1.

(a)

$$i_R = \frac{v}{R} = \frac{15}{200} = 75\ mA$$
$$i_C = -(i_L + i_R) = -(-45 + 75) = -30\ mA$$

(b)

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 200 \cdot 200 \cdot 10^{-9}} = \frac{1}{8 \cdot 10^{-5}} = 12500$$
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50 \cdot 10^{-3} \cdot 200 \cdot 10^{-9}}} = 10^4 = 10000$$
$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -12500 \pm 7500 = -5000, -20000$$
$$v(t) = A_1 e^{-5000t} + A_2 e^{-20000t} ------ eq.1$$

Initial condition:

$$v(0) = 15 = A_1 + A_2 ------- eq.2$$
$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri_L(0)}{RC} = -\frac{15 + 200 \cdot (-45) \cdot 10^{-3}}{200 \cdot 200 \cdot 10^{-9}} = -150000$$

From eq.1:

$$\frac{dv}{dt} = -5000A_1 e^{-5000t} - 20000A_2 e^{-20000t}$$

At t = 0:

$$-150000 = -5000A_1 - 20000A_2 ------- eq.3$$

Solve eq.2 and eq.3:

$$\begin{cases} A_1 = 10 \\ A_2 = 5 \end{cases}$$

⇨ $v(t) = 10e^{-5000t} + 5e^{-20000t}$ (V) , t > 0

(c)

$$i_L = \frac{1}{L}\int_{-\infty}^{t} v\, dt = \frac{1}{50 \cdot 10^{-3}}(10e^{-5000t} + 5e^{-20000t})$$
$$= -0.04e^{-5000t} - 0.005e^{-20000t}\ (A) \quad t > 0$$

2.

a) $\frac{d^2v_C}{dt^2} + \frac{1}{RC}\frac{dv_C}{dt} + \frac{1}{LC} = 0$

$2 \cdot \alpha = \frac{1}{RC}$

${\omega_o}^2 = \frac{1}{LC}$

$\alpha < \omega_o \;\; \rightarrow$ UNDERDAMPED

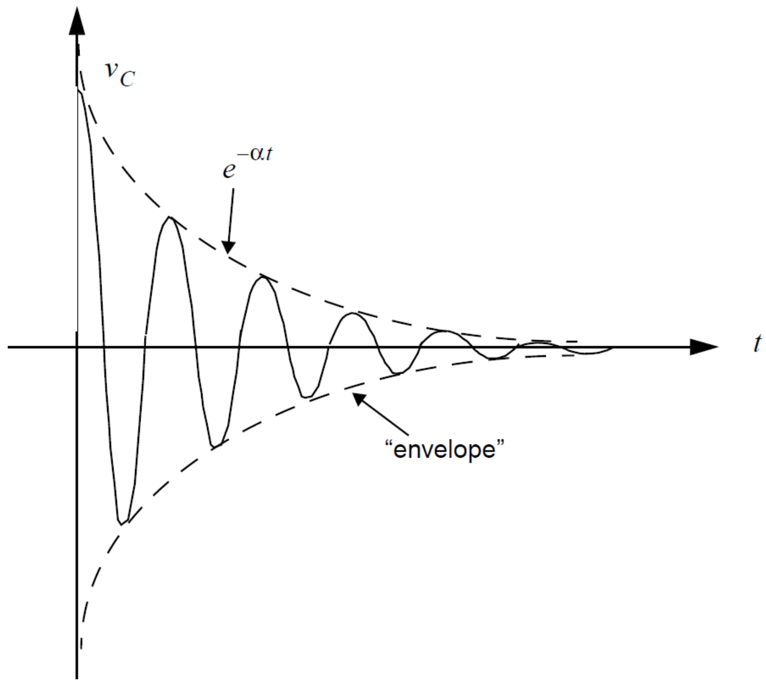
b) $v_C = K\, e^{-\alpha t} \cdot \cos(\omega_d \cdot t + \phi)$

$\omega_d = \sqrt{{\omega_o}^2 - \alpha^2}$

$\phi = \tan^{-1}(\frac{\alpha}{\omega_d})$

$\omega_o = 10 \times 10^6$

$\alpha = 3.33 \times 10^6$

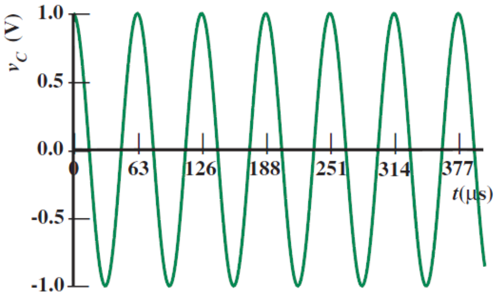


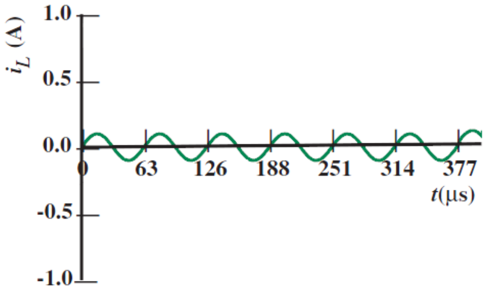
3.

$$\omega_\circ = \frac{1}{\sqrt{LC}} = 10^5 \quad \text{rad/s}$$

$$v_C(t) = v_C(0)\cos(\omega_\circ t) = 1.0\cos(10^5 t)$$

$$i_L(t) = \sqrt{\frac{C}{L}}\, v_C(0)\sin(\omega_\circ t) = 0.1 \times \sin(10^5 t).$$





4.

$$\frac{v_1}{8k} + \frac{v_1 - v_2}{6k} + \left(\frac{1}{24}\mu F\right)\frac{dv_1}{dt} = 0$$

$$v_1 - \left(\frac{1}{18}\mu F\right)\frac{dv_2}{dt}(6000) - v_2 = 0 \quad \rightarrow \quad v_1 = v_2 + \frac{1}{3000}\frac{dv_2}{dt}$$

Rearrange equation as follow:

$$v_2 = A\, e^{-1000t} + B\, e^{-9000t}$$

Initial conditions allow us to find constants A and B:

$$A + B = 0 \quad \rightarrow \quad \text{from } v_2(0) = 0$$

$$A + B - \frac{1}{3}A - 3B = 1 \quad \rightarrow \quad \text{from } v_1(0) = 1\text{Volt}$$

$$A = \frac{3}{8}$$

$$B = -\frac{3}{8}$$

$$v_2 = \frac{3}{8}(e^{-1000t} - e^{-9000t})$$