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Electric Circuits
Lecture 2 Resistive Networks

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Lecture Outline

- Review
- Chapter 2 in the textbook
  - Kirchoff's laws (KCL, KVL)
  - Voltage and current dividers
  - Series and parallel simplification
  - Dependent sources
  - Circuit analysis examples

Review

- Lumped circuit element
  - Element described by its v-i relation
  - Power consumed by element is $v \cdot i$
- Lumped matter discipline (LMD)

1) - $\frac{\partial \phi_B}{\partial t} = 0$ outside elements

2) - $\frac{\partial q}{\partial t} = 0$ inside elements

3) - Signal speeds of interest should be way lower than speed of light.

- Maxwell's equations simplify to algebraic KVL & KCL under LMD.
  - Kirchhoff's Voltage Law (KVL): $\sum_k v_k = 0$ for loop.
  - Kirchhoff's Current Law (KCL): $\sum_j i_j = 0$ for node.

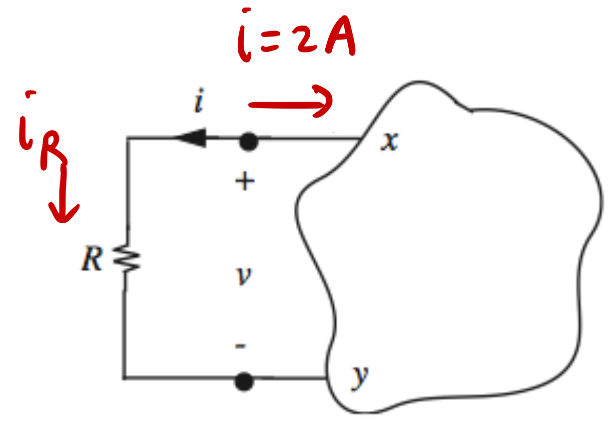
Example 1 –
Terminal Variables and Power into a Resistor

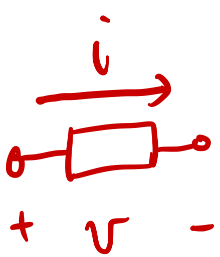
- A resistor (10 Ohms) is connected to an arbitrary circuit at points x and y. Assume the current flowing into the network at node x is 2 A.

$i_R = -2\,A$

$v = R \cdot i_R = 10 \cdot (-2) = -20\,V$

$p = v \cdot i_R = (-20) \cdot (-2) = 40$ Watt (Joule/sec)





$p = v \cdot i$

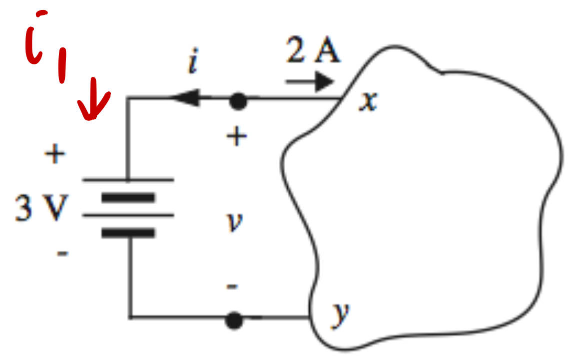
Example 2 –
Power Supplied by a Battery

- The same example but replaced with a 3-V battery.

$\bar{i}_1 = -2A$

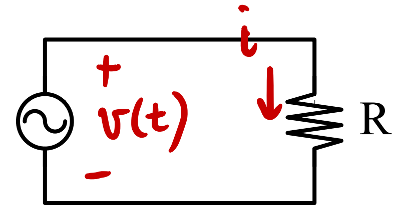
$v = 3V$

$p = v \cdot \bar{i} = -6W$



Example 3 – AC (alternating current) Power

$\begin{cases} dc \\ ac \end{cases}$

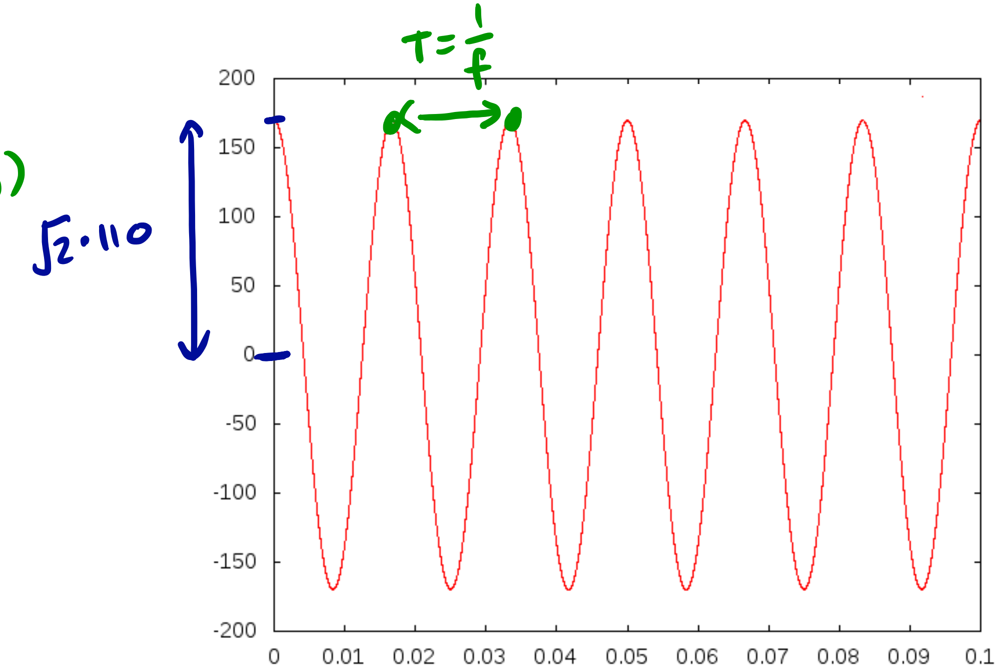


- 110-V 60-Hz AC voltage , $R = 50\Omega$

$v(t) = \underbrace{\sqrt{2}\cdot 110}_{\text{amplitude (V)}} \cdot \sin(2\pi \cdot \underbrace{60}_{\text{frequency (Hz)}} t)$

$i(t) = \frac{v(t)}{50}$

$p(t) = \frac{110^2}{50}\left(1 - \cos(2\pi\cdot 120 t)\right)$



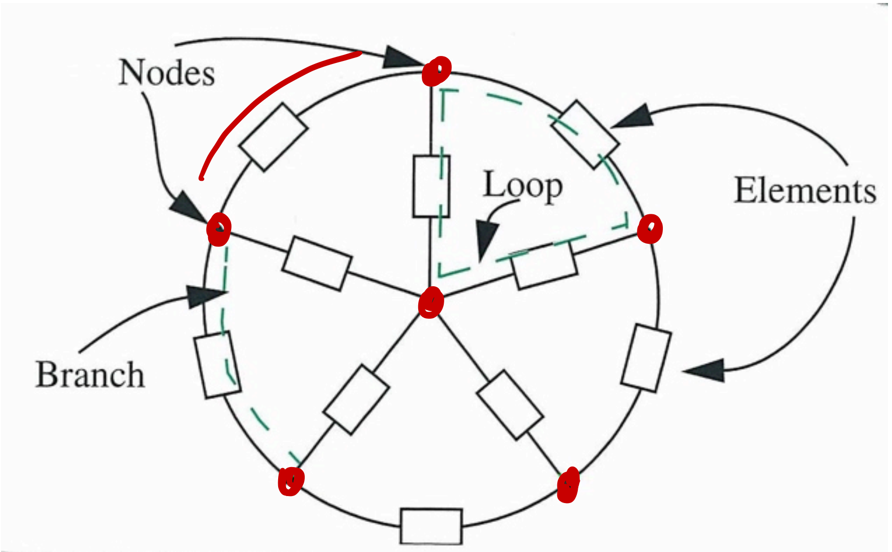
- What would be the power dissipated by the resistor if the voltage was a constant value of 110 V?

$v(t) = 110V \qquad p(t) = 110 \cdot \frac{110}{50}$

Chapter 2 Resistive Networks

Terminology

- Element is accessed through its terminals.
- Node: the junction point where the terminals of two or more elements are connected.
- Branch: the connection between nodes.
- Loop: a closed path through a circuit along its branches.

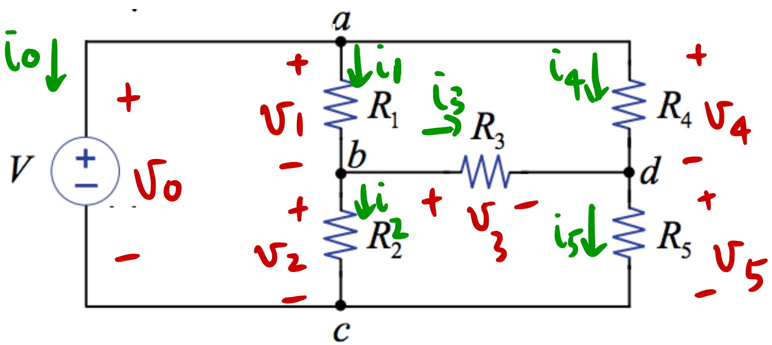


6 nodes

10 branches (10 elements)

Basic KVL/KCL Method of Circuit Analysis

- Analyzing a circuit means to find out all the element v's and i's.



6 elements, 12 unknowns

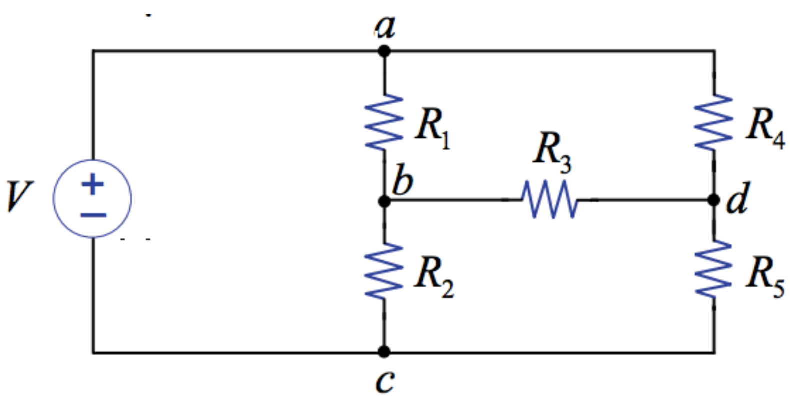
($v_0, v_1, \dots v_5, i_0, i_1, \dots i_5$)

1. Label all elements' v's and i's.
2. Write element v-i relationship.
   - For voltage source
   - For current source
3. Write KCL for all nodes.
4. Write KVL for all loops.

- Basically lay out all equations…

Basic KVL/KCL Method of Circuit Analysis

- Goal: find out all element v's and i's (12 unknowns).
- Step 2: Write v-i relation for all elements.



6 equations

$v_0 = V$

$v_1 = i_1 \cdot R_1$

$v_2 = i_2 \cdot R_2$

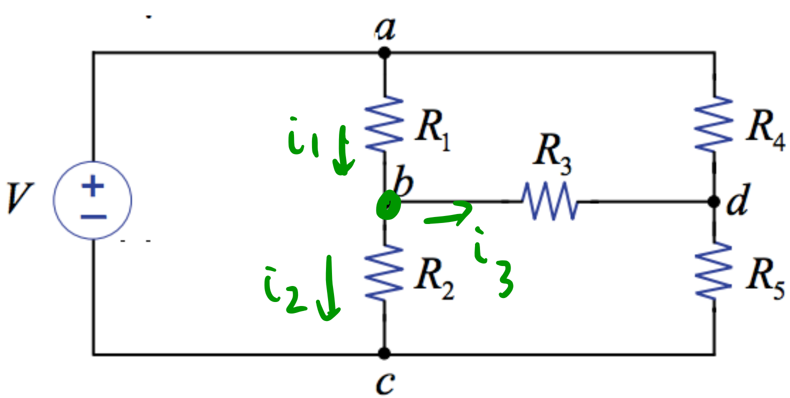
$v_3 = i_3 \cdot R_3$

$v_4 = i_4 \cdot R_4$

$v_5 = i_5 \cdot R_5$

Basic KVL/KCL Method of Circuit Analysis

- Goal: find out all element v's and i's (12 unknowns).
- Step 3: Apply KCL at the nodes.
  - Use convention: e.g., sum currents leaving the node.



node a: $i_0 + i_1 + i_4 = 0$

node b: $i_2 + i_3 - i_1 = 0$

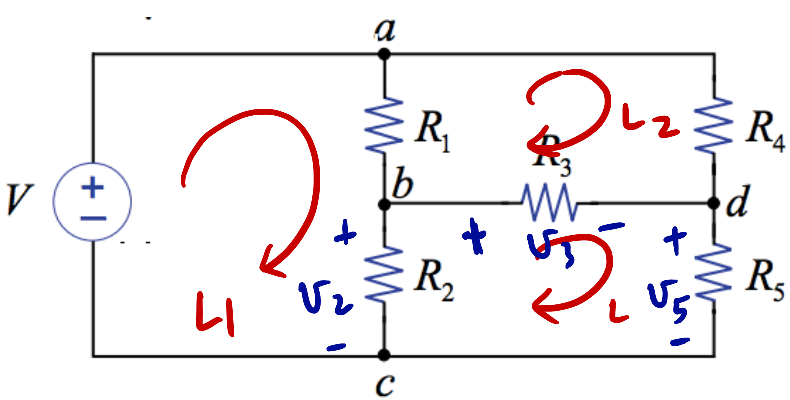
node c: $-i_0 - i_2 - i_3 = 0$

node d: $-i_3 - i_4 + i_5 = 0$

4 equations, 3 independent equations

Basic KVL/KCL Method of Circuit Analysis

- Goal: find out all element v's and i's (12 unknowns).
- Step 4: Apply KVL for the loops
  - Use convention: e.g., as you go around the loop, assign first encountered sign to each voltage.



$L_1: \; v_1 + v_2 - v_0 = 0$

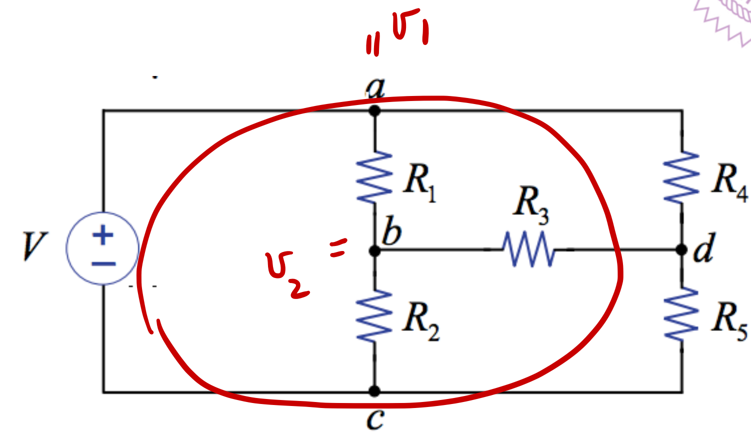
$L_2: \; v_4 - v_3 - v_1 = 0$

$L_3: \; v_5 - v_2 + v_3 = 0$

3 independent equations

Basic KVL/KCL Method of Circuit Analysis

- 12 equations for 12 unknowns.
  - 1. Element v-i relationships



$$v_0 = V_0 \qquad v_3 = i_3 \cdot R_3$$
$$v_1 = i_1 \cdot R_1 \qquad v_4 = i_4 \cdot R_4$$
$$v_2 = i_2 \cdot R_2 \qquad v_5 = i_5 \cdot R_5$$

  - 2. KCL at the nodes

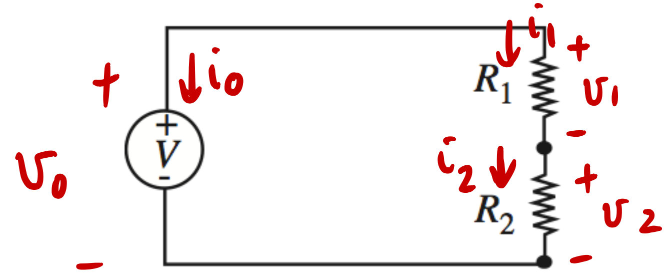
$$a: \quad i_0 + i_1 + i_4 = 0$$
$$b: \quad i_2 + i_3 - i_1 = 0$$
$$d \; \cancel{c}: \quad i_5 - i_3 - i_4 = 0$$
$$c \; \cancel{d}: \quad -i_0 - i_2 - i_5 = 0 \;\checkmark$$

3. KVL for the loops

$$L1: \quad -v_0 + v_1 + v_2 = 0$$
$$L2: \quad v_1 + v_3 - v_4 = 0$$
$$L3: \quad v_3 + v_5 - v_2 = 0$$
$$L4: \quad -v_0 + v_4 + v_5 = 0$$

- Circuit components are connected to each other on various nodes in the circuit.
  - (Using node voltages, instead of component voltages, as the main variables can reduce the computation complexity.)

Voltage Divider



- An isolated loop with >2 resistors and a voltage source in series.

1. Element relationship laws:

$v_0 = V_0$

$v_1 = i_1 \cdot R_1$

$v_2 = i_2 \cdot R_2$

2. KCL at nodes.

node a : $i_1 + i_0 = 0$

node b : $i_1 - i_2 = 0$

3. KVL for loops.

$-V_0 + v_1 + v_2 = 0$

$\Rightarrow$

$v_1 = V_0 \cdot \dfrac{R_1}{R_1 + R_2}$

$v_2 = V_0 \cdot \dfrac{R_2}{R_1 + R_2}$

$i_1 = i_2 = \dfrac{V_0}{R_1 + R_2}$

Voltage Divider

- Voltage division
  - The two resistors divide the voltage V in proportion to their resistance.

$$\frac{v_1}{v_2} = \frac{R_1}{R_2}$$

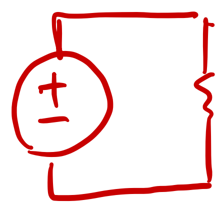
- Power into each resistor

$$P_1 = v_1 \cdot i_1 = \frac{V_0^2 \cdot R_1}{(R_1 + R_2)^2}$$

$$v_0 = V_0$$

$$P_2 = v_2 \cdot i_2 = \frac{V_0^2 \cdot R_2}{(R_1 + R_2)^2}$$

- Resistors in series
  - Equivalent resistance



R_{eq}

$$R_{eq} = \sum_i R_i$$