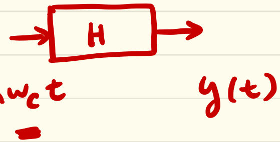
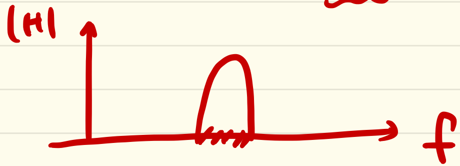
Frequency response



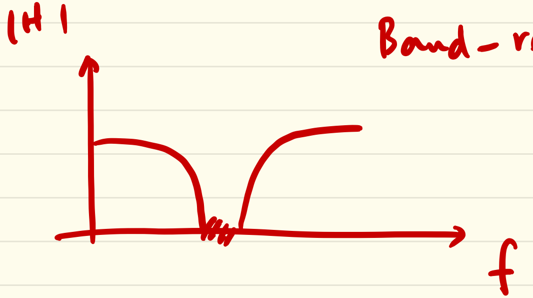
Low-pass

Band-pass

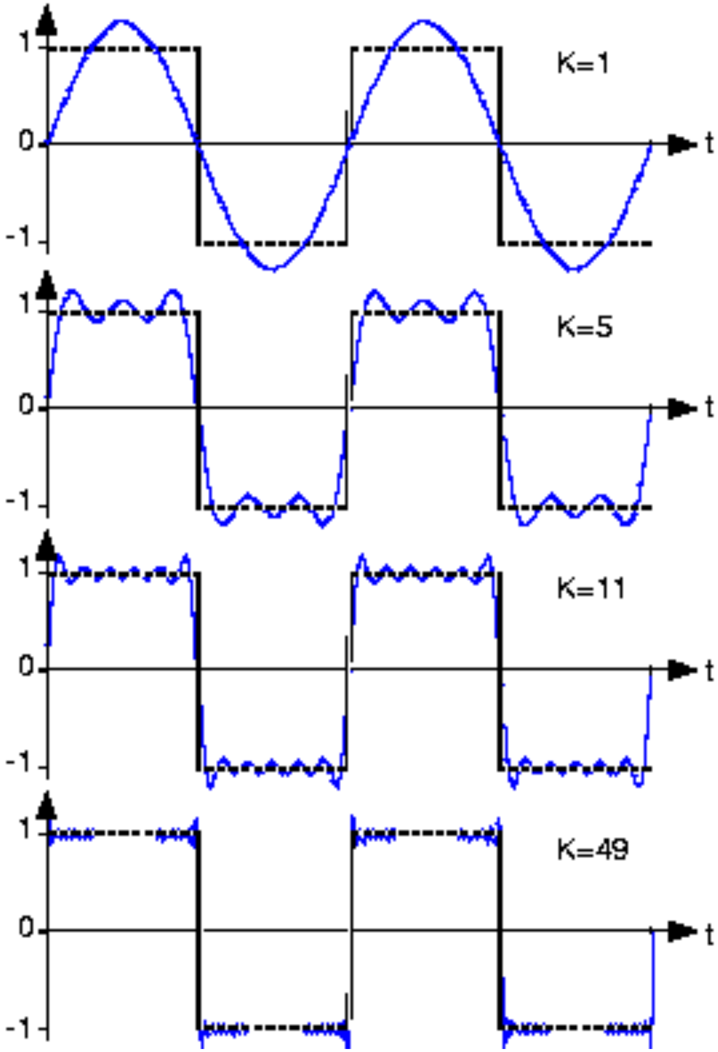


High-pass

Band-reject

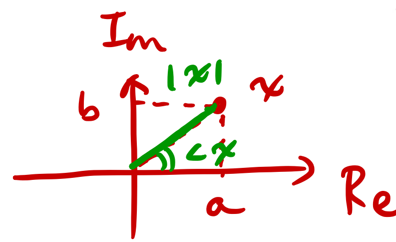


Square Wave



- For example, a square wave can be constructed by multiple (ideally infinite) sine waves at different frequencies.

Complex Numbers (Appendix C).



- $x = a + jb = |x| \angle x = \sqrt{a^2 + b^2} \cdot \tan^{-1} \frac{b}{a}$

  $= |x| \cos \angle x + j\,|x| \sin \angle x$

- x, y: complex numbers

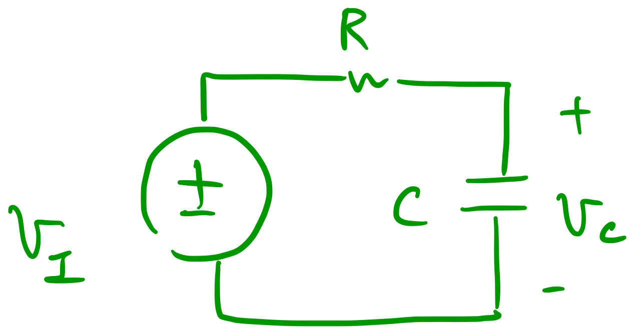
  $A = x \cdot y \Rightarrow |A| = |x| \cdot |y|, \quad \angle A = \angle x + \angle y$

  $B = \frac{x}{y} \Rightarrow |B| = \frac{|x|}{|y|}, \quad \angle B = \angle x - \angle y$

- $x(t) = |x| \cos(\omega t + \angle x) = \mathrm{Re}\left[|x| \cdot e^{j(\omega t + \angle x)} \right] \longrightarrow \overbrace{\underbrace{|x| \angle x}_{\text{complex amplitude}} \cdot\, e^{st}}^{\text{Phasor}}$

Sinusoidal Response of RC Network

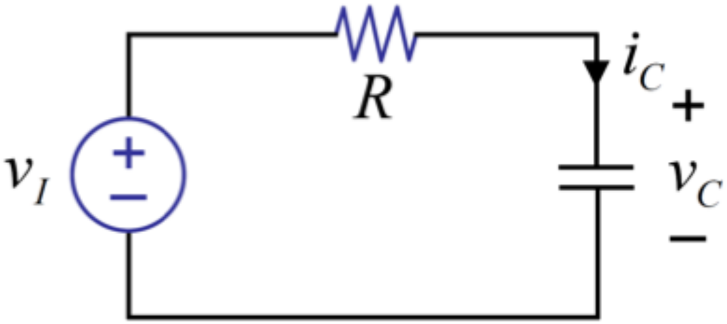
- Find $v_C(t)$ for t ≥ 0. Assume $v_I(t) = V_i \cos(\omega t)$ for t ≥ 0 and $v_I(t) = 0$ for t < 0. Initial condition $v_C(0) = 0$.



Our Usual Approach

1. Write DE for circuit by applying node method.
2. Find particular solution v_P by guessing and trial & error.
3. Find homogeneous solution $v_{H.}$
4. Total solution is $v_P + v_H$, then solve for remaining constants using initial conditions .

Step 1: Set Up the Differential Equation



1.. KCL @ v_c.

$$\frac{v_c - v_I}{R} + C\frac{dv_c}{dt} = 0$$

$$\Rightarrow \quad RC\frac{dv_c}{dt} + v_c = v_I = V_i \cdot \cos\omega t \quad \text{for } t \geq 0$$

Step 2: Find the Particular Solution

$$RC\frac{dv_P}{dt} + v_P = V_i\cos(\omega t)$$

Assume

1. $v_p = V_i \cdot \cos\omega t$ 代入: $-RC \cdot V_i \cdot \omega \cdot \sin\omega t + V_i \cdot \cos\omega t \neq V_i \cdot \cos\omega t$

2. $v_p = V_i \cdot \sin\omega t$ ✗

3. $v_p = V_i \cdot \cos(\omega t + \phi)$ ✗

Sneaky Approach

$$\boxed{RC\frac{dv_P}{dt} + v_P = V_i\cos(\omega t)}$$

- Instead of input $v_I(t) = V_i \cos\omega t$

→ Find particular solution for another input $v_{IS}(t) = V_i \exp(st)$

$$A\cos\omega t \rightarrow \boxed{\text{Linear}} \rightarrow \underline{\underline{y}} = \mathrm{Re}(\tilde{y})$$

$$A \cdot e^{j\omega t} \rightarrow \boxed{\text{Linear}} \rightarrow \tilde{y}$$

$$\mathrm{Im}(A e^{j\omega t}) \longrightarrow \mathrm{Im}(\tilde{y})$$

$$RC \cdot \frac{d\tilde{v}_p}{dt} + \tilde{v}_p = V_i \cdot e^{st} \quad (e^{st} = e^{j\omega t})$$

Assume $\tilde{v}_p = V_p \cdot e^{st}$

$$\Rightarrow RC \cdot V_p \cdot s \cdot e^{st} + V_p \cdot e^{st} = V_i \cdot e^{st}$$

$$\Rightarrow e^{st}(sRCV_p + V_p - V_i) = 0$$

$$\Rightarrow V_p = \frac{V_i}{1 + sRC}, \quad s \neq \frac{-1}{RC}$$

Sneaky Approach

- Assume $s = j\omega$, then particular solution for input $v_{IS}(t) = V_i \exp(st)$

$$\tilde{v}_p = V_p \cdot e^{st} = \underbrace{\frac{V_i}{1+sRC}}_{\text{complex amplitude}} \underbrace{e^{st}}_{\text{complex exponential}}$$

- Finding the particular solution to $v_{IS}(t) = V_i \exp(j\omega t)$ was easy.
- From Euler relation $v_{Is}(t) = V_i \exp(j\omega t) = V_i \cos(\omega t) + j \sin(\omega t)$

- An inverse superposition argument, assuming system is real and linear.

Sneaky Approach

$v_P(t)$ particular response to $V_i \cos \omega t$

$v_{PS}(t)$ particular response to $V_i \exp(j\omega t)$

- Let's try to find v_P from v_{PS}:

$$v_p(t) = \mathrm{Re}(\tilde{v}_p) = \mathrm{Re}\left(\frac{V_i}{1+j\omega RC} \cdot e^{j\omega t} \right)$$

$$= \mathrm{Re}\left(\frac{V_i}{\sqrt{1+(\omega RC)^2}} \cdot e^{j\tan^{-1}(-\omega RC)} \cdot e^{j\omega t} \right)$$

$$= \frac{V_i}{\sqrt{1+(\omega RC)^2}} \cdot \cos\left(\omega t + \tan^{-1}(\omega RC) \right)$$

Step 3: Homogeneous Solution

- Recalled from Chapter 10, the homogeneous solution for RC circuit v_H: $v_H(t) = A \cdot e^{-t/RC}$

- The total solution:

$$v_C(t) = v_P(t) + v_H(t) = \frac{V_i}{\sqrt{1+(\omega RC)^2}} \cdot \cos(\omega t + \tan^{-1}(\omega RC)) + A \cdot \exp(\frac{-t}{RC})$$

Given

$v_c(t=0) = 0 \Rightarrow A = -\frac{V_i}{\sqrt{1+(\omega RC)^2}} \cdot \cos\left(-\tan^{-1}(\omega RC)\right)$

Sinusoidal Steady State

Exercise 12.2

- The total solution

$$v_C(t) = \frac{V_i}{\sqrt{1+(\omega RC)^2}}\cos(\omega t+\phi) - \frac{V_i}{\sqrt{1+(\omega RC)^2}}\cos\phi \cdot e^{\frac{-t}{RC}}$$

where $\phi = \tan^{-1}(-\omega RC)$

- We are usually interested only in the particular solution for sinusoids, i.e., after the transients have died.

動態 ⟷ steady-state

- When $t \to \infty$

$$\underbrace{v_C(t) = \frac{V_i}{\sqrt{1+(\omega RC)^2}}\cdot\cos(\omega t+\tan^{-1}(\omega RC))}_{\text{Steady-state response}}$$