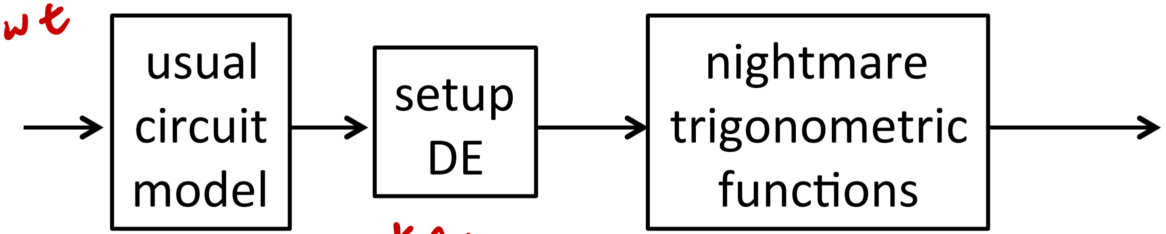
Review: Sinusoidal Steady State Analysis Approach

$V_i \cdot \cos\omega t$



KCL

KVL

node method

$\downarrow$

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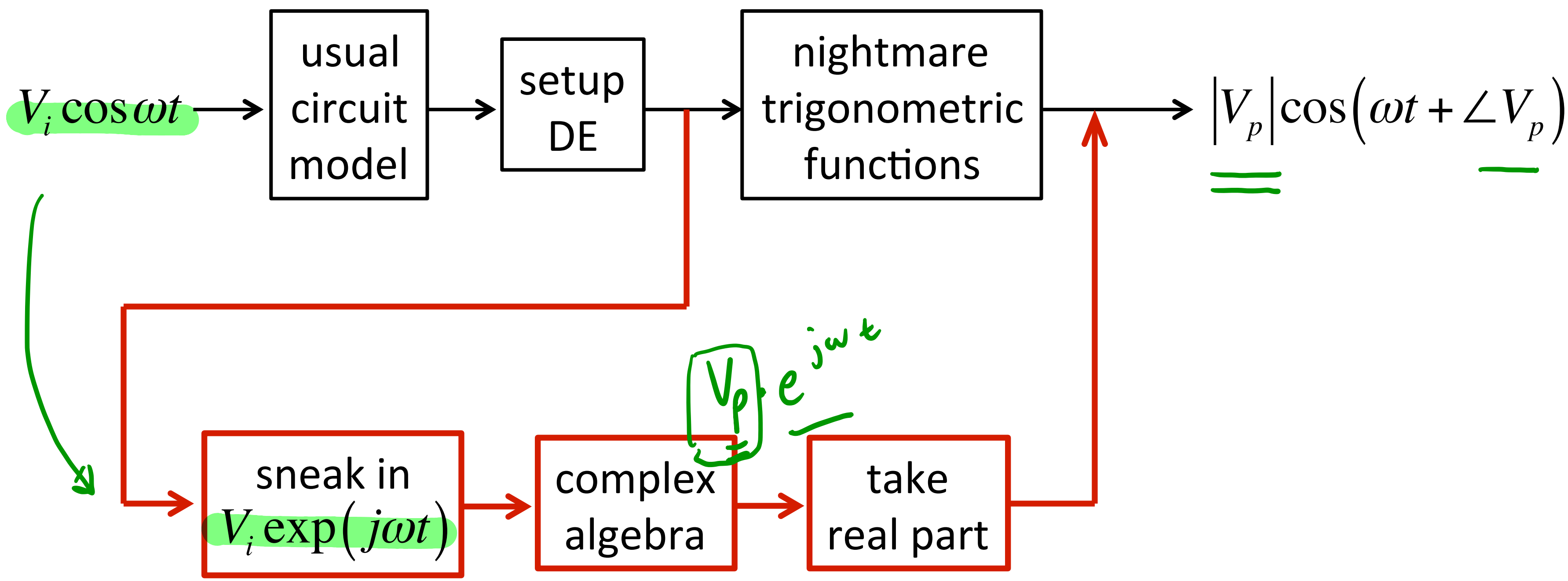
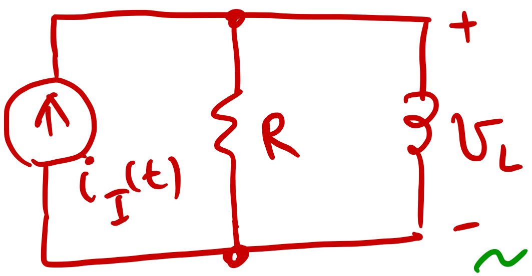


$$V_p = \frac{V_i}{1 + j\omega RC}$$ complex amplitude

- V_p contains all the information we need.
  - Complex amplitude gives the amplitude and phase of the output cosine.

Review

Practice



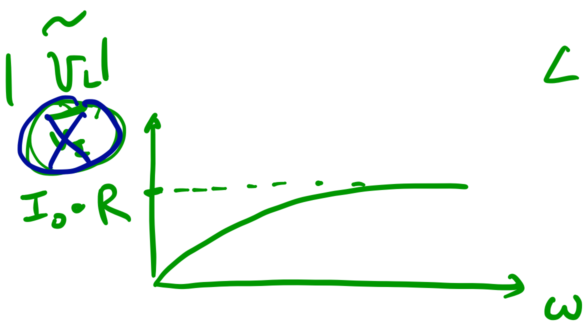
Find SSS of $v_L(t)$.

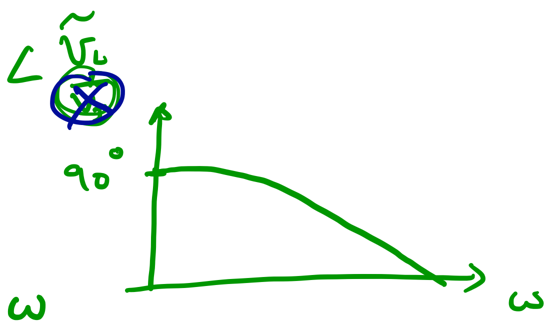
$i_I(t) = I_0 \cos\omega t \rightarrow \tilde{i}_I = I_0 \cdot e^{j\omega t}$

$i_I(t) \longrightarrow \tilde{i}_I \rightarrow \tilde{v}_L \longrightarrow v_L(t)$

$\tilde{i}_I = \frac{\tilde{v}_L}{R} + \frac{1}{L}\int \tilde{v}_L \, dt \rightarrow \frac{d}{dt} \rightarrow \tilde{v}_L$

$\Rightarrow v_L(t) = Re(\tilde{v}_L) = \frac{I_0 \cdot \omega R L}{\sqrt{R^2+\omega^2 L^2}} \cos\left(\omega t + \tan^{-1}\left(\frac{R}{\omega L}\right)\right)$





Review

Frequency response

1) magnitude plot

① sketch low freq asympotote $\omega \to 0$

② 〃 high freq $\omega \to \infty$

③ Two asympototes intersect at corner freq.

2) phase plot

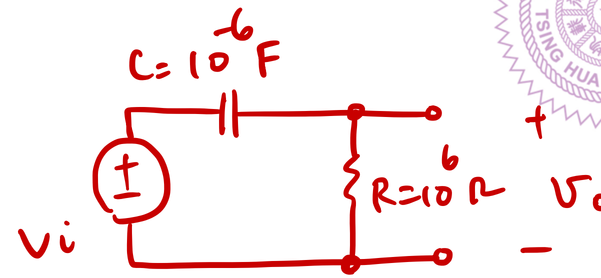
① 同 上 $\left(\frac{1}{10}\omega_c\right)$

② 向 上 $(10\omega_c)$

③ At corner freq, the phase is 45° or -45°

Review

Plot $|H(j\omega)|$ and $\angle H(j\omega)$, $H(j\omega) = \dfrac{V_o}{V_i}$



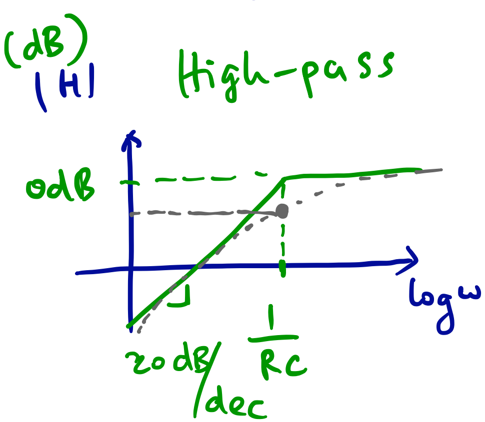
Assume $v_i(t) = 5\cos\omega t \rightarrow 5 \cdot e^{j\omega t}$

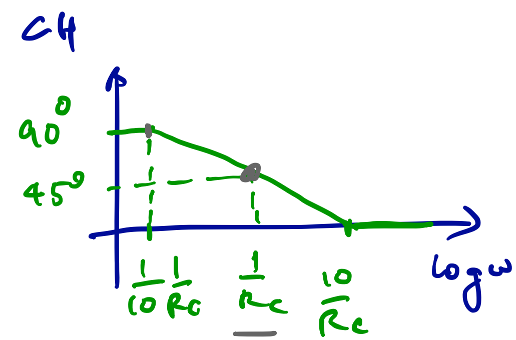
$$C \frac{d(v_i - v_o)}{dt} = \frac{v_o}{R} \Rightarrow \frac{dv_i}{dt} = v_o + \frac{dv_o}{dt} = 5 \cdot j\omega \cdot e^{j\omega t} \qquad \tan^{-1}\frac{\omega}{\omega^2}$$

particular

$$v_o = Ae^{j\omega t} \Rightarrow A = \frac{5j\omega}{1+j\omega} \quad , \quad H = \frac{v_o}{v_i} = \frac{\frac{5j\omega}{1+j\omega}}{5} = \frac{j\omega}{1+j\omega} = \frac{\omega^2 + j\omega}{(\quad)}$$

$$|H| = \frac{\omega}{\sqrt{1+\omega^2}} \quad , \quad \angle H = \tan^{-1}\left(\frac{1}{\omega}\right)$$





$\omega_c = \dfrac{1}{RC}$

There You Have It!

1. Replace the (sinusoidal) source by their complex (or real) amplitude.

$V_i \cos\omega t \rightarrow V_i e^{j\omega t}$

$R \rightarrow R, \quad C \rightarrow \frac{1}{j\omega C}, \quad L \rightarrow j\omega L$

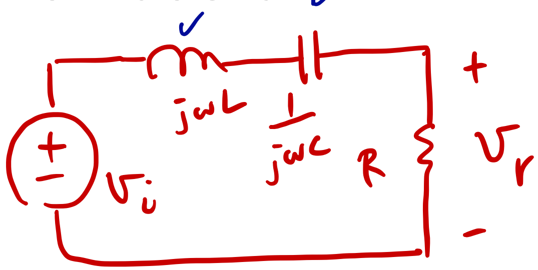
2. Replace circuit elements by their impedances. The resulting diagram is called the impedance model of the network.
3. Determine the complex amplitude of voltages and currents in the circuit using any standard circuit analysis method.
4. Obtain the time variables from the complex amplitudes. For example, for a voltage signal with complex amplitude of V_o :

$\Rightarrow \quad v_o(t) = |V_o| \cdot \cos(\omega t + \angle V_o)$

Another Example – Recall Series RLC Network

- Remember, we only want the steady-state response to sinusoid.

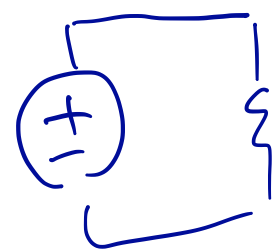
$v_i = V_i \cdot \cos\omega t \rightarrow V_i \cdot e^{j\omega t}$



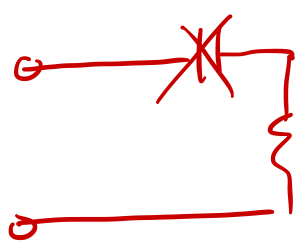
$v_r = |V_r| \cdot \cos(\omega t + \angle V_r)$

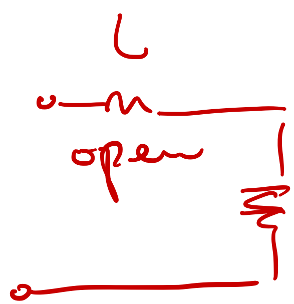
$$V_r = V_i \cdot \frac{R}{j\omega L + \frac{1}{j\omega C} + R}$$

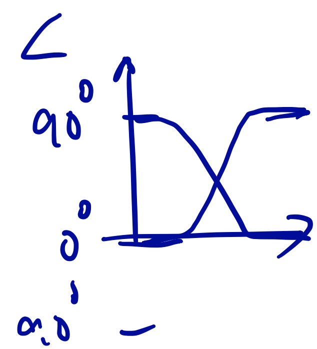
$\omega L = \frac{1}{\omega C}$



$j\omega L + \frac{1}{j\omega C} = 0$







Series RLC Network

H

- The transfer function $\dfrac{V_r}{V_i} = \dfrac{\dfrac{sR}{L}}{s^2 + \dfrac{sR}{L} + \dfrac{1}{LC}} = \dfrac{\dfrac{j\omega R}{L}}{-\omega^2 + \dfrac{j\omega R}{L} + \dfrac{1}{LC}}$

- Review complex algebra in Appendix C of textbook!!

Graphically

- Magnitude plot $\left|\dfrac{V_r}{V_i}\right| = \dfrac{\omega RC}{\sqrt{\left(1-\omega^2 LC\right)^2 + (\omega RC)^2}}$

ω

- At low ω:

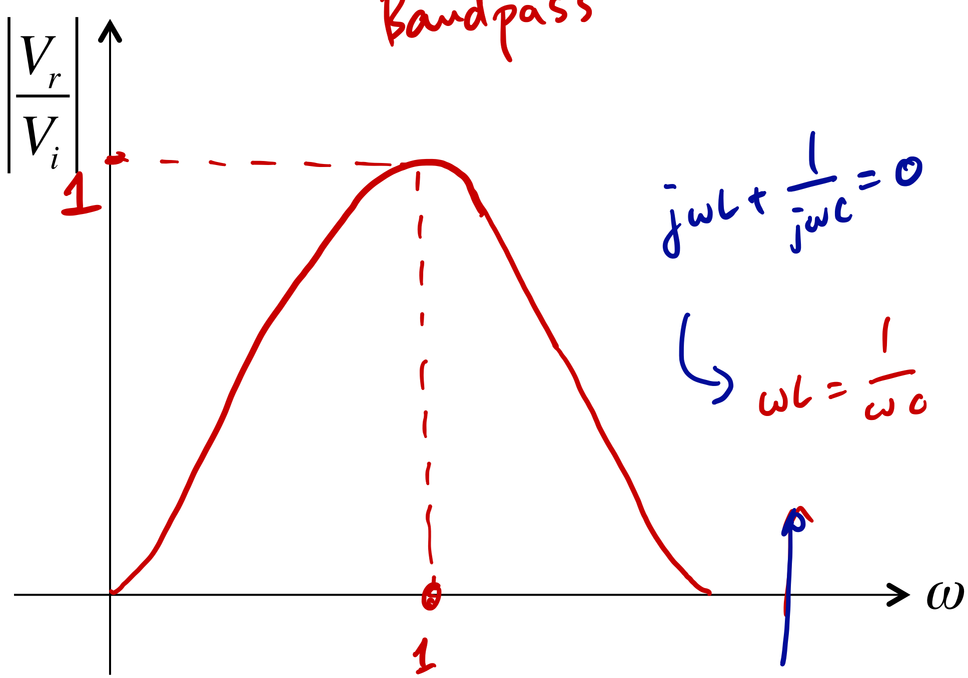
$|H| \approx \omega RC$

- At high ω:

$|H| \approx \dfrac{\omega RC}{\omega^2 LC} = \dfrac{R}{\omega L}$

- At $\omega = \dfrac{1}{\sqrt{LC}}$, $|H| \approx \dfrac{\omega RC}{\omega RC} = 1$

Bandpass



$j\omega L + \dfrac{1}{j\omega C} = 0$

$\hookrightarrow \omega L = \dfrac{1}{\omega C}$

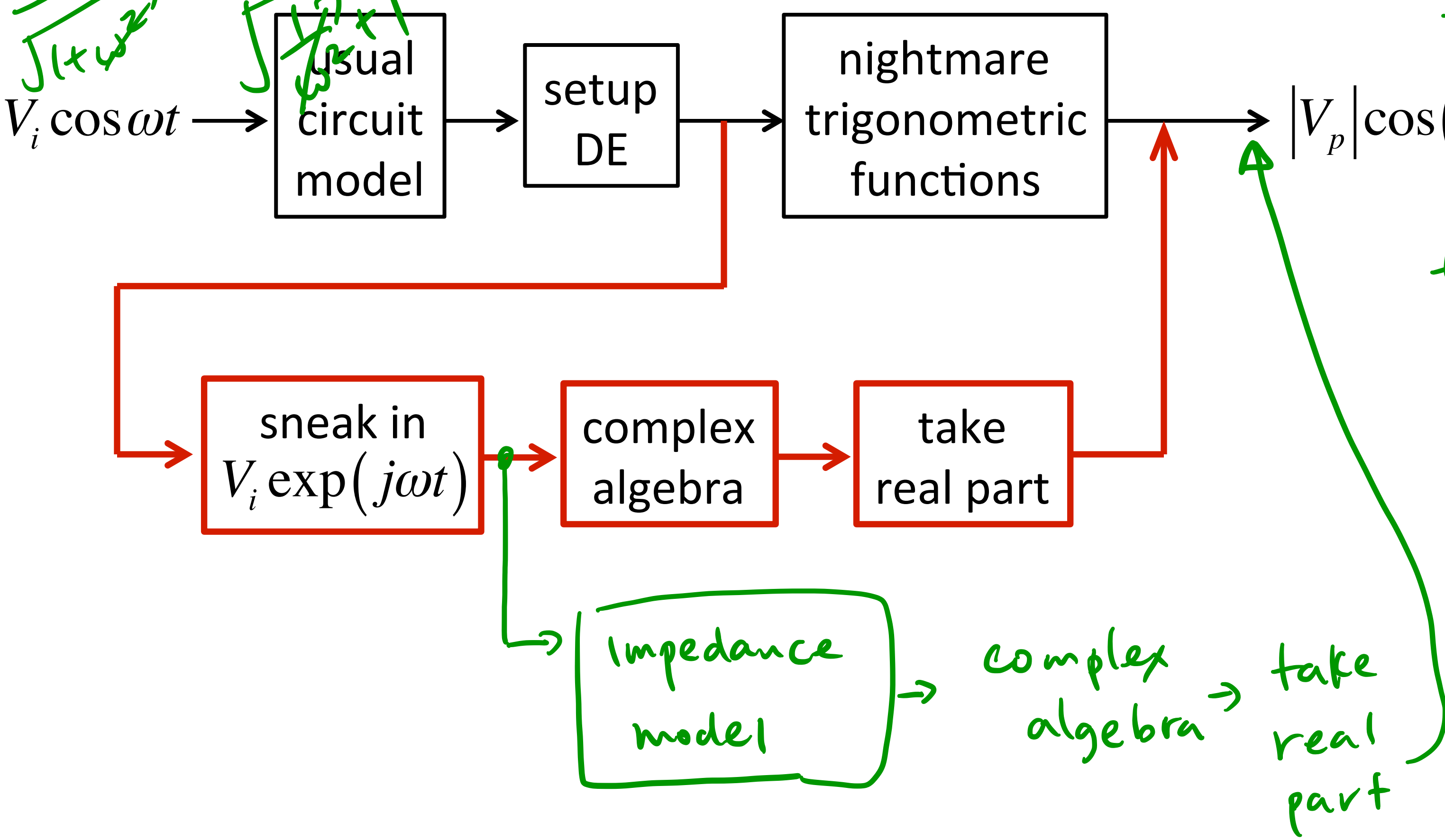
$\dfrac{1}{\sqrt{LC}}$: resonant freq

- Passes signals of frequencies in a middle band.

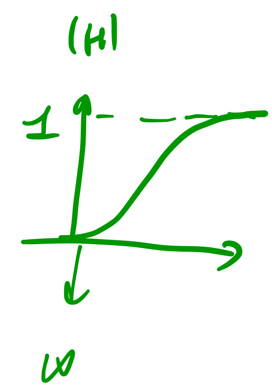
The Big Picture Again

$\frac{\omega}{\sqrt{1+\omega^2}} = \frac{1}{\sqrt{\frac{1}{\omega^2}+1}} = 1$ $\quad 20 \log 1 = 0\ \mathrm{dB}$

$\frac{j\omega(1-j\omega)}{(\ \ )(\ \ )} = \frac{\omega^2 + j\omega}{}$



$\tan^{-1} \frac{\omega}{\omega^2}$



- No differential equations!! No trigonometry!!