V_L = V_I \cdot e^{-R/L \cdot t}$$

- Energy, power
(Joule) (Watt)

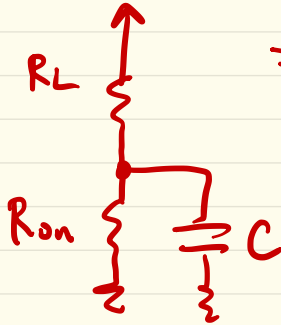
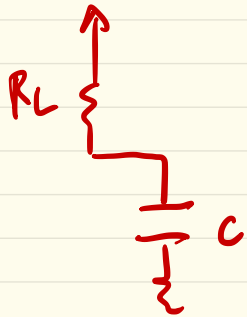
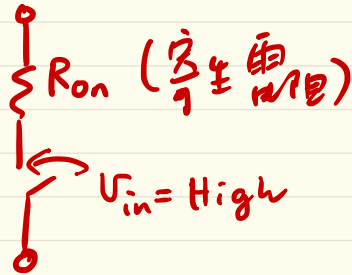
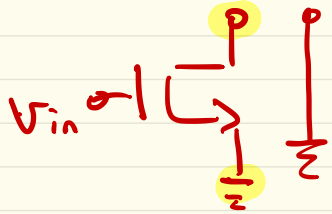
$$E = \int_T p \, dt$$



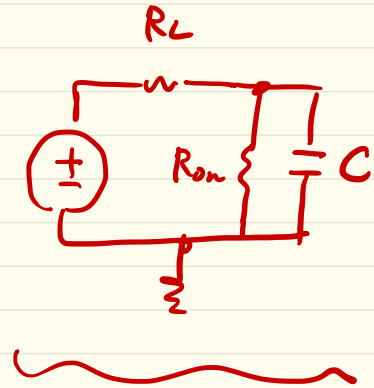
P_{static} : static current from power supply,
 $P_{dynamic}$: charging/discharging

$$P = \frac{V_s^2}{2(R_L + R_{on})} + \text{static}$$

$$C V_s^2 \cdot f \cdot \frac{R_L^2}{(R_L + R_{on})^2} + \text{dynamic}$$



\Rightarrow



• 2nd-order circuits (only LC)

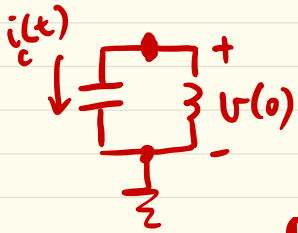


~

1) • Homogeneous sol (Ae^{st})

2) • Particular sol $A_1 e^{s_1 t}$ + $A_2 e^{s_2 t}$

3) Total = 1) + 2)



Given $v(0)$, $\frac{dv(0)}{dt} = 0$

Particular sol = 0

$$\frac{d^2 v(t)}{dt^2} + \frac{1}{LC} v(t) = 0$$

Assume $v(t) = Ae^{st} \Rightarrow s^2 + \frac{1}{LC} = 0 \Rightarrow s = \pm j\omega_0$, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\Rightarrow v(t) = A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t} = v(0) \cdot \cos \omega_0 t$$

$$i_c(t) = -\sqrt{\frac{C}{L}} \cdot v(0) \cdot \sin \omega_0 t$$

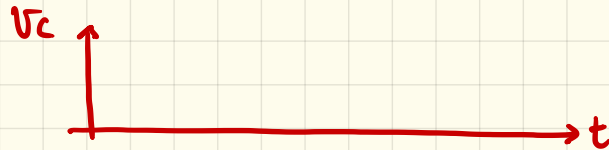
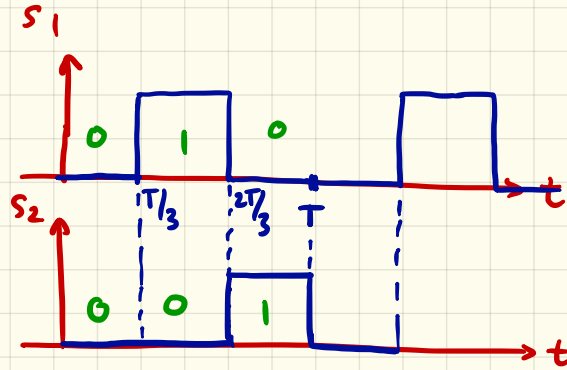
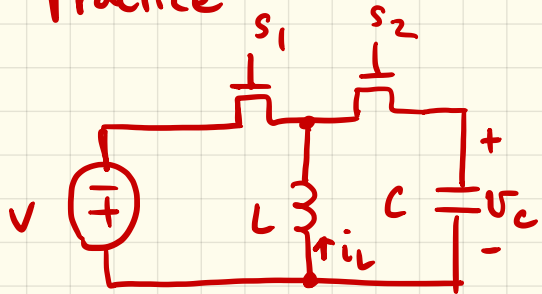
$$\frac{V_{\text{peak}}}{i_{\text{peak}}} = \sqrt{\frac{L}{C}} : \text{characteristic impedance}$$

$$\frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} : \text{oscillation frequency, } \sqrt{LC} : \text{time constant}$$

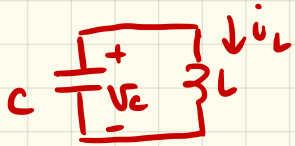
Summary:

- 1) $v(t)$ and $i(t)$ are both sinewave, with $\frac{\pi}{2}$ phase difference
- 2) Initial conditions affect only their amplitude and absolute phase.

Practice 1



Practice 2



$$C = 1 \mu\text{F}, L = 100 \mu\text{H},$$

$$i_L(t=0^-) = 0 \text{ A}, V_C(t=0^-) = 1 \text{ V}.$$

Find i_L and V_C .