3/4/2019 (Mon)

$f = 60 Hz$

~~$f = \frac{1}{60} Hz$~~

amplitude

Vamp

$V_{rms} = \frac{1}{\sqrt{2}} V_{amp}$

- $v(t) = \sqrt{2} \cdot 110 \cdot \sin(2\pi \cdot 60t)$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \quad , \; T = \frac{1}{60} s$$

$$= \sqrt{60\int_0^{\frac{1}{60}} 2 \cdot 110^2 \cdot \sin^2(2\pi 60t)\,dt}$$

$$= \sqrt{60 \cdot \int_0^{\frac{1}{60}} 2 \cdot 110^2 \cdot \frac{1}{2}\left(1 - \cos(2 \cdot 2\pi 60t)\right)dt}$$

$$= \sqrt{60 \cdot 110^2 \cdot \int_0^{\frac{1}{60}} 1 - \cos(2 \cdot 2\pi 60t)\,dt} = \sqrt{60 \cdot 110^2 \cdot (t - \sin 4\pi 60t)\Big|_0^{\frac{1}{60}}}$$

$$= \sqrt{60 \cdot 110^2 \cdot \frac{1}{60}} = 110 \text{ V}$$

- Review:

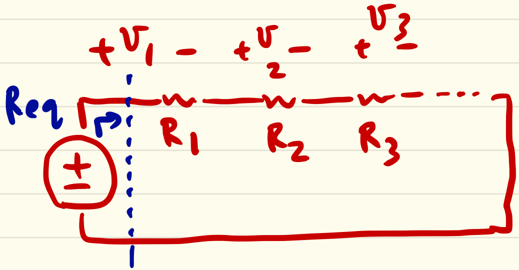
- KCL, KVL

1) label v's , i's

2) write v-i relationship of each component

3) write KVL, KCL

4) Solve KVL, KCL

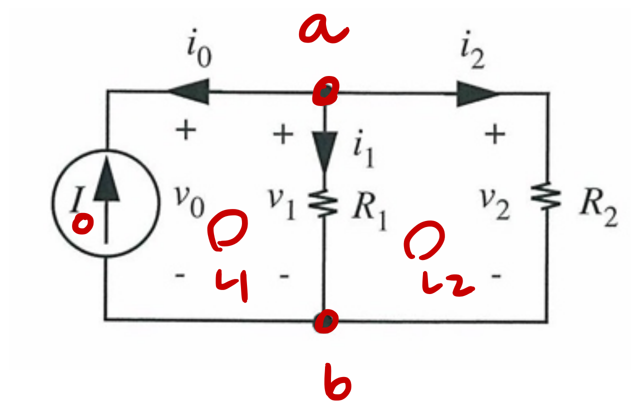


- Voltage divider: resistors in series

$$\frac{v_1}{v_2} = \frac{R_1}{R_2} \qquad \frac{P_1}{P_2} = \frac{R_1}{R_2}$$

$$R_{eq} = \sum_i R_i$$

Current Divider



- A node with >2 resistors and a current source in parallel.

1. Element relationship laws

$i_0 = -I_0$

$v_1 = i_1 \cdot R_1$

$v_2 = i_2 \cdot R_2$

2. KCL at nodes

node a: $i_0 + i_1 + i_2 = 0$

3. KVL for loops

L_1: $-v_0 + v_1 = 0$

L_2: $-v_1 + v_2 = 0$

$\Rightarrow$

$$i_1 = I_0 \frac{R_2}{R_1 + R_2}$$

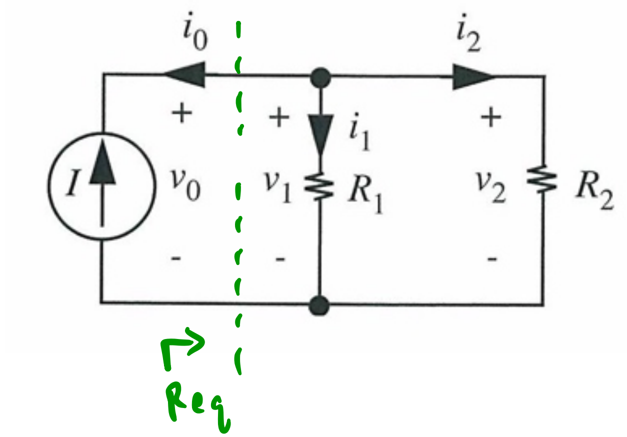
$$i_2 = I_0 \frac{R_1}{R_1 + R_2}$$

$$v_0 = v_1 = v_2 = \frac{I_0}{\frac{1}{R_1} + \frac{1}{R_2}}$$

let $\frac{1}{R_1} = G_1$, $\frac{1}{R_2} = G_2$, $v_1 = v_2 = \frac{I_0}{G_1 + G_2}$

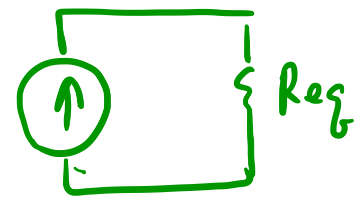
R: resistance (Ω), G: conductance ($1/\Omega$)

Current Divider



- Current division
  - The two resistors divide the current I in proportion to their conductance.

$$\frac{i_1}{i_2} = \frac{G_1}{G_2} = \frac{R_2}{R_1}$$



- Power into each resistor

$$P_1 = v_1 \cdot i_1 = \frac{I_0^2 \cdot G_1}{(G_1+G_2)^2}, \quad P_2 = v_2 \cdot i_2 = \frac{I_0^2 \cdot G_2}{(G_1+G_2)^2}$$

$$\frac{P_1}{P_2} = \frac{G_1}{G_2}$$

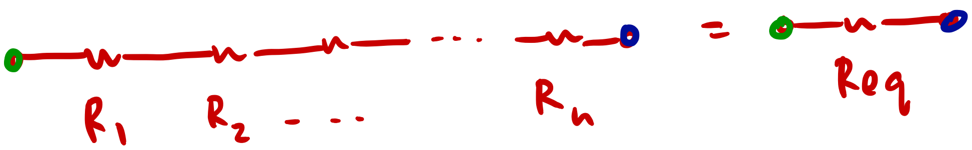
- Resistors in parallel
  - Equivalent conductance

$$G_{eq} = \sum_i G_i \qquad R_{eq} = \frac{1}{\sum_i \frac{1}{R_i}}$$

$$G_{eq} = G_1 + G_2, \quad R_{eq} = \frac{1}{G_{eq}} = \frac{1}{G_1 + G_2} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

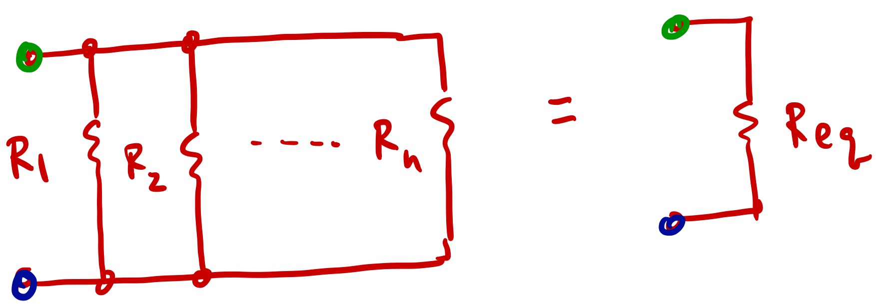
Element Combination Simplification

- Resistors in series



$$R_{eq} = \sum_i R_i \ , \ i = 1, 2, \dots n$$

- Resistors in parallel

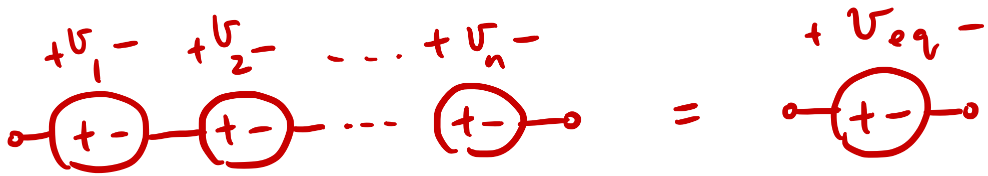


$$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{\sum_i G_i} \ , \ i = 1, 2, \dots n$$

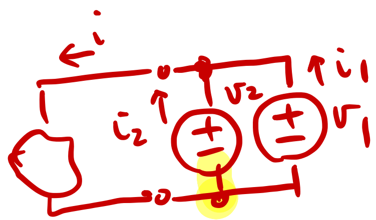
$$\frac{1}{R_{eq}} = \sum_k \frac{1}{R_k} \ , \ k = 1, 2, \dots n$$

Element Combination Simplification

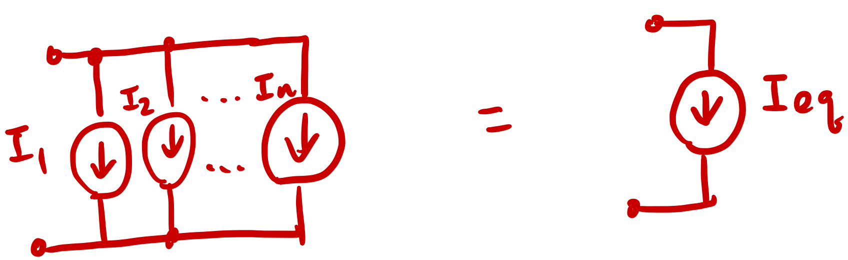
- Voltage sources in series



$v_{eq} = \sum_k v_k \,, \quad k = 1, 2, \ldots n$

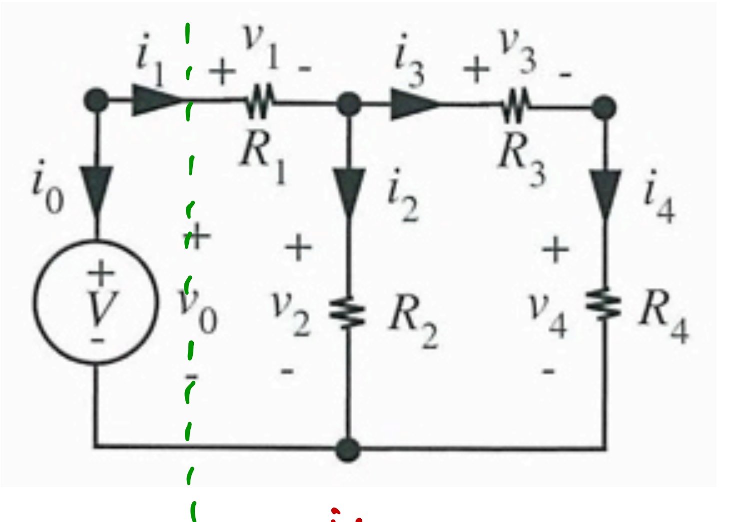


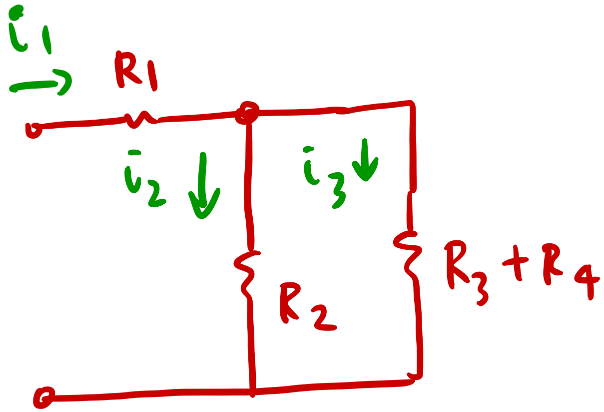
- Current sources in parallel



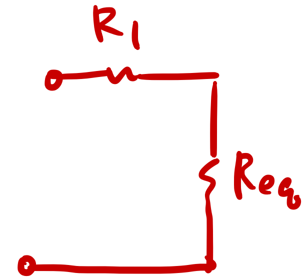
$I_{eq} = \sum_k I_k \,, \quad k = 1, 2, \ldots n$

Circuit Analysis with Element Combination Technique

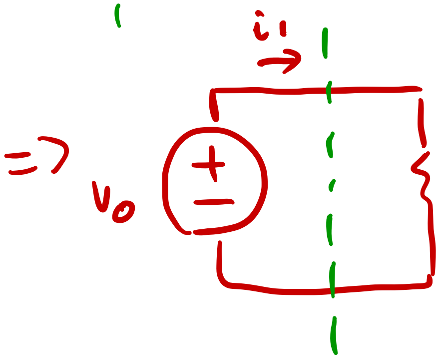




$\Rightarrow$



$$R_{eq1} = \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}$$



$$R_{eq} = R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}, \quad i_1 = \frac{V_0}{R_{eq}}$$

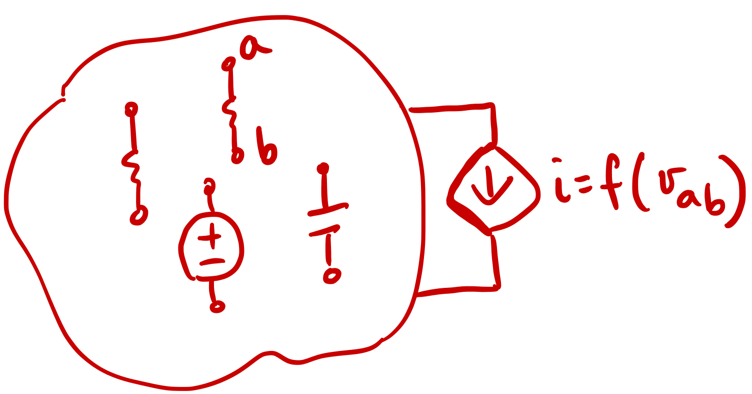
$$v_1 = i_1 \cdot R_1$$

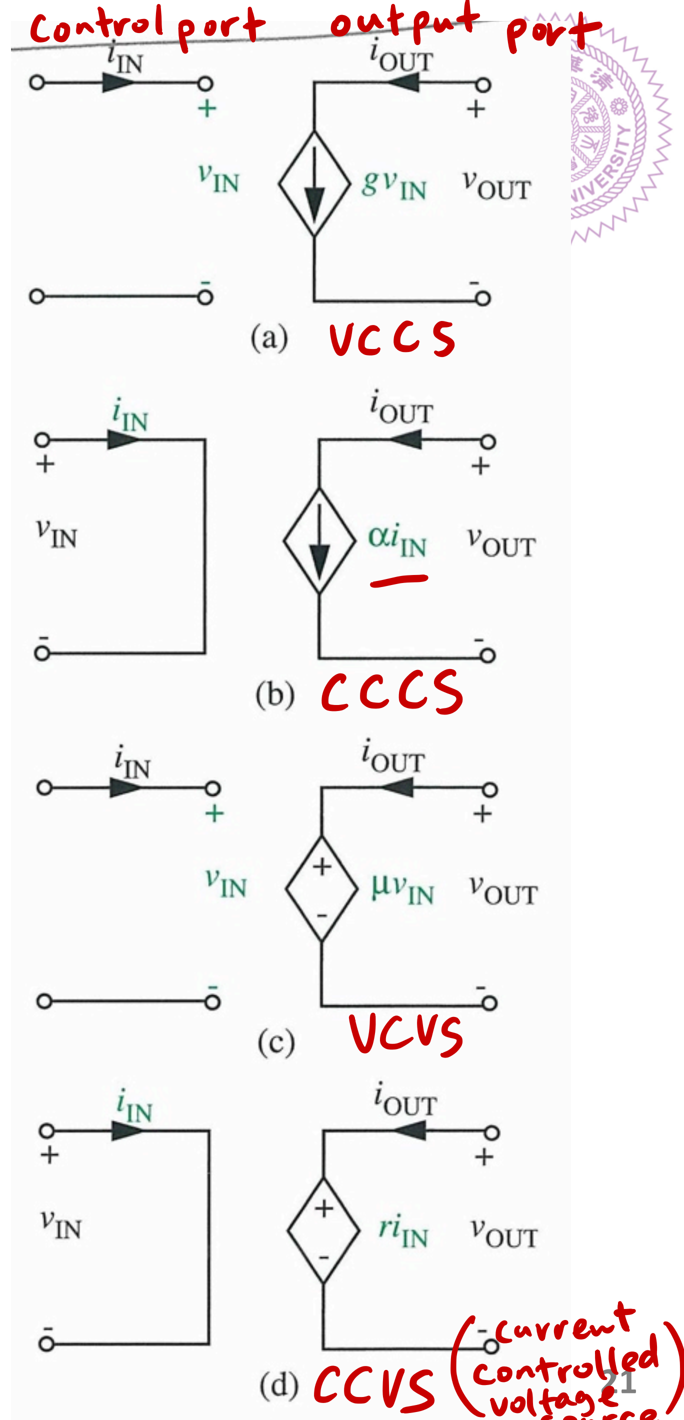
$$v_2 = V_0 - v_1, \quad i_2 = \frac{v_2}{R_2}$$

$$i_3 = i_4 = i_1 - i_2, \quad v_3 = i_3 \cdot R_3, \quad v_4 = i_4 \cdot R_4$$

Dependent Sources (can be linear or nonlinear) 2-port element

- Independent sources: the values (voltages, currents) are independent of circuit operation. (1 port element)
- Dependent sources: the values are controlled by some other parameters in the system.



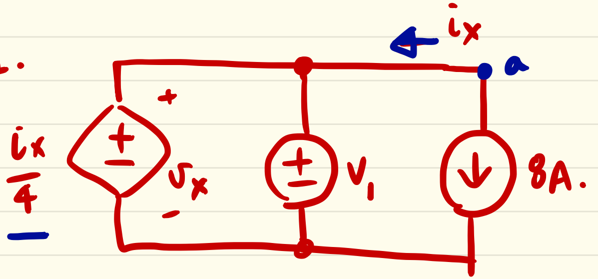


Dependent source

Example 1.

Find V_1.

CCVS



KCL: $8 + i_x = 0$

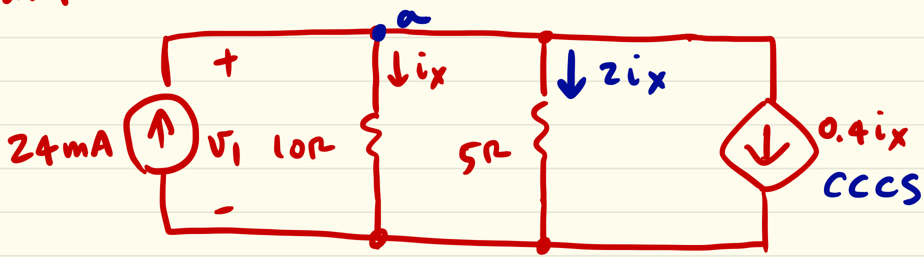
$$v_x = \frac{i_x}{4}$$

$$i_x = -8\,A \quad , \quad v_x = \frac{-8}{4} = -2\,V$$

$$V_1 = -2\,V$$

Example 2.

Find v_1



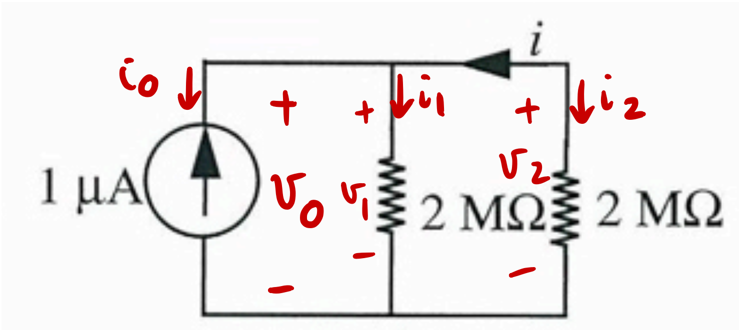
KCL

node a: $24\,mA = i_x + 2i_x + 0.4i_x$

$\Rightarrow i_x = 7.05\,mA$

$v_1 = i_x \cdot 10 = 70.5\,mV$

Circuit Analysis Example I



1) $i_o = -1\ \mu A$

$v_1 = i_1 \cdot 2M$

$v_2 = i_2 \cdot 2M$

2) KCL: $i_o + i_1 + i_2 = 0$

3) KVL: $v_o = v_1$

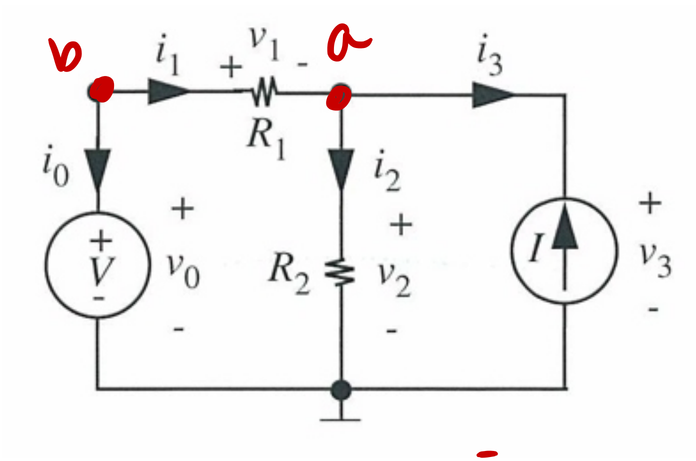
$v_1 = v_2$

4) Solve equations

$i_1 = i_2 = -\frac{1}{2} i_o = 0.5\ \mu A$

$v_o = v_1 = v_2 = 0.5\ \mu A \cdot 2M\Omega = 1\ V$

Circuit Analysis Example II



1) $v_0 = V$

$v_1 = i_1 R_1$

$v_2 = i_2 R_2$

$i_3 = -I$

2) KCL :

node a : $i_1 - i_2 - i_3 = 0$

node b : $i_0 + i_1 = 0$

3) KVL

L1 : $v_0 - v_1 - v_2 = 0$

L2 : $v_2 - v_3 = 0$

4)

$$i_0 = -i_1 = \frac{R_2}{R_1 + R_2} I - \frac{1}{R_1 + R_2} V$$

$$i_2 = \frac{R_1}{R_1 + R_2} I + \frac{1}{R_1 + R_2} V$$

$$i_3 = -I$$

$$v_0 = V$$

$$v_1 = -\frac{R_1 R_2}{R_1 + R_2} I + \frac{R_1}{R_1 + R_2} V$$

$$v_2 = v_3 = \frac{R_1 R_2}{R_1 + R_2} I + \frac{R_2}{R_1 + R_2} V$$