EE2210 Lecture 8B:
Damped Second-Order Systems

Chapter 12 of textbook

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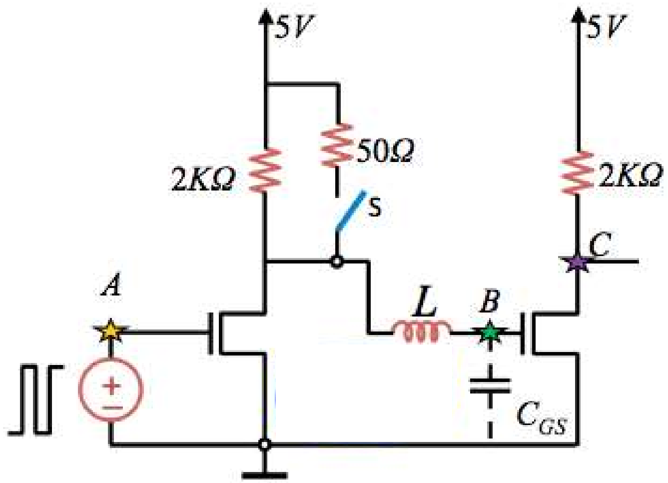
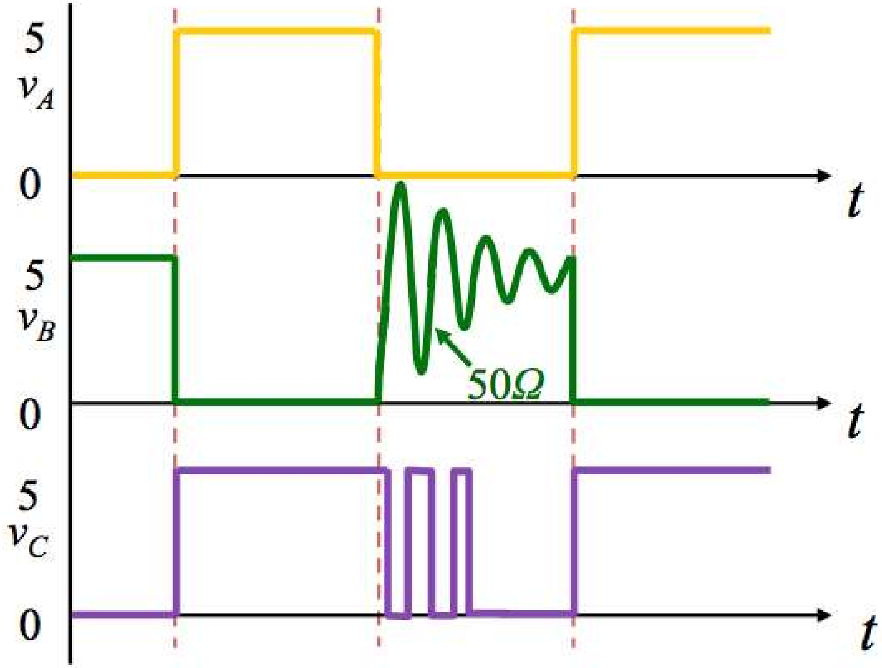
Review

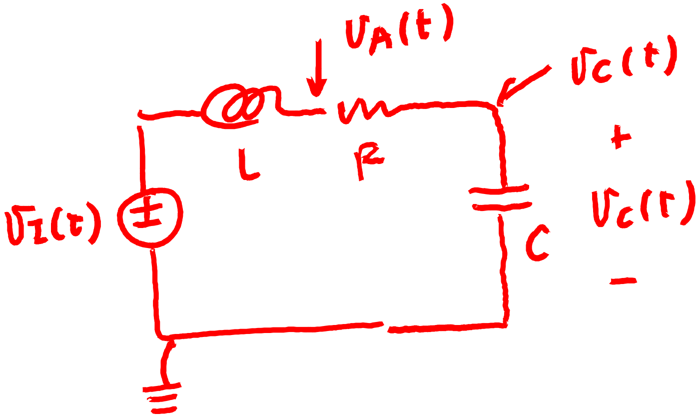




LC Circuit in Last Lecture…

- We solved
- For input
- and initial conditions
- Total solution:

Let's Analyze the RLC Network (Damped Oscillator)



two energy-storage element

$v_C(t)$, $i_L(t)$

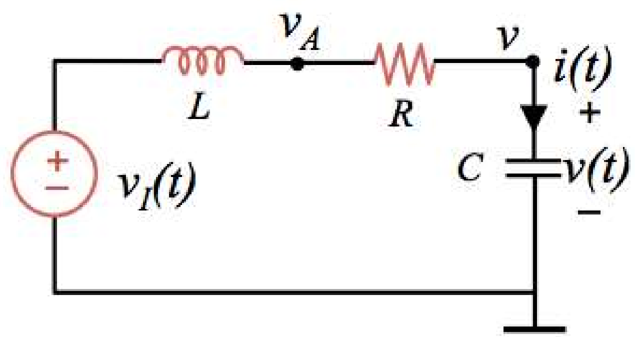
- **Node method:**

KCL @ v_C → ↓ $\frac{v_A - v_C}{R} = C\frac{dv_C}{dt}$

@ v_A → → $\frac{1}{L}\int_{-\infty}^{t}(v_I - v_A)\,dt = \frac{v_A - v_C}{R}$

we focus on $v_C(t)$, need to get rid of v_A

Setup the Differential Equation



- **Need to get rid of v_A**

$$\frac{v_A - v}{R} = C\frac{dv}{dt} \Rightarrow v_A = RC\frac{dv}{dt} + v$$

$$\frac{1}{L}\int_{-\infty}^{t}(v_I - v_A)dt = \frac{v_A - v}{R}$$

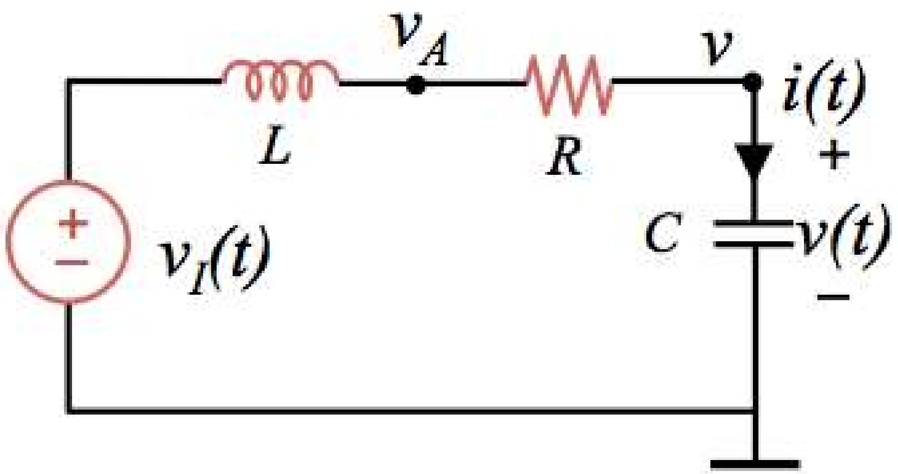
$$\frac{1}{L}\int_{-\infty}^{t}(v_I - v_A)\,dt = C\frac{dv}{dt}$$

$$\frac{1}{L}(v_I - v_A) = C\frac{d^2v}{dt^2}$$

代入 v_A $\quad \frac{1}{L}\left(v_I - RC\frac{dv}{dt} - v\right) = C\frac{d^2v}{dt^2}$

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{LC}v_I$$

Setup the Differential Equation Differently



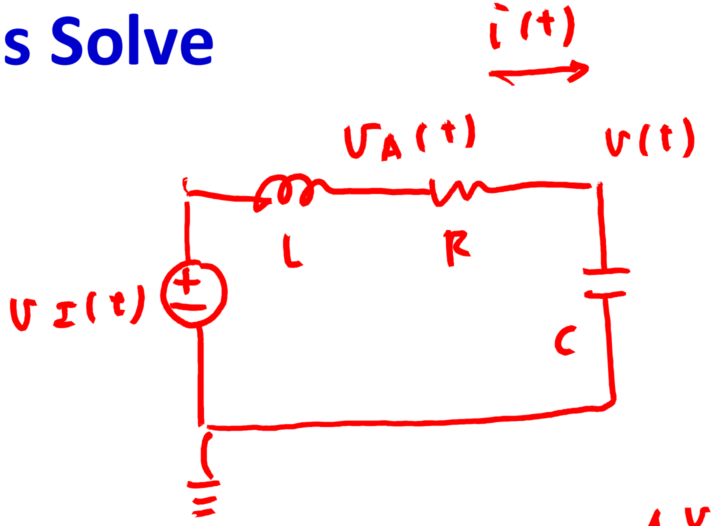
Method of Particular and Homogeneous Solutions

- **Four-step procedure**

$$\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{1}{LC}v=\frac{1}{LC}v_I$$

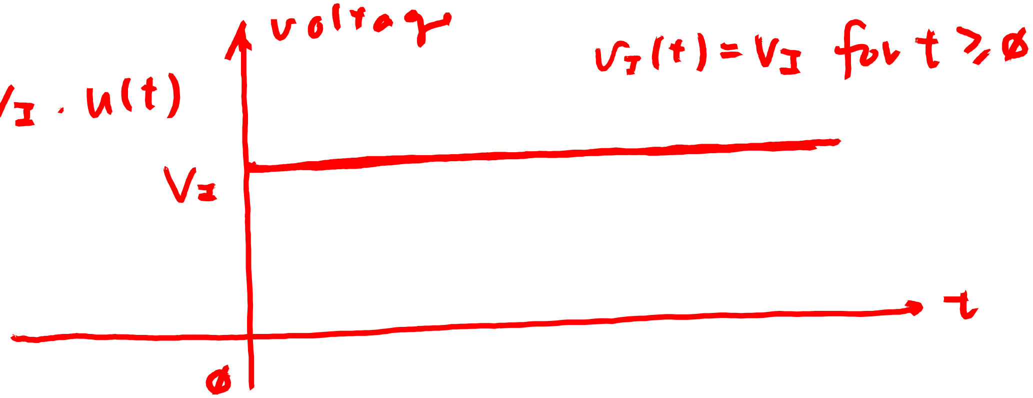
1. **Find the particular solution** $v_P(t)$
2. **Find the homogeneous solution** $v_H(t)$
   - Four-step procedure
3. **The total solution is the sum of the particular solution and homogeneous solution**
4. **Use initial condition to solve for the remaining constraints**

Let's Solve



$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{LC}v_I$$

For $v_I(t) = V_I \cdot u(t)$



For $v(0) = 0$, $i(0) = 0$ } zero-state response

Want to solve $v(t)$ for $t \geq 0$

1. Particular Solution

V_I

$$\boxed{\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{LC}v_I}$$

猜 $v_p(t) = V_I$

2. Homogeneous Solution

- Look for solution to

$$\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{1}{LC}v=\frac{1}{LC}v_I$$

$$\frac{d^2 v_H(t)}{dt^2}+\frac{R}{L}\frac{d v_H(t)}{dt}+\frac{1}{LC}v_H(t)=0$$

- Four-step method

2A 猜 $v_H(t)=A\cdot e^{st}$ $A\neq 0$ (再求 A, s)

2B $\cancel{A}\cdot s^2\cdot \cancel{e^{st}}+\frac{R}{L}\cancel{A}\cdot s\cdot \cancel{e^{st}}+\frac{1}{LC}\cancel{A}\cdot \cancel{e^{st}}=0$

char. eq. $s^2+\frac{R}{L}s+\frac{1}{LC}=0$

2C 得 s_1, s_2

2D $v_H(t)=A_1\cdot e^{s_1 t}+A_2\cdot e^{s_2 t}$

2. Homogeneous Solution

$$\frac{d^2 v_H(t)}{dt^2} + \frac{R}{L}\frac{dv_H(t)}{dt} + \frac{1}{LC}v_H(t) = 0$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\Rightarrow \quad s^2 + 2\cdot\frac{R}{2L}s + \frac{1}{LC} = 0$$

$$\Rightarrow \quad s^2 + 2\alpha s + \omega_0^2 = 0$$

$$\left|\begin{array}{l} \alpha = \dfrac{R}{2L} \\ \omega_0^2 = \dfrac{1}{LC} \end{array}\right.$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

3. Total Solution

4. Find unknowns from initial conditions

Let's Stare at the Total Solution for a While Longer

$$v(t) = V_I + A_1 e^{-\alpha t} e^{\left(\sqrt{\alpha^2 - \omega_0^2}\right)t} + A_2 e^{-\alpha t} e^{\left(-\sqrt{\alpha^2 - \omega_0^2}\right)t}$$

- **There are 3 possible cases:**

$\alpha = \frac{R}{2L}$

$\omega_0^2 = \frac{1}{LC}$

$\alpha > \omega_0$ over-damped

$\alpha = \omega_0$ critically-damped

$\alpha < \omega_0$ underdamp