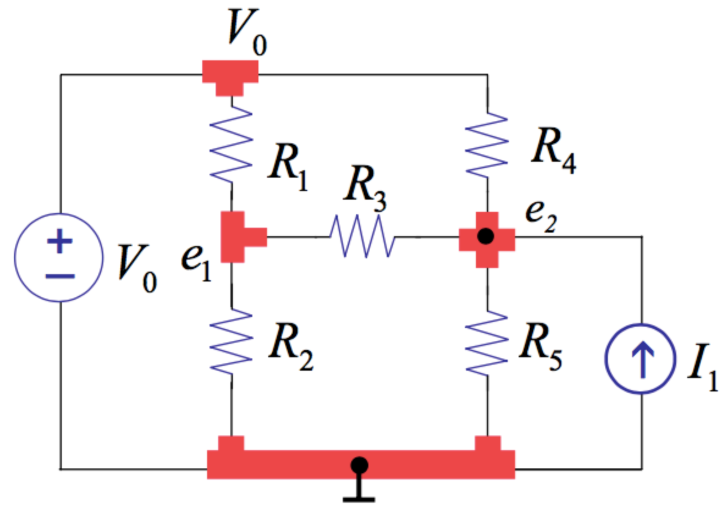
3/6/2019 (wed)

Revisit Step 4

$$
\begin{cases}
e_1(G_1+G_2+G_3)+e_2(-G_3)=V_0(G_1) \\
e_1(-G_3)+e_2(G_3+G_4+G_5)=V_0(G_4)+I_1
\end{cases}
$$



- In matrix form :

$$
\left[\begin{array}{c|c} G_1+G_2+G_3 & -G_3 \\ \hline -G_3 & G_3+G_4+G_5 \end{array}\right]
\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}
=
\begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}
=
\begin{bmatrix} G_1 & 0 \\ G_4 & 1 \end{bmatrix}
\begin{bmatrix} V_0 \\ I_0 \end{bmatrix}
$$

conductivity matrix

unknown node voltages

sources

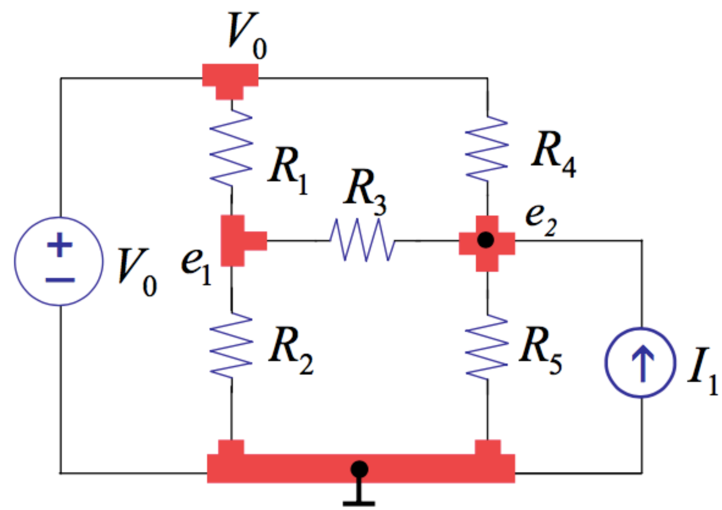
Revisit Step 4

$$\left[\begin{array}{c|c} G_1+G_2+G_3 & -G_3 \\ \hline -G_3 & G_3+G_4+G_5 \end{array}\right] \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} G_1V_0 \\ G_4V_0+I_1 \end{bmatrix}$$

$$e_1 = \frac{(G_3+G_4+G_5)(G_1V_0)+(G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3G_4+G_3G_5}$$

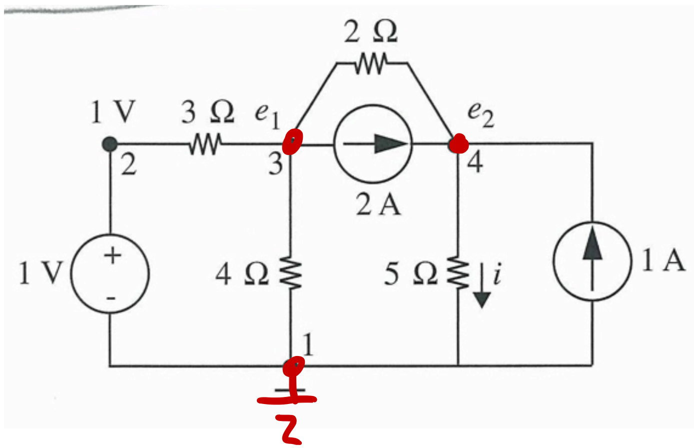
$$e_2 = \frac{(G_3)(G_1V_0)+(G_1+G_2+G_3)(G_4V_0+I_1)}{G_1G_3+G_1G_4+G_1G_5+G_2G_3+G_2G_4+G_2G_5+G_3G_4+G_3G_5}$$

- The same denominator for e_1 and e_2.
- No negative terms in denominator.
- Linear in V_0 and I_1.



More Example – Node Method

- Determine the current i.



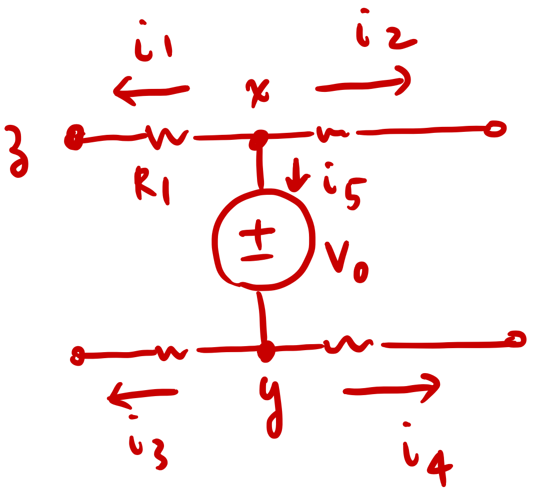
1. Choose node 1 = ground (0 V)
2. Label the nodes (e_1, e_2)
3. KCL @ e_1: $\frac{e_1 - 1}{3} + \frac{e_1}{4} + \frac{e_1 - e_2}{2} + 2 = 0$

   KCL @ e_2: $-2 + \frac{e_2 - e_1}{2} + \frac{\overset{e_2}{\cancel{e_1}}}{5} - 1 = 0$

4. Solve KCL ⇒ $e_1 = 0.05\,V$, $e_2 = 4.75\,V$

5. $i = \frac{e_2}{5} = 0.95\ A$

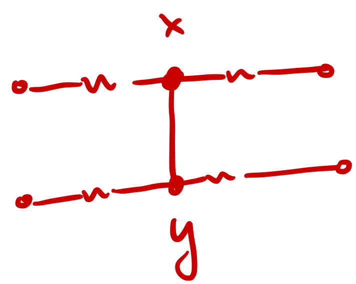
Floating Independent Sources



KCL @x $\quad i_1 + i_2 + i_5 = 0$

$+ \quad$ @y $\quad i_3 + i_4 - i_5 = 0$

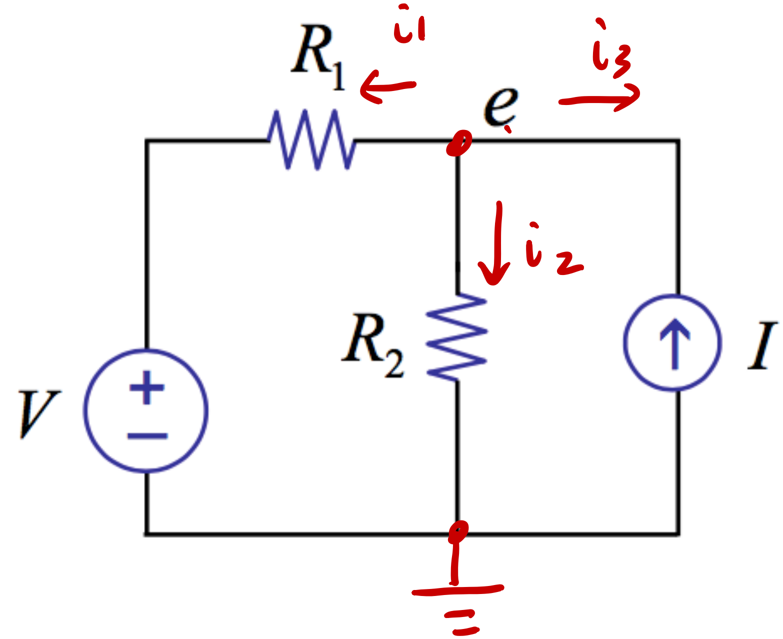
$\Rightarrow \quad i_1 + i_2 + i_3 + i_4 = 0$



x, y : supernode

Linearity 線性度

- Consider this circuit



- Write node equation.

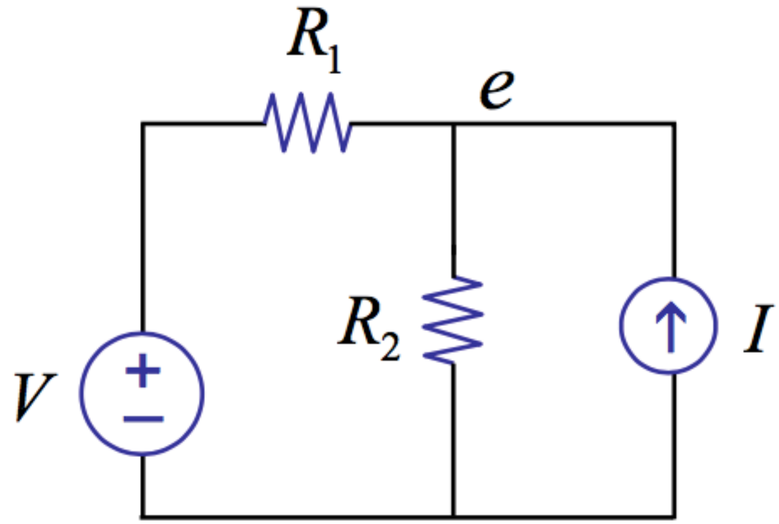
KCL @ e ; $\frac{e-V}{R_1} + \frac{e}{R_2} - I = 0$

e, V, I have linear relationship (no e^2, eV, I^2, $\sqrt{e}$, ...)

Linearity

- Node equation

$$\frac{e-V}{R_1}+\frac{e}{R_2}-I=0$$ linear in e, V, I



$$\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\cdot e=\frac{V}{R_1}+I$$

$$\Rightarrow e=\underbrace{\frac{R_2}{R_1+R_2}V}_{e_1}+\underbrace{\frac{R_1R_2}{R_1+R_2}I}_{e_2}$$

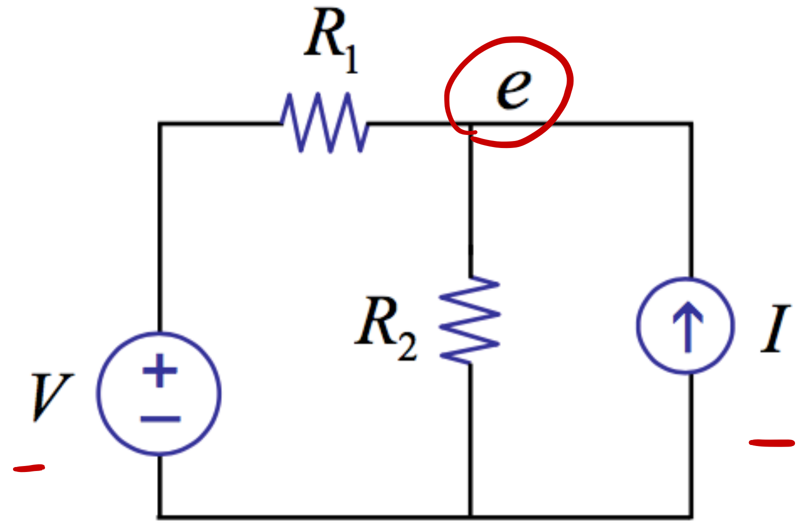
$$\Rightarrow \underbrace{(G_1+G_2)}_{\text{conductivity matrix}}\cdot\underbrace{e}_{\text{unknown}}=\underbrace{\frac{V}{R_1}+I}_{\text{source matrix}}$$

source matrix (linear sum of all sources)

Linearity ✗

$$\frac{e-V}{R_1}+\frac{e}{R_2}-I=0$$

$$\left[\frac{1}{R_1}+\frac{1}{R_2}\right]\cdot e=\frac{V}{R_1}+I$$

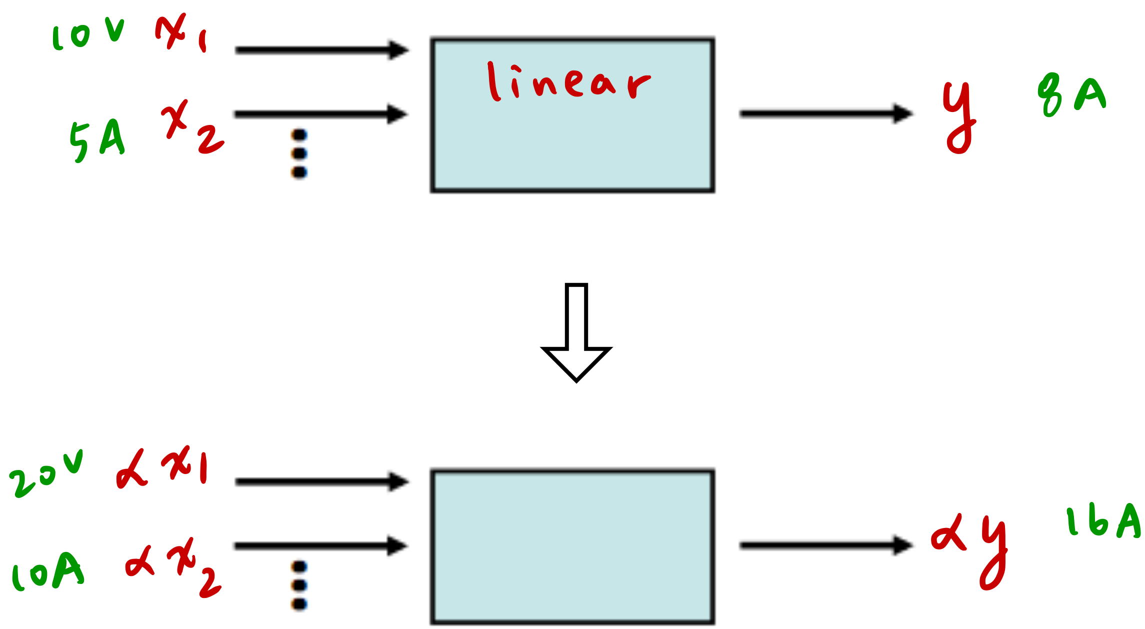


Linearity

R, L, C, independent source

- Linearity
  1. - Homogeneity
  2. - Superposition

Homogeneity (scaling)

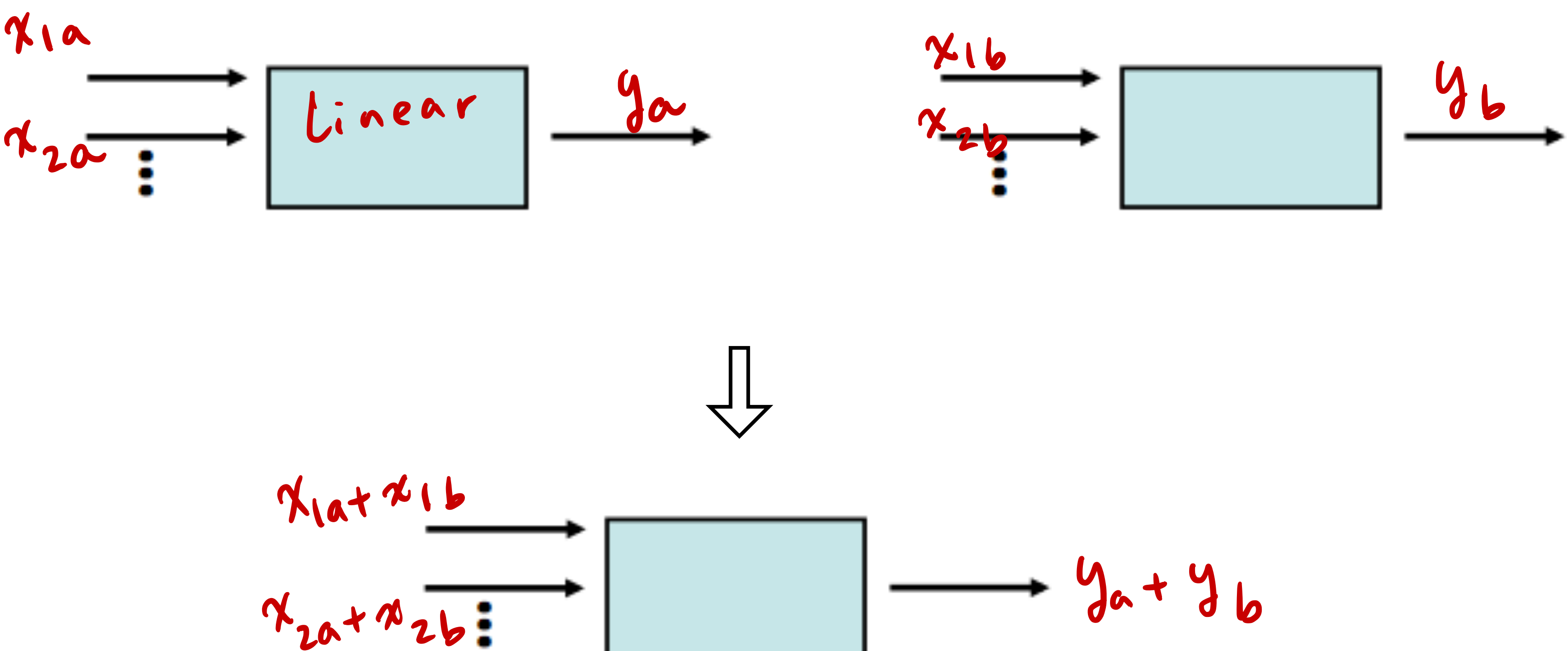


α: scaling factor

$$y = f(x)$$

$$\alpha y = f(\alpha x)$$

Superposition



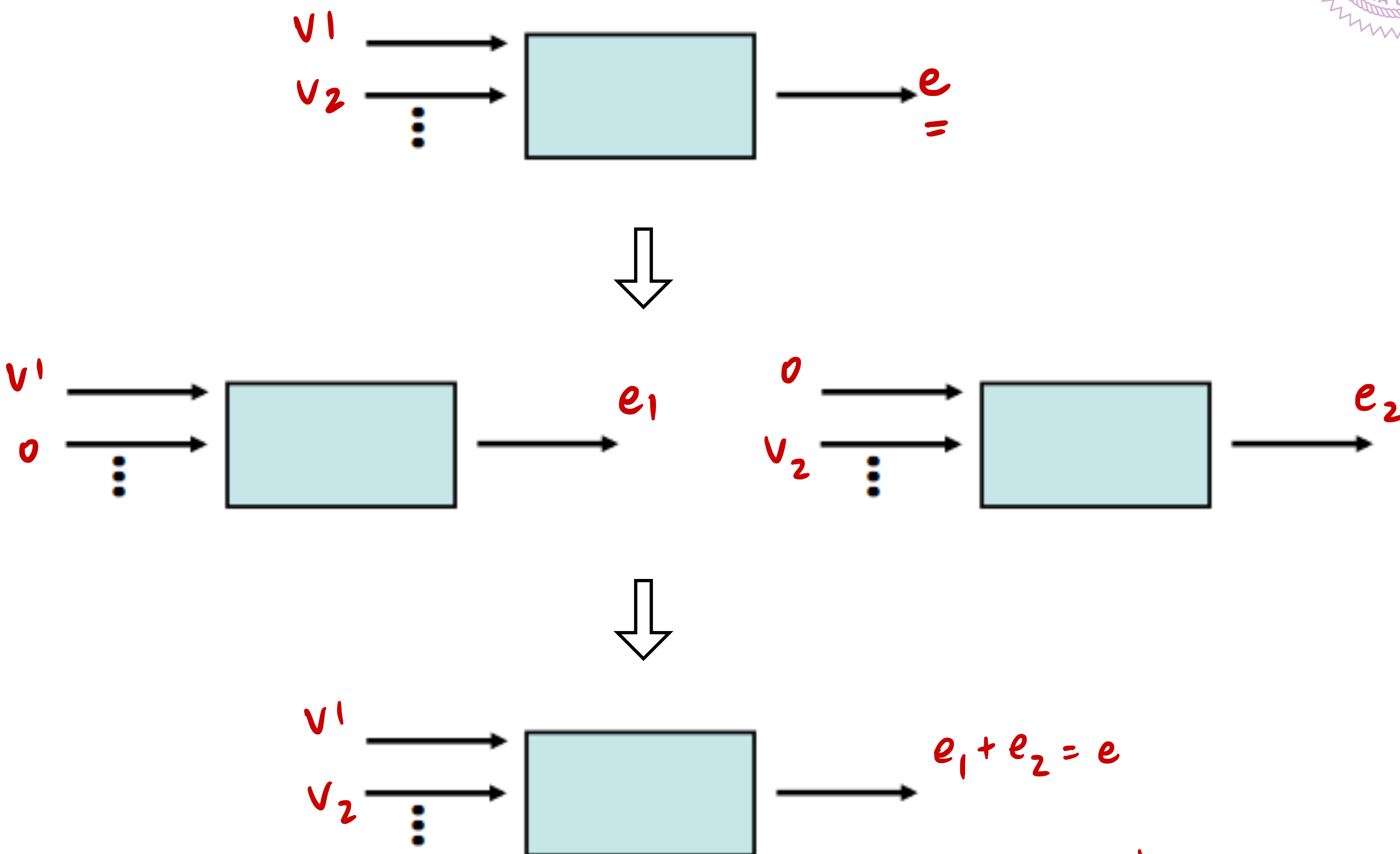
$y_a = f(x_a)$

$y_b = f(x_b)$

$\Rightarrow y_a + y_b = f(x_a + x_b)$

$f(ax_1 + bx_2) = a f(x_1) + b f(x_2)$

Specific Superposition Example



e_1, e_2: partial response

e: total response

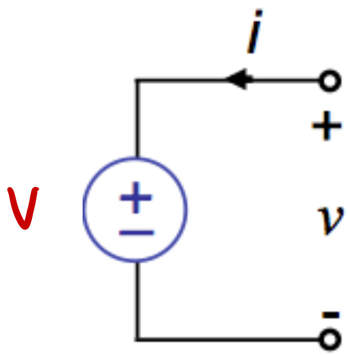
Method 4: Superposition

1. Find the partial responses of the circuit to each source acting along.
2. Sum the individual responses.

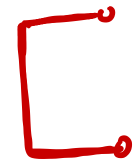
- Dependent sources remain active at all time. Only the independent sources are turned ON and OFF.

Each Source Acting Along Means…

- Voltage source

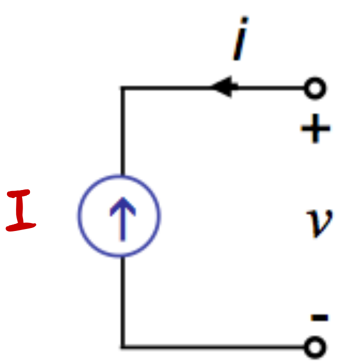


$V = 0$



short

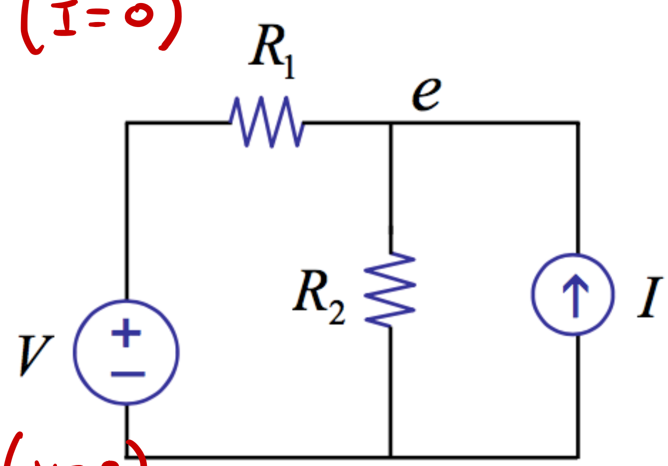
- Current source



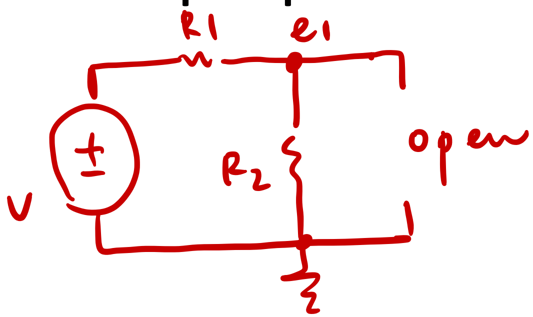
$I = 0$

open

Example – Analysis Using Superposition

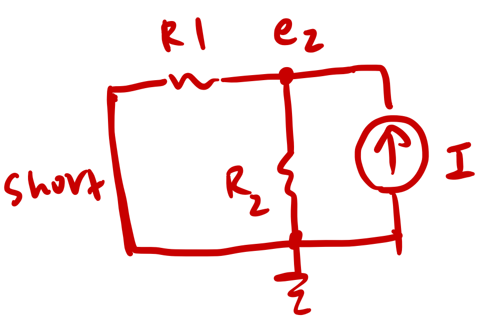


- Superposition – with V acting alone $(I=0)$



$$e_1 = \frac{R_2}{R_1+R_2} V$$

- Superposition – with I acting alone $(V=0)$

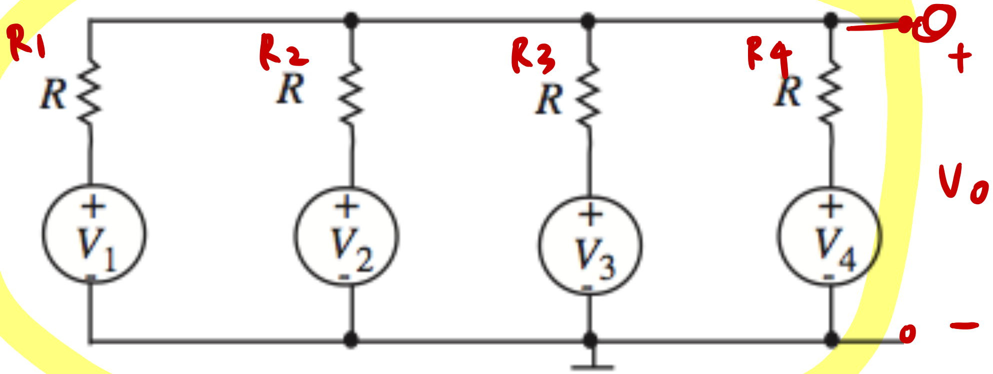


$$e_2 = (R_1 /\!/ R_2) \cdot I$$

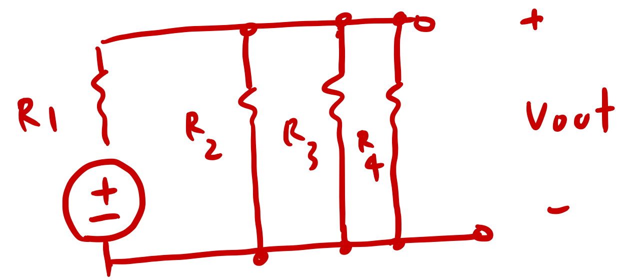
- Sum two partial responses

Total response $e = e_1 + e_2 = \frac{R_2}{R_1+R_2} V + \frac{R_1 R_2}{R_1+R_2} I$

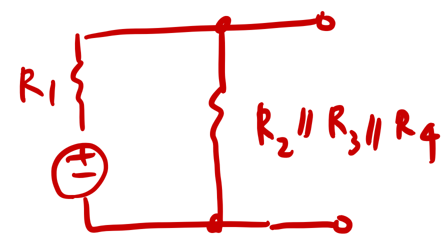
Example – Resistive Adder Circuit



Find V_{out}



=>



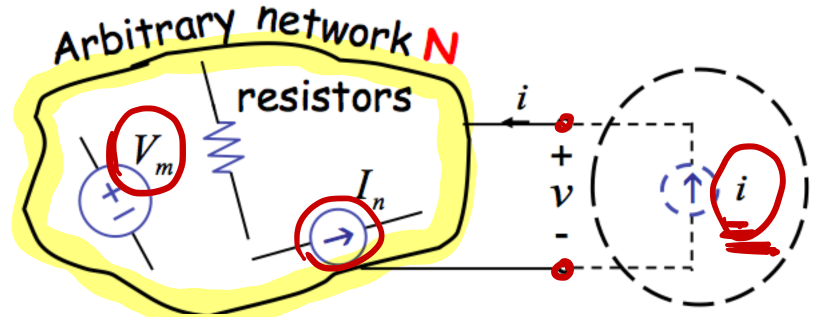
1) Partial response due to V_1

$$V_{out,1} = \frac{R_2 \| R_3 \| R_4}{R_1 + (R_2 \| R_3 \| R_4)} \cdot V_1 = \frac{1}{4} V_1$$

2) Total response

$$V_{out} = \frac{1}{4}V_1 + \frac{1}{4}V_2 + \frac{1}{4}V_3 + \frac{1}{4}V_4$$

Yet Another Method?



- Arbitrary network

By superposition

$$v = \underbrace{\sum_m \alpha_m V_m}_{\#1} + \underbrace{\sum_n \beta_n I_n}_{\#2} + \underbrace{Ri}_{\#3}$$

#1, #2: depend only on the network N. independent of external source

#3: due to external source. (all V_m, $I_m = 0$)

$v = V_{TH} + R_{TH} \cdot i$

1. Independent of external excitation and behave like a voltage.
   - Let's call it 'v_{TH}'
2. Independent of external excitation and behave like a resistor.
   - Let's call it 'R_{TH}'