EE2210 Electric Circuits, Spring 2018
Practice problems solution (Lecture11-Lecture14)

1.

ANS:

$$(8+\mathrm{j}7) = 10.63e^{41.18°j}$$
$$(0.3-\mathrm{j}0.1) = 0.316e^{-18.43°j}$$
$$\text{MAG} = 16.8$$
$$\text{PHASE} = 13.75 \text{ deg}$$

2.

ANS:

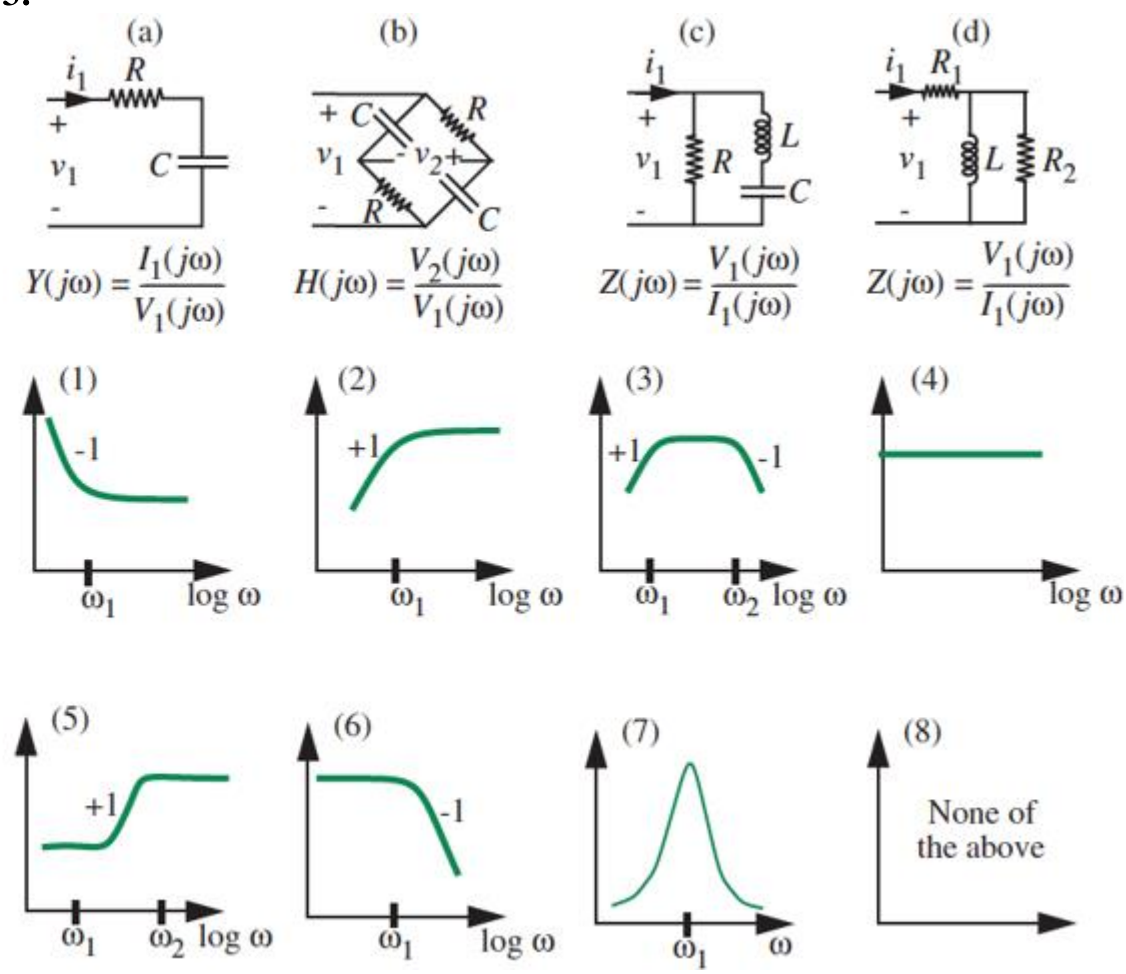
a)

$$5.83e^{j59°}\cdot 4e^{j50°}\cdot 7e^{-j20°} = 163.26e^{j89°} \rightarrow 2.84+\mathrm{j}163$$

b)

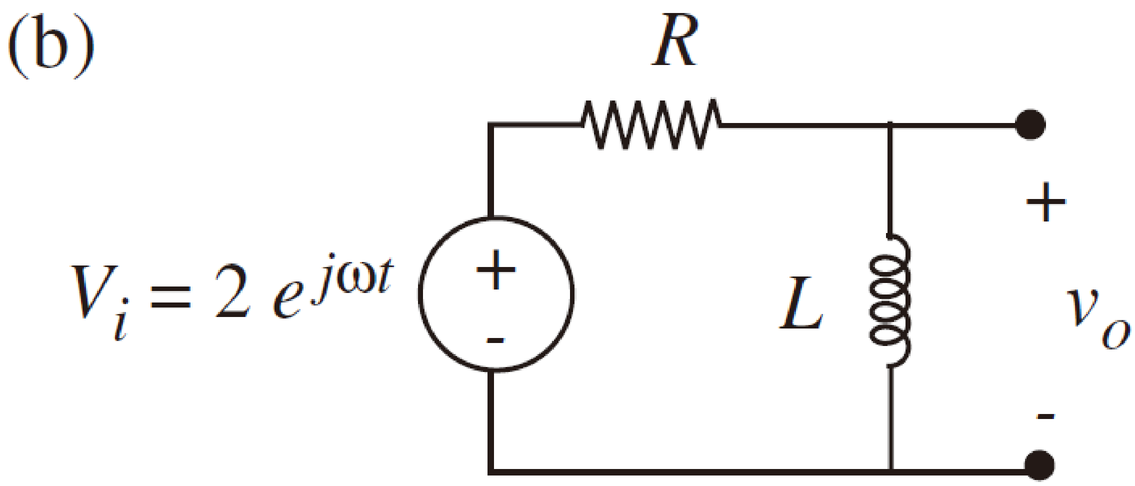
$$10e^{j70°} \rightarrow 3.42+\mathrm{j}9.4$$

3.



ANS: (a)2(b)4(c)8(d)5

4.

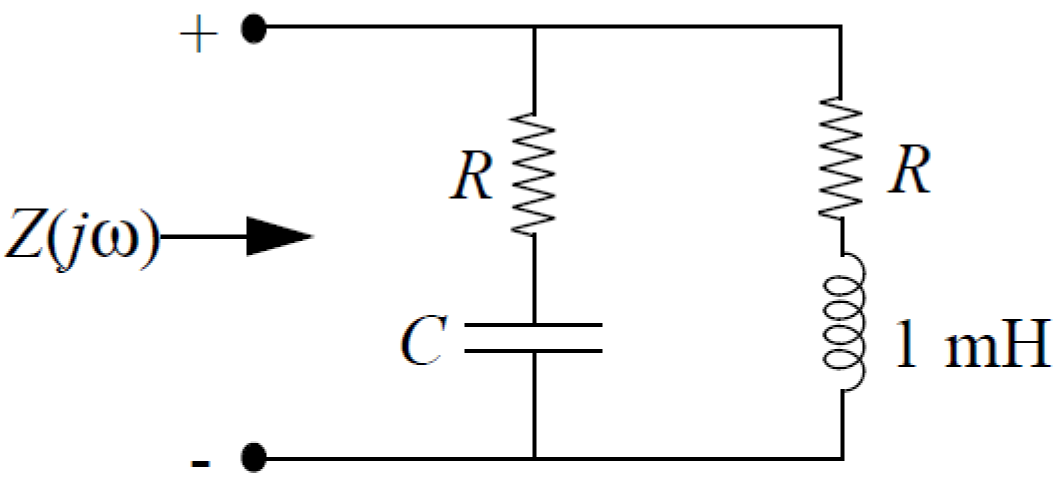


ANS:

$$\frac{V_o}{V_i} = \frac{L_s}{L_s + R} = \frac{\omega L}{\sqrt{(\omega L)^2 + R^2}} e^{j\emptyset}$$

$$\emptyset = \tan^{-1}\left(\frac{R}{\omega L}\right)$$

5.



ANS:

$$Z = \frac{(R + \frac{1}{C_s})(R + L_s)}{2R + \frac{1}{C_s} + L_s} = \frac{R(LCs^2 + \left(\frac{L}{R} + RC\right)s + 1)}{LCs^2 + 2RCs + 1}$$

In order for Z to aleays be purely real,

$$\left(\frac{L}{R} + RC\right) = 2RC$$
$$L = R^2C$$

Then

$$Z = R = 2000$$

Independent of ω.

$$0.001 = 2000^2\, C$$
$$C = 2.5 \cdot 10^{-10}\ Farads$$

ANS: $R = 2000$ and $C = 2.5 \cdot 10^{-10}\ Farads$

6.

ANS:

$$\mathrm{H(s)} = \frac{1}{LCs^2 + RCs + 1} = \frac{1}{(1 - \omega^2 LC) + j\omega RC}$$

7.

ANS:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{\frac{T}{2}} V^2 dt} = \sqrt{\frac{1}{T}\int_0^{\frac{T}{2}} \left(V_m \sin\left(\frac{2\pi}{T}t\right)\right)^2 dt} = \sqrt{\frac{V_m{}^2}{T}\int_0^{\frac{T}{2}} \sin^2\left(\frac{2\pi}{T}t\right) dt}$$
$$= \sqrt{\frac{V_m{}^2}{T}\frac{T}{2\pi}\int_0^{\frac{T}{2}} \sin^2\left(\frac{2\pi}{T}t\right) d\frac{2\pi}{T}t}$$
$$= \sqrt{\frac{V_m{}^2}{T}\frac{T}{2\pi}\int_0^{\frac{T}{2}} \frac{1 - \cos\left(2 * \frac{2\pi}{T}t\right)}{2} d\frac{2\pi}{T}t}$$
$$= \sqrt{\frac{V_m{}^2}{2\pi}[\frac{2\pi}{T}\frac{t}{2} - \frac{\sin(2 * \frac{2\pi}{T}t)}{4}]_0^{T/2}} = \sqrt{\frac{V_m{}^2}{2\pi}[\frac{\pi}{2}]} = \frac{V_m}{2}$$

請注意積分項的變數變換!

Note:

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

8.

ANS:

$$\omega = 50$$

$$Z = R + j\omega L + \frac{1}{j\omega C} = 4 + \mathrm{j} * 50 * 0.24 - \mathrm{j}\frac{1}{50 * 0.0025} = 5.66\angle 45^{O}\Omega$$

$$I_0 = \frac{V}{Z} = \frac{0.1\angle -90^{O}}{5.66\angle 45^{O}} = 17.67\angle -135^{O}\ mA$$

$$i_0 = 17.67\cos(50\mathrm{t} - 135^{O})\ mA$$

9.

ANS:

(a)

$$Z_{ab} = j\omega L + R\|\left(\frac{1}{j\omega C}\right) = j\omega L + \frac{-\frac{jR}{\omega C}}{R - \frac{j}{\omega C}} = j\omega L + \frac{-jR}{\omega CR - j} = j\omega L + \frac{-jR(\omega CR + j)}{(\omega CR)^2 + 1}$$

Pure resistive→ $\mathrm{Im}(Z_{ab}) = 0$

$$\therefore \omega L - \frac{\omega CR^2}{(\omega CR)^2 + 1} = 0 \rightarrow \omega^2 = \frac{\left(\frac{CR^2}{L}\right) - 1}{(CR)^2} = 900 * 10^8$$

$$\omega = 300\ krad/s$$

(b)

$$Z_{ab}(300 * 10^3) = j48 + \frac{100(-j133.33)}{100 - j133.33} = 64\ \Omega$$

10.

ANS:

a) $\mathrm{H(jw)} = \frac{v_o}{v_i} = \frac{1/sC\|R_L}{R+1/sC\|R_L} = \frac{R_L}{sRCR_L+(R+R_L)}$

b) $|\mathrm{H(jw)}| = \frac{R_L}{\sqrt{wRCR_L{}^2+(R+R_L)^2}}$ |H(jw)| is maximum at w=0

c) $|H(jw)|_{max} = \frac{R_L}{R+R_L}$

d) $|H(jw)| = \frac{R_L}{\sqrt{2}(R+R_L)} = \frac{1/RC}{\sqrt{w_c{}^2+[(R+R_L)/RR_LC]^2}}$ $w_c = \frac{1}{RC}(1 + \left(\frac{R}{R_L}\right))$

e) $w_c = \frac{1}{(10^3)(10^{-7})}\left[1 + \left(\frac{10^3}{10^4}\right)\right] = 10000(1 + 0.1) = 11000\frac{rad}{s}$

$$\mathrm{H}(j0) = \frac{10000}{11000} = 0.9091\angle 0^{o}$$

$$\mathrm{H}(jw_c) = \frac{10000}{11000 + j11000} = 0.6428\angle -45^{o}$$

$$\mathrm{H(j0.1}w_c) = \frac{10000}{11000 + j1100} = 0.9046\angle - 5.71^{o}$$

$$\mathrm{H}(\mathrm{j}10w_c)\frac{10000}{11000 + j110000} = 0.0905\angle{-84.29^o}$$