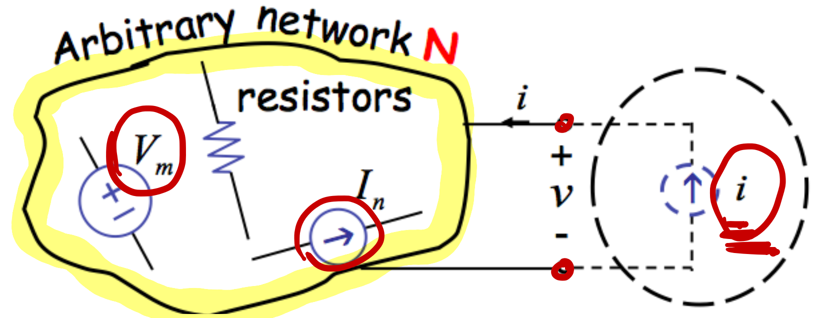
3/11/2019 (Mon)

Yet Another Method?



- Arbitrary network

By superposition

$$\underline{v} = \underbrace{\sum_m \alpha_m V_m}_{\#1} + \underbrace{\sum_n \beta_n I_n}_{\#2} + \underbrace{Ri}_{\#3}$$

#1, #2: depend only on the network N. independent of external source

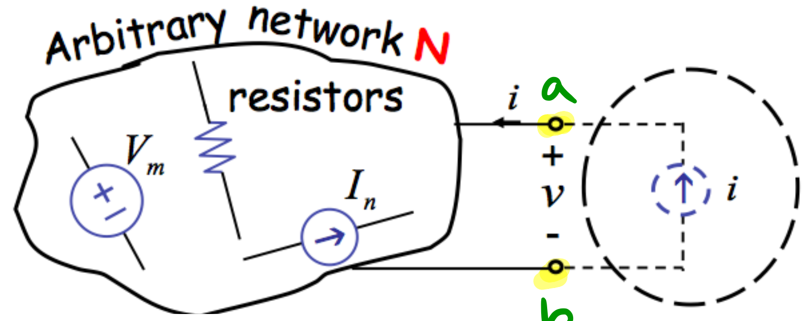
#3: due to external source. (all V_m, $I_m = 0$)

$v = V_{TH} + R_{TH} \cdot i$

1. Independent of external excitation and behave like a voltage.
   - Let's call it 'v_{TH}'
2. Independent of external excitation and behave like a resistor.
   - Let's call it 'R_{TH}'

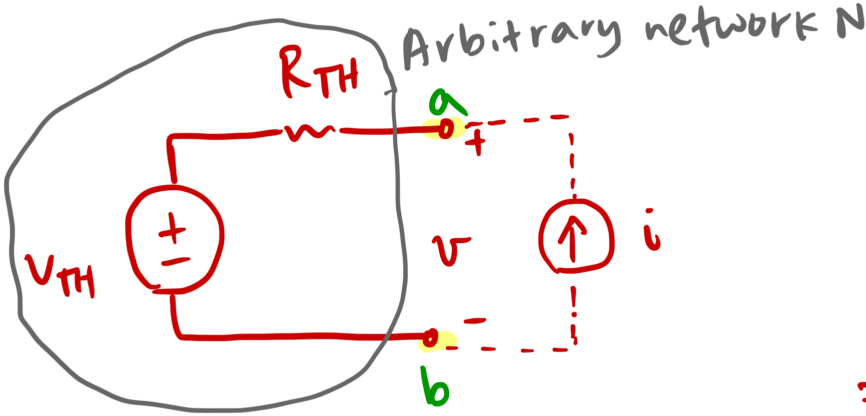
Arbitrary Network

$$v = \sum_m \alpha_m V_m + \sum_n \beta_n I_n + Ri$$



$$v = v_{TH} + R_{TH} \cdot i$$

- In other words, as far as the external world is concerned (for the purpose of the *i*-*v* relation), 'arbitrary network N' is indistinguishable from:



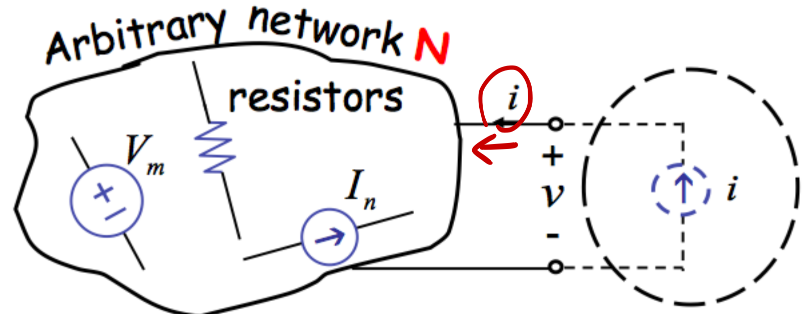
$$V_{TH} = v_{open}\Big|_{i=0}$$

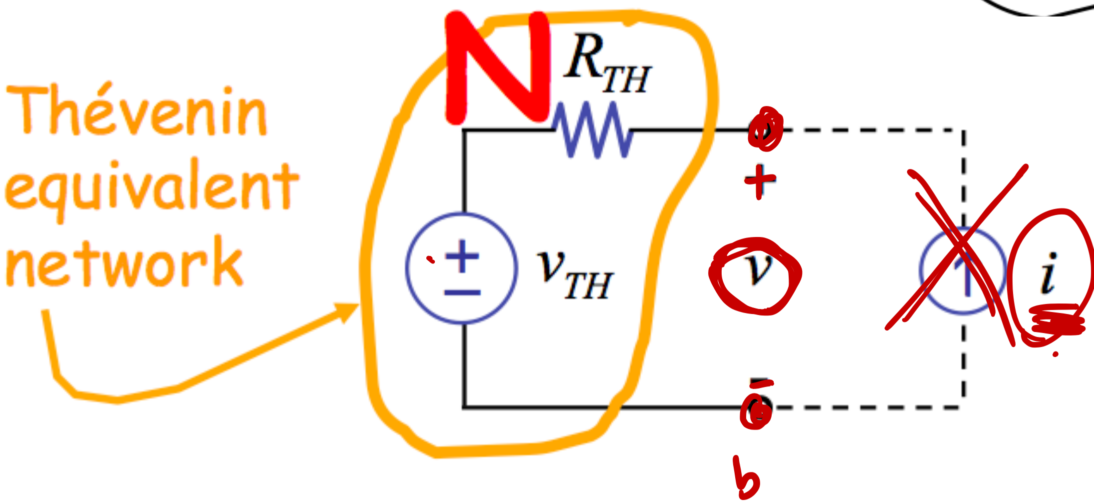
$$R_{TH} = \frac{v}{i}\Big|_{\text{internal independent source}=0}$$

$\Rightarrow$ Thevenin Equivalent circuit

Arbitrary Network

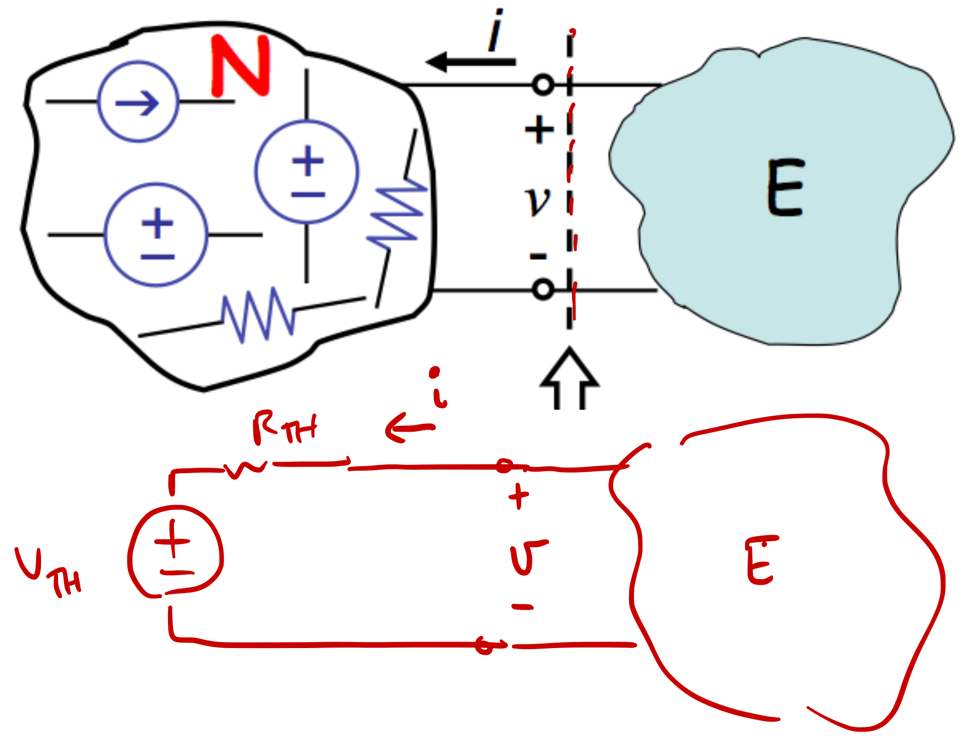
$$v = v_{TH} + R_{TH}\, i$$





- How to derive v_{TH} and R_{TH}?
- v_{TH} → Open circuit voltage seen at terminal pair (aka port).
- R_{TH} → Resistance of network seen from port (with V_m's and I_n's set to 0).

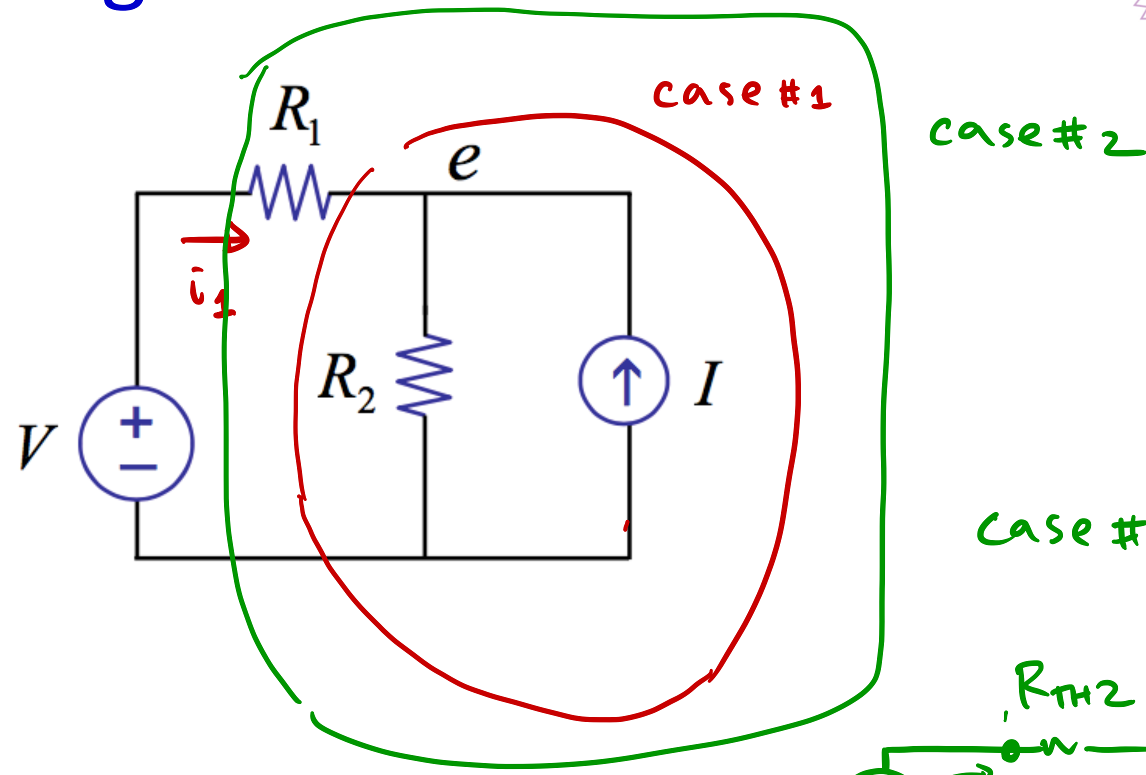
Method 5: The Thévenin Method



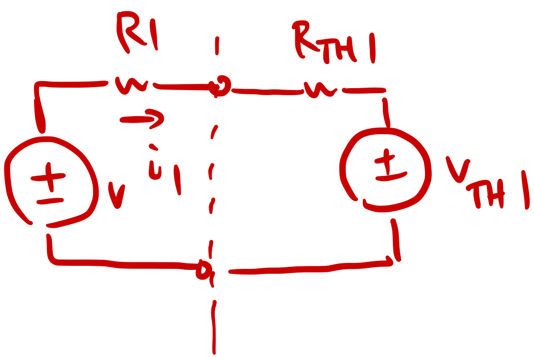
- Replace network N with its Thévenin equivalent
- Solve with external network E

Example – Using Thévenin Method

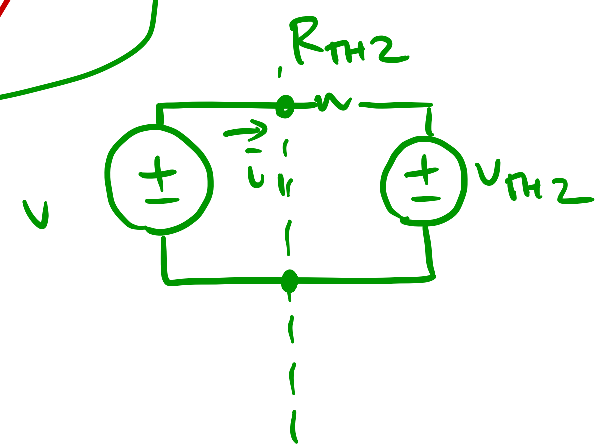
Find i_1.



case #1

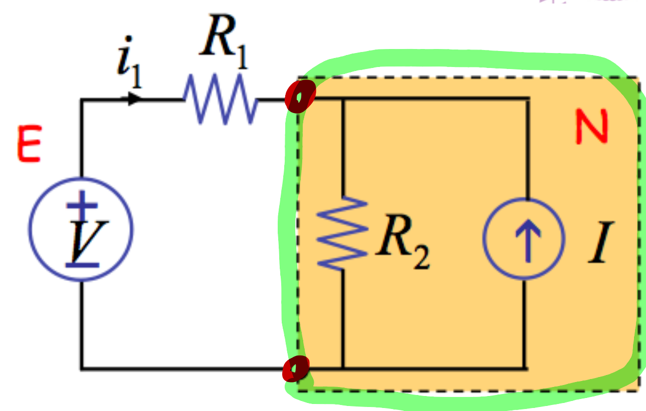


case #2



Example – Using Thévenin Method

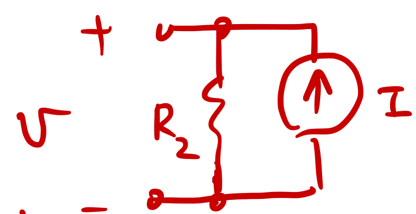
- Case 1



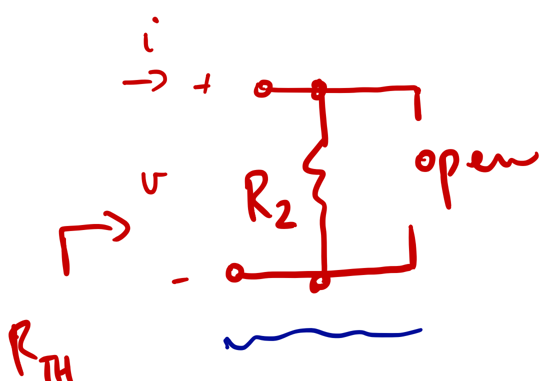
$$V_{TH} = IR_2$$
$$R_{TH} = R_2$$

1) V_{TH}: open circuit v of network N

$V_{TH} = I \cdot R_2$

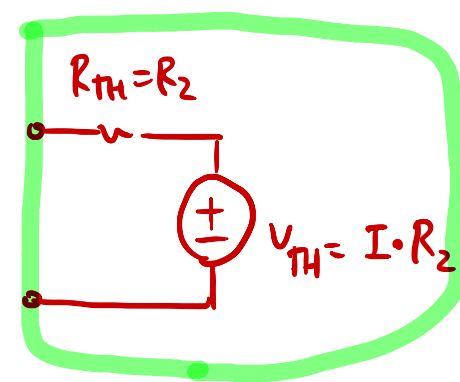


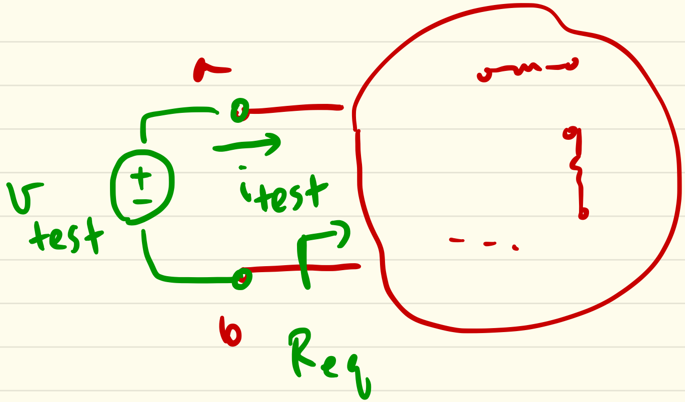
2) R_{TH}: set all independent source = 0, measure $\frac{v}{i}$



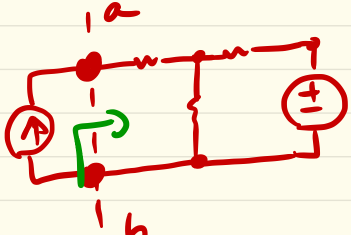
$R_{TH} = R_2$

⇒

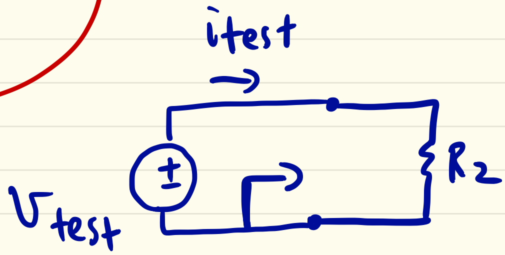




$$R_{eq} = \frac{v_{test}}{i_{test}}$$



$R_{eq} = ?$

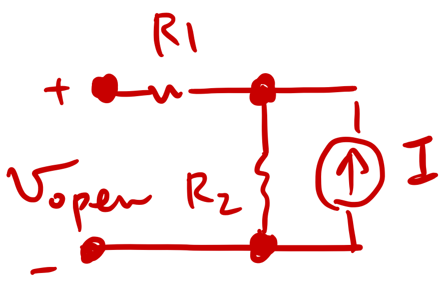


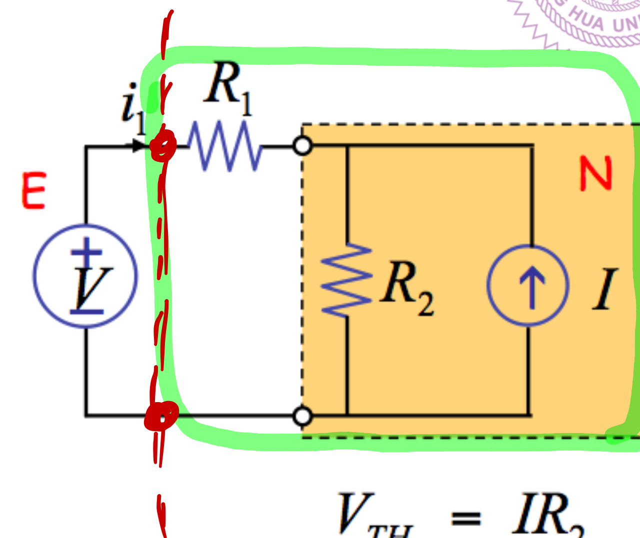
$R_{eq} = ?$

$$\frac{v_{test}}{i_{test}} = R_2$$

Example – Using Thévenin Method

- Case 2





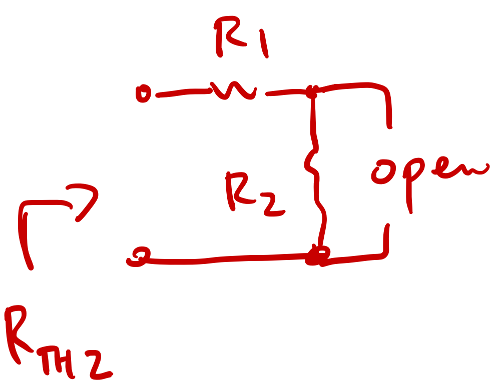
1) $V_{TH2} =$

$V_{open} = I \cdot R_2$

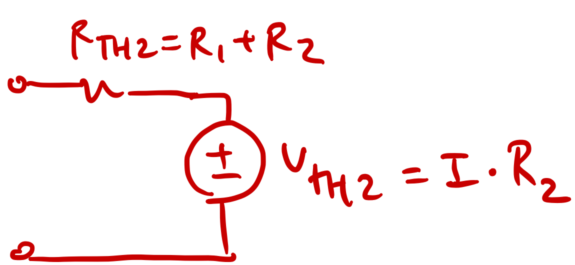
$V_{TH} = IR_2$

$R_{TH} = R_2$

2) $R_{TH2} = R_1 + R_2$

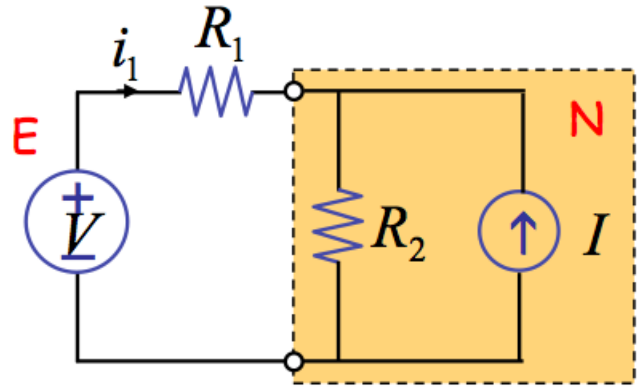


⇒



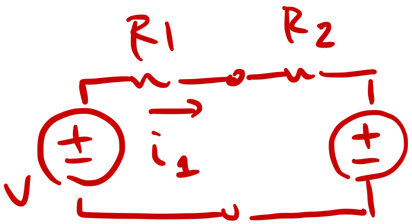
Example – Using Thévenin Method

- Solve with external network E



$$V_{TH} = IR_2$$
$$R_{TH} = R_2$$

Case #1

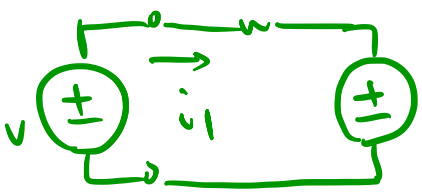


$V_{TH1} = I \cdot R_2$

$$i_1 = \frac{V - V_{TH1}}{R_1 + R_2} = \frac{V - I \cdot R_2}{R_1 + R_2}$$

Case # 2

$R_{TH2} = R_1 + R_2$



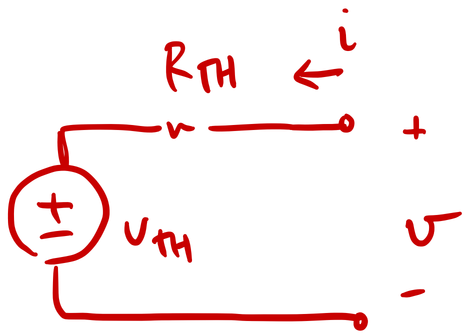
$V_{TH2} = I \cdot R_2$

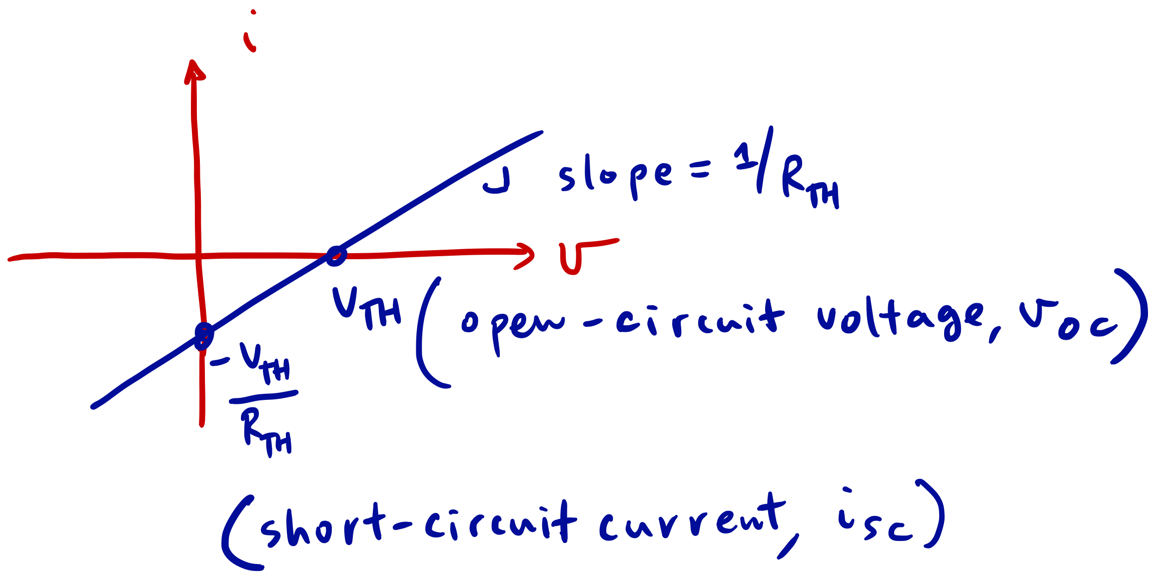
$$i_1 = \frac{V - V_{TH2}}{R_{TH2}} = \frac{V - I \cdot R_2}{R_1 + R_2}$$

Example – Using Thévenin Method

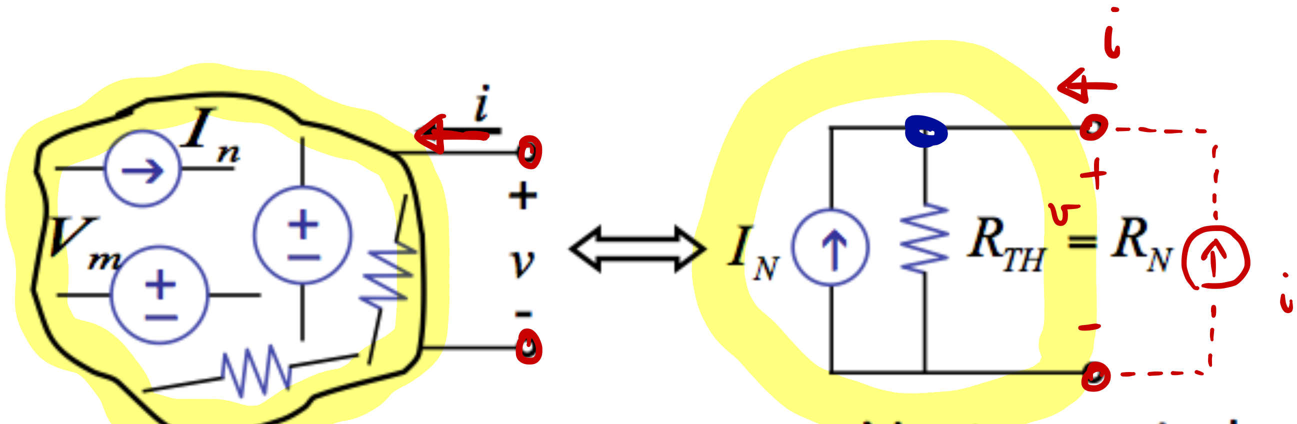
- Graphically…

$v = V_{TH} + R_{TH} \cdot i$





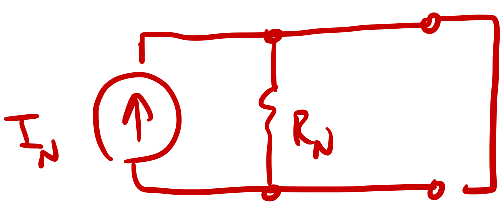
Method 6: The Norton Method



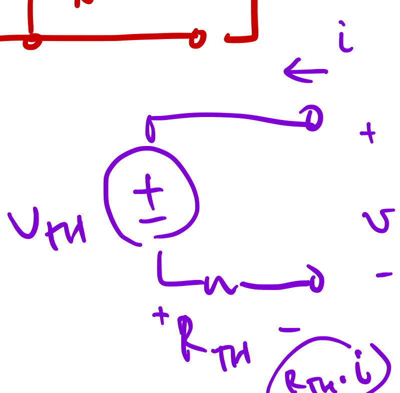
Norton equivalent

(thevenin: $v = V_{TH} + R_{TH} \cdot i$)

Norton: $i = -I_N + \frac{v}{R_N}$



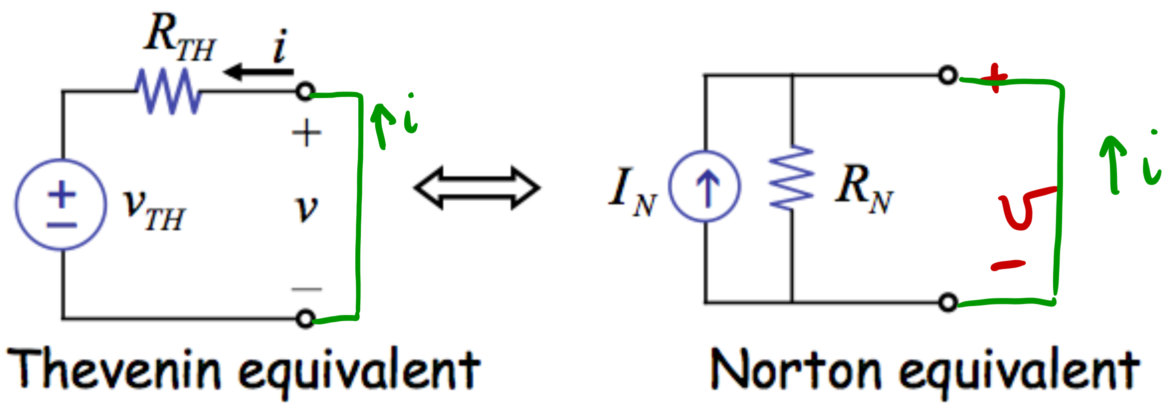
I_N: short circuit current seen at port, $-I_N = i_{sc} = i$



$v = R_{TH} \cdot i + V_{TH}$

$R_N = R_{TH}$, set independent source $= 0$

Thévenin and Norton



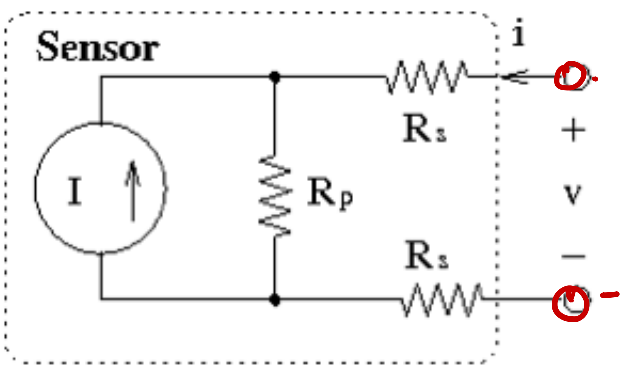
Equivalent resistance $\quad R_{TH} = R_N$

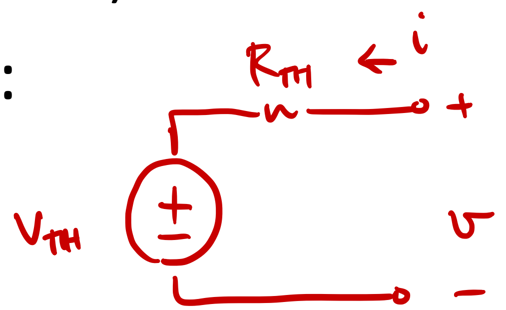
open-circuit voltage, v_{oc} $\quad V_{TH} = I_N \cdot R_N$

short-circuit current, i_{sc} $\quad -\frac{V_{TH}}{R_{TH}} = -I_N$

Example – Thévenin and Norton

- A light sensor is modeled as a current source that produces a current proportional to the intensity of light.
  - Leakage through the sensor is modeled as R_P.
  - Resistance in the contacts (wires) is modeled as R_S.
- Thévenin equivalent circuit:





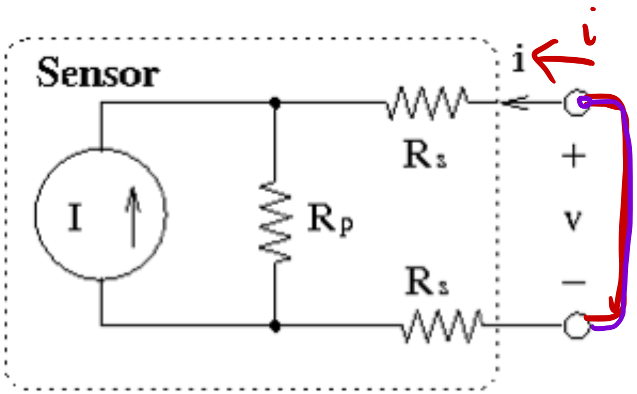
$$v = V_{TH} + R_{TH} \cdot i$$

$$V_{TH} = I \cdot R_P$$

$$R_{TH} = 2R_S + R_P$$

Example – Thévenin and Norton

- Norton equivalent circuit:

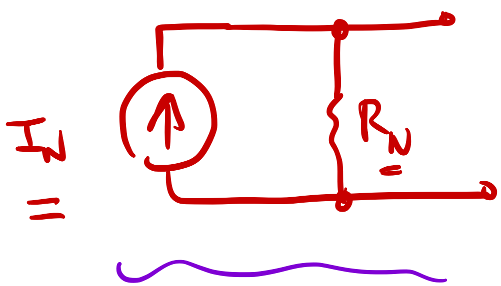


$$i = -I_N + \frac{v}{R_N}$$

$$I_N = I \cdot \frac{R_p}{R_p + 2R_s}\Bigg|_{v=0}$$

$(= -i)$

$$R_N = R_p + 2R_s$$



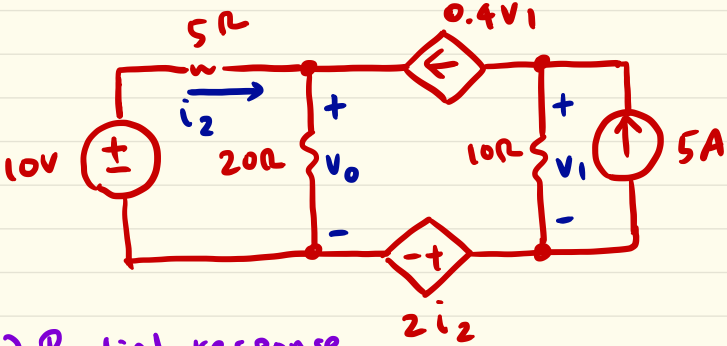
Summary

- Basic circuit analysis methods
  - KCL, KVL
  - Element combination rules
  - Node method
  - Superposition
  - Thévenin
  - Norton

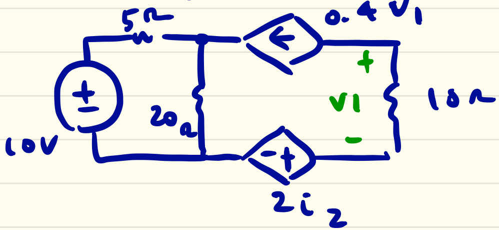
For all lumped elements

For linear cirmits only

Example. Use superposition. Find v_o

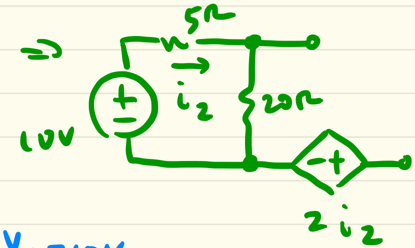


1) Find partial response due to 10V source



$$\frac{-v_1}{10} = 0.4v_1 \Rightarrow v_1 = 0\,V$$

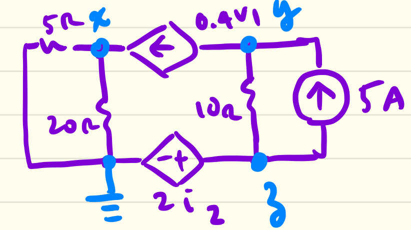
$\Rightarrow$



$$i_2 = \frac{10V}{20+5} = \frac{2}{5}A$$

$$v_{o,1} = \frac{2}{5} \times 20 = 8V$$

2) Partial response due to 5A source



KCL @ x : $\frac{v_x}{5} + \frac{v_x}{20} - 0.4v_1 = 0$

@ y : $0.4v_1 + \frac{v_y - v_z}{10} - 5 = 0$

$v_z = 2i_2$, $v_1 = v_y - v_z$

$\Rightarrow$ $v_1 = 10V$

$v_x = v_{o,2} = 16V$

3) Total response

$$v_o = v_{o,1} + v_{o2} = 24V$$