EE2210 Electric Circuits, Spring 2018

Practice problems (Lecture5-Lecture8)

Exercise 9.2

(a)

$$\frac{1uF \cdot 2uF}{1uF + 2uF} = \frac{2}{3}\ uF$$

(b)

$$\frac{1uF \cdot 10pF}{1uF + 10pF} = 9.9\ pF$$

(c)

$$\frac{40pF \cdot 1uF}{40pF + 1uF} = 38.5\ pF$$

(d)

$$1mH + 2mH = 3\ mH$$

(e)

$$2mH + 1uH = 2.001\ mH$$

(f)

$$\frac{2mH \cdot 1mH}{2mH + 1mH} + 1uH \cong 2/3\ mH$$

Exercise 9.4

$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(t)dt$

Assume initial capacitor voltage = $v_{initial}$ when t = 0.

$0 < \mathrm{t} < \frac{T}{2}$, $\quad v_c(\mathrm{t}) = \frac{1}{C}\int_{-\infty}^{t} I_0 dt = \frac{1}{C} \cdot I_0 \cdot t + v_{initial}$

$\mathrm{t} = \frac{T}{2}$, $\quad v_c(\mathrm{t}) = (\frac{1}{C} \cdot I_0 \cdot \frac{T}{2}) + \frac{Q_0}{C}$

$\frac{T}{2} < \mathrm{t} < T$, $\quad v_c(\mathrm{t}) = (v_{initial} + \frac{1}{C} \cdot I_0 \cdot \frac{T}{2} + \frac{Q_0}{C}) + (\frac{1}{C} \cdot I_0 \cdot \left(t - \frac{T}{2}\right))$

Capacitor voltage is given as V_0 at t = T,

⇨ At t = T, $V_0 = (v_{initial} + \frac{1}{C} \cdot I_0 \cdot \frac{T}{2} + \frac{Q_0}{C}) + (\frac{1}{C} \cdot I_0 \cdot \left(T - \frac{T}{2}\right))$

⇨ $v_{initial} = V_0 - \left(\frac{1}{C} \cdot I_0 \cdot \frac{T}{2} + \frac{Q_0}{C}\right) - \left(\frac{1}{C} \cdot I_0 \cdot \frac{T}{2}\right) = V_0 - \left(\frac{1}{C} \cdot I_0 \cdot \mathrm{T} + \frac{Q_0}{C}\right)$, at t = 0.

Exercise 9.5

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)dt$$

For voltage expression,

$$0 < \mathrm{t} < \frac{T}{2}, \qquad V_L(\mathrm{t}) = \frac{2}{T}(-V_0)t + V_0$$

$$\frac{T}{2} < \mathrm{t} < \mathrm{T}, \qquad V_L(\mathrm{t}) = \frac{2}{T}(V_0)t - V_0$$

Assume initial inductor current = $i_{initial}$ when t = 0.

$$0 < \mathrm{t} < \frac{T}{2}, \qquad i_L(\mathrm{t}) = \frac{1}{L}\int_{-\infty}^{t}(\frac{2}{T}(-V_0)t + V_0)dt = \frac{1}{L}\cdot\frac{(-V_0)}{T}\cdot t^2 + \frac{V_0}{L}\cdot t + i_{initial}$$

$$\mathrm{t} = \frac{T}{2}, \qquad i_L(\mathrm{t}) = 0 + \frac{\Lambda_0}{L}$$

$$\frac{T}{2} < \mathrm{t} < T,$$

$$i_L(\mathrm{t}) = (i_{initial} + (\frac{1}{L}\cdot\frac{2}{T}\cdot(-V_0)\cdot\frac{1}{2}\cdot\left(\frac{T}{2}\right)^2 + \frac{V_0}{L}\cdot\frac{T}{2}) + \frac{\Lambda_0}{L}) + (\frac{1}{L}\cdot\frac{2}{T}\cdot V_0\cdot\frac{1}{2}\cdot(T^2 - \left(\frac{T}{2}\right)^2) - \frac{V_0}{L}\cdot(t - \frac{T}{2})$$

⇨ $i_L(\mathrm{t}) = \frac{1}{L}\cdot\frac{V_0}{T}(t-T)^2 + \frac{V_0 T}{2L} + \frac{\Lambda_0}{L} + i_{initial}$

Inductor current is given as I_0 at t = T,

⇨ At t = T, $I_0 = \frac{1}{L}\cdot\frac{V_0}{T}(T-T)^2 + \frac{V_0 T}{2L} + \frac{\Lambda_0}{L} + i_{initial}$

$i_{initial} = I_0 - \frac{\Lambda_0}{L} - \frac{V_0 T}{2L}$, at t = 0.

Problem 9.1

$V_{c1}(t) = 4u(t)$

$\mathrm{i}(t) = \mathrm{C}\,\frac{dv(t)}{dt}$

$4u\mathrm{C}\ \delta(\mathrm{t}) = C_1\frac{dv(t)}{dt} = C_1\frac{4\cdot du(t)}{dt} = C_1 \cdot 4\delta(\mathrm{t})$

⇨ $C_1 = u\mathrm{C}$

$\mathrm{q}(\mathrm{t}) = C_1V_{c1} = C_2V_{c2}$

⇨ $C_2 = 4uC$

Problem 9.6

For ideal linear inductor:

$$V_t(\mathrm{t}) = \frac{d(L(t)\cdot i(t))}{dt}$$

$$V_t(\mathrm{t}) = \frac{d((L_0+L_1\sin(\omega t))\cdot I)}{dt}$$

⇨ $V_t(\mathrm{t}) = IL_1 \cdot \omega \cdot \cos(\omega t)$

Exercise 10.1

The inductor first acts as an open circuit and eventually becomes a wire:

$$t \geq 0 \begin{cases} initially: i_t(t) = 0\ (open\ circuit) \\ finally: i_1(t) = \dfrac{V_s(t)}{3k} + i_S(t) = \dfrac{4}{3}\ mA \end{cases}$$

Assume $i_S(t)$ source points down.

$$i_i(t) = (Final\ value) + (Initial\ value - Final\ value)e^{-t/\tau}$$

$$i_i(t) = 4/3\left(1 - e^{-\frac{t}{\tau}}\right)\ [mA]$$

$$\tau = \frac{L}{R} = \frac{1}{3}ms$$

Ans. $i_i(t) = 4/3\left(1 - e^{-\frac{t}{\tau}}\right)$ mA for t ≥ 0; $\tau = \frac{1}{3}ms$

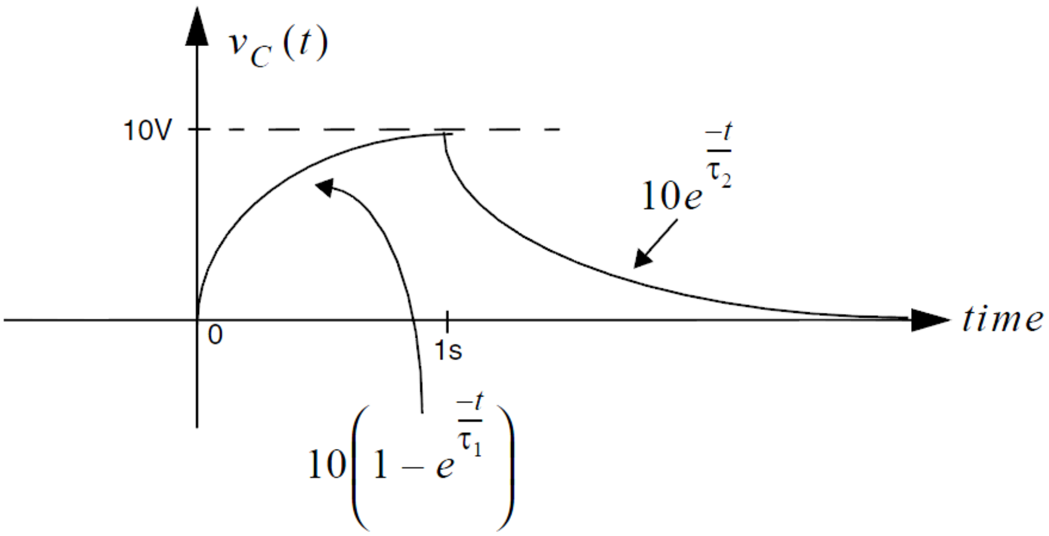
Exercise 10.4

Assume $v_c = 0$ for t < 0. When the switch is closed at t = 0, v_c rises from 0 to

$11 \cdot \frac{10k}{10k+1k} = 10\text{V}$ with $\tau_1 = [1\text{k}||10\text{k}] \cdot \text{C}$

$\tau_1 = 9.09$ ms

When the switch is opened,
v_c falls exponentially back to zero with $\tau_2 = 10k \cdot C = 1$ second.

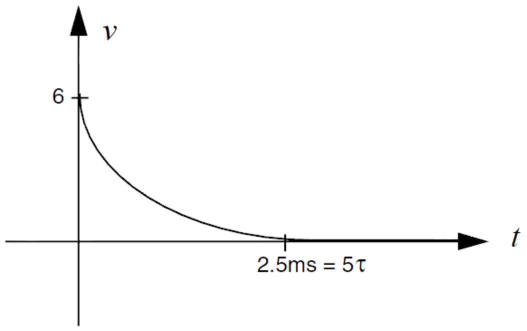


Exercise 10.5

(a)

$\tau = [1k||1k] \cdot C = 500us$

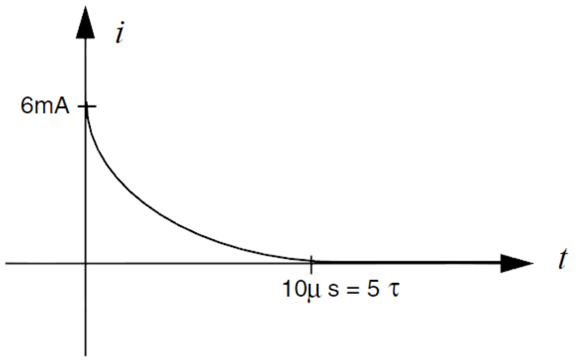
$v = 6e^{-t/\tau}$



(b)

$\tau = [1k||1k] \cdot L = 2us$

$i = (6 \cdot 10^{-3})e^{-t/\tau}$

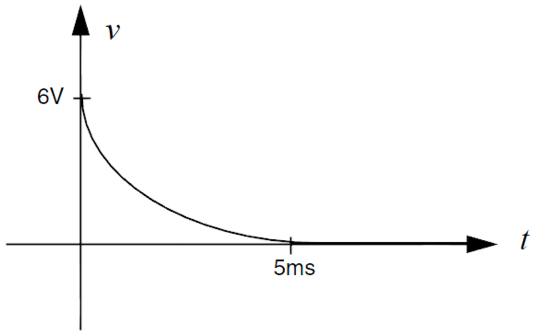


(c)

v(0) = 6

$\tau = R \cdot C = (1k\Omega)(1uF) = 1ms$

$v = 6e^{-t/\tau}$

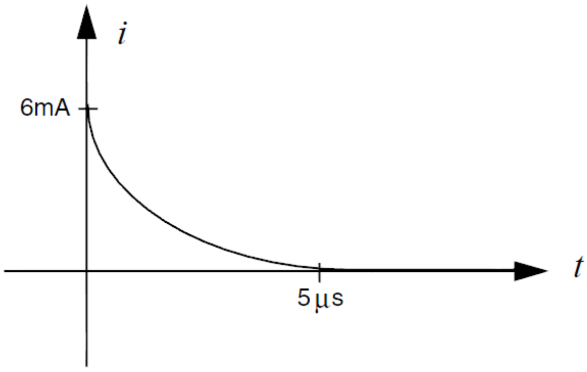


(d)

$\mathrm{i}(0) = \frac{6V-0}{1000\Omega} = 6\ mA$

$\tau = \mathrm{L}/1\mathrm{k} = 1\mathrm{us}$

$\mathrm{i} = 0.006e^{-t/\tau}$



Problem 10.24

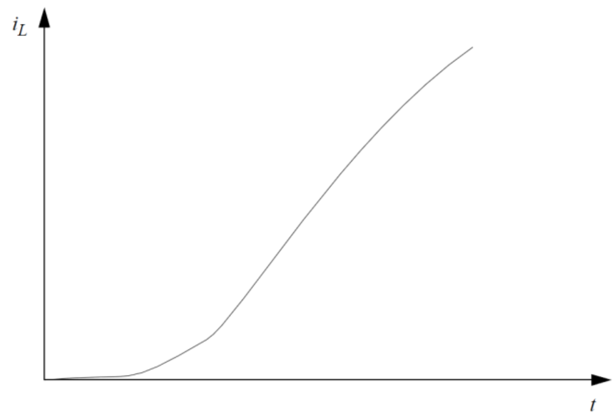
(a)

$\tau = L/R$

$i_L(0^+) = 0$

$i_L(t) = \int \frac{K_1}{R}\left(1 - e^{-\frac{t}{\tau}}\right) dt = \frac{K_1 t}{R} + \frac{K_1 \tau}{R} e^{-\frac{t}{\tau}} - \frac{K_1 \tau}{R}$

$i_L(t) = \frac{K_1 t}{R} - \frac{K_1 \tau}{R}(1 - e^{-\frac{t}{\tau}})$



(b)

$i_L = K_2 t$

$v_L = L\frac{di_L}{dt} = \mathrm{L}K_2$

$v_R = Ri_L = \mathrm{R}K_2 t$

$v_I = v_L + v_R = \mathrm{L}K_2 + \mathrm{R}K_2 t$