Electric Circuits
Lecture 3 Network Theorems

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Lecture Outline

- Review
  - KCL, KVL (method 1)
  - Element combination (method 2)
- Chapter 3 in the textbook
  - Node analysis (method 3)
  - Linearity
  - Superposition (method 4)
  - Thévenin and Norton (method 5 and 6)

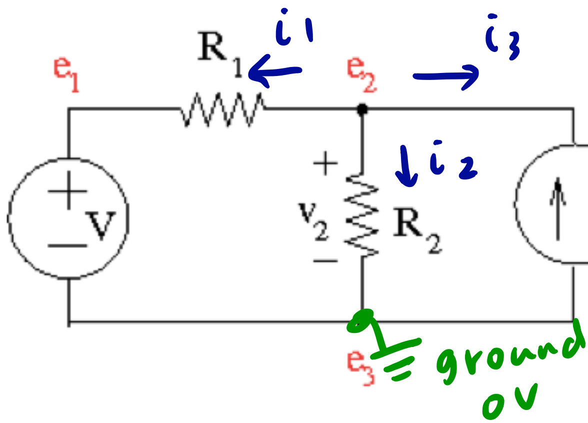
Chapter 3 Network Theorems

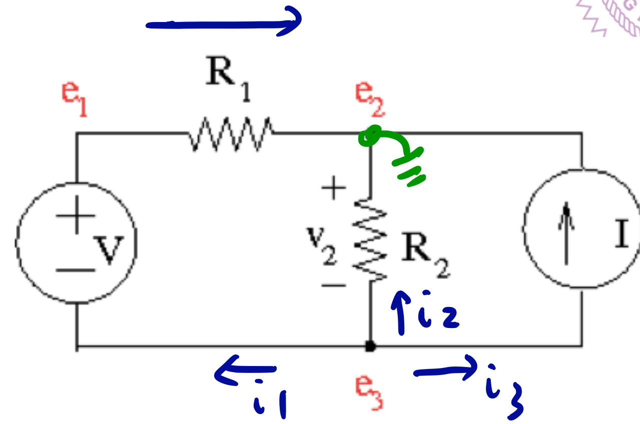
Node Voltage

- Node voltage: the potential difference between the given node and a reference node, which is usually ground (zero V).
  - Current flows from the node with higher potential to the node with lower potential.
  - It is common to choose the node that has the maximum number of elements connected to it as the reference node.

Example – Reference Node

$$i_1 = \frac{e_1 - e_2}{R_1} = \frac{e_1}{R_1} = \frac{e_3 + V}{R_1}$$





- Given $V = 5V$, $I = 3A$, $R_1 = 3\Omega$, $R_2 = 5\Omega$

Case 1: Set e_3 as reference node

KCL at e_2: $i_1 + i_2 + i_3 = 0$

$$\Rightarrow \frac{e_2 - e_1}{R_1} + \frac{e_2}{R_2} - I = 0$$

$$\Rightarrow \frac{e_2 - 5}{3} + \frac{e_2}{5} - 3 = 0 \Rightarrow e_2 = 8.75V$$

Case 2: Set e_2 as reference node

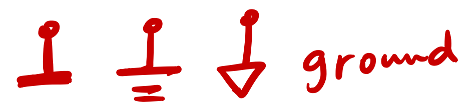
KCL at e_3: $i_1 + i_2 + i_3 = 0$

$$\Rightarrow \frac{e_3 + V}{R_1} + \frac{e_3}{R_2} + I = 0$$

$$\Rightarrow \frac{e_3 + 5}{3} + \frac{e_3}{5} + 3 = 0 \Rightarrow e_3 = -8.75V$$

Method 3: Node Analysis

- The most efficient utilization of KCL and circuit elements' laws to solve for the voltages and currents of all elements of a given circuit.

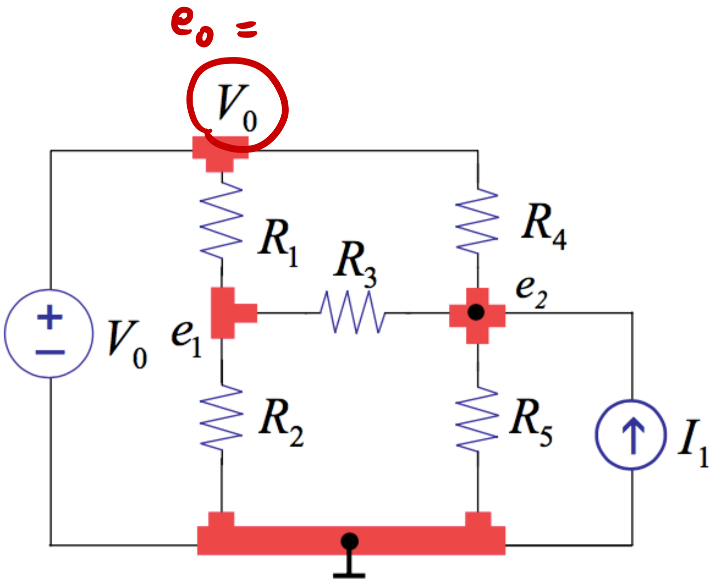


1. Select a node as as reference (ground) node.
2. Label node voltages to the remaining nodes with respect to ground.
   - These are the primary unknowns.
3. Apply KCL to all but the ground node.
   Use device laws to express the branch currents in terms of node voltages.
4. Solve the $n-1$ equations for node voltages.
5. Calculate all branch voltages and currents based on elements' laws.

Example – Using Node Method

1. Select a node as as reference (ground) node.
2. Label node voltages to the remaining nodes with respect to ground.

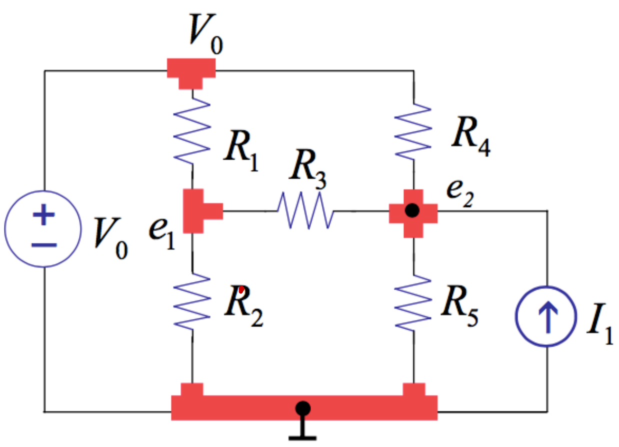
4 nodes



Example – Using Node Method

3. Apply KCL to all but the ground node. Use device laws to express the branch currents in terms of node voltages.

- For convenience, write $G_i = \frac{1}{R_i}$
- To avoid mistakes use convention: e.g., always sum currents leaving a node



KCL @ e_1: $\frac{e_1 - V_0}{R_1} + \frac{e_1 - e_2}{R_3} + \frac{e_1}{R_2} = 0$

$\Rightarrow (e_1 - V_0) G_1 + (e_1 - e_2) G_3 + e_1 \cdot G_2 = 0$ (1)

KCL @ e_2: $(e_2 - V_0) \cdot G_4 + (e_2 - e_1) \cdot G_3 + e_2 \cdot G_5 - I_1 = 0$ (2)

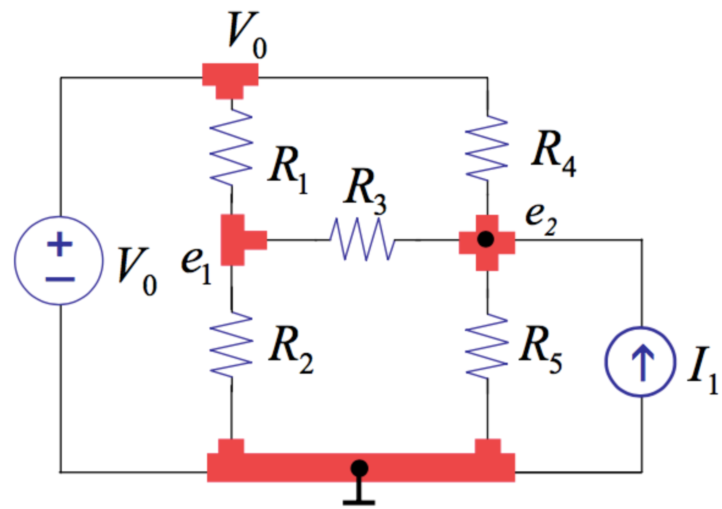
From (1) and (2), express e_1, e_2 as a function of $V_0, I_1, G_1, G_2, G_3, G_4, G_5$

Example – Using Node Method

4. Solve the n – 1 equations for node voltages

KCL at e_1 $(e_1 - V_0)G_1 + (e_1 - e_2)G_3 + (e_1 - 0)G_2 = 0$

KCL at e_2 $(e_2 - e_1)G_3 + (e_2 - V_0)G_4 + (e_2 - 0)G_5 - I_1 = 0$



$$\begin{cases} e_1(G_1+G_2+G_3) + e_2(-G_3) = V_0 \cdot G_1 \\ e_1(-G_3) + e_2(G_3+G_4+G_5) = V_0 \cdot G_4 + I_1 \end{cases}$$

- linear equations with 2 unknowns
- Check the units

Example – Using Node Method

5. Calculate all branch voltages and currents based on elements' laws.

$v = i \cdot R$