Energy Storage Elements

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Motivation

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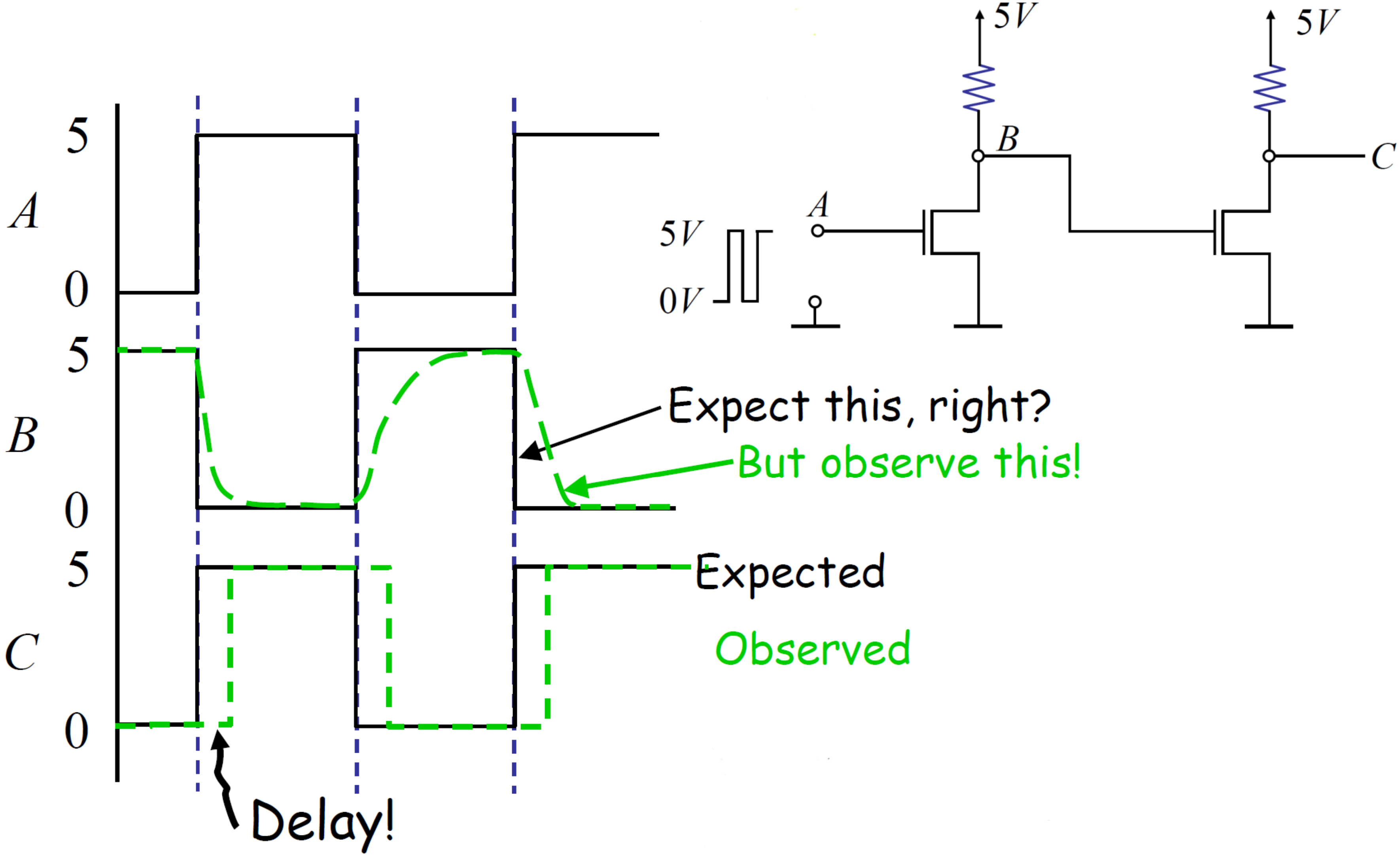
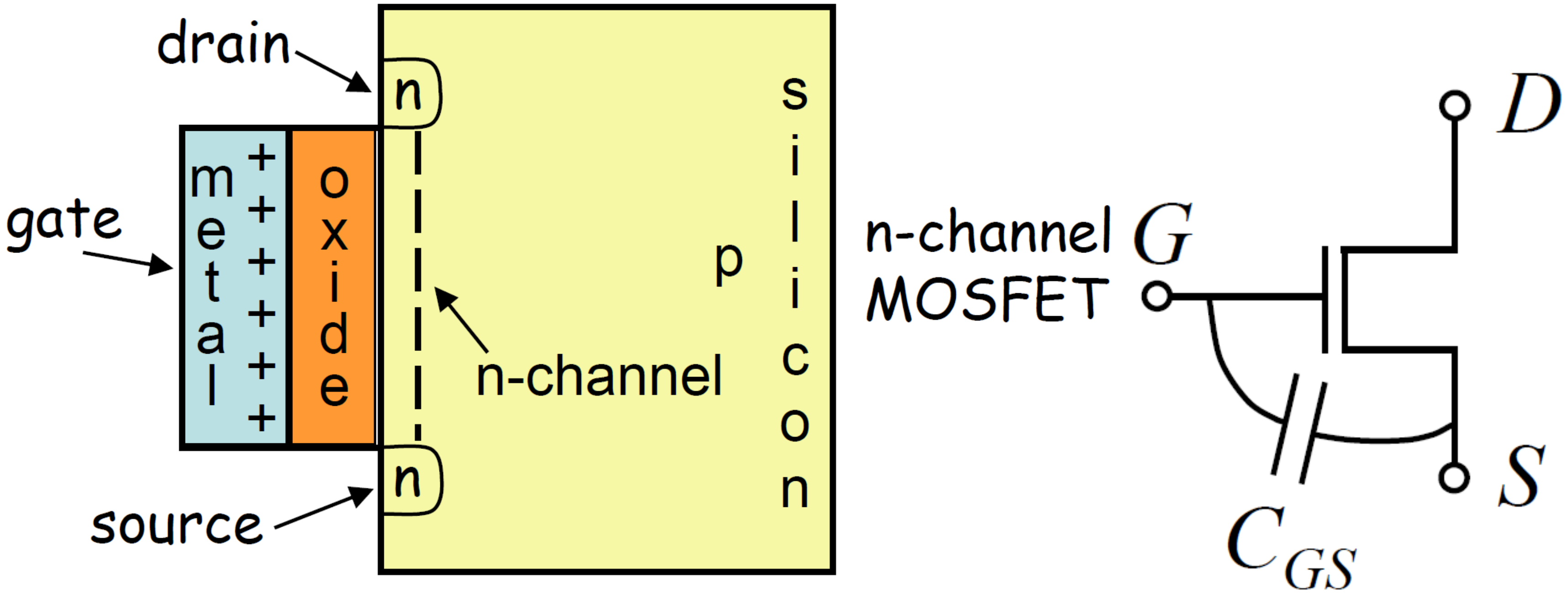
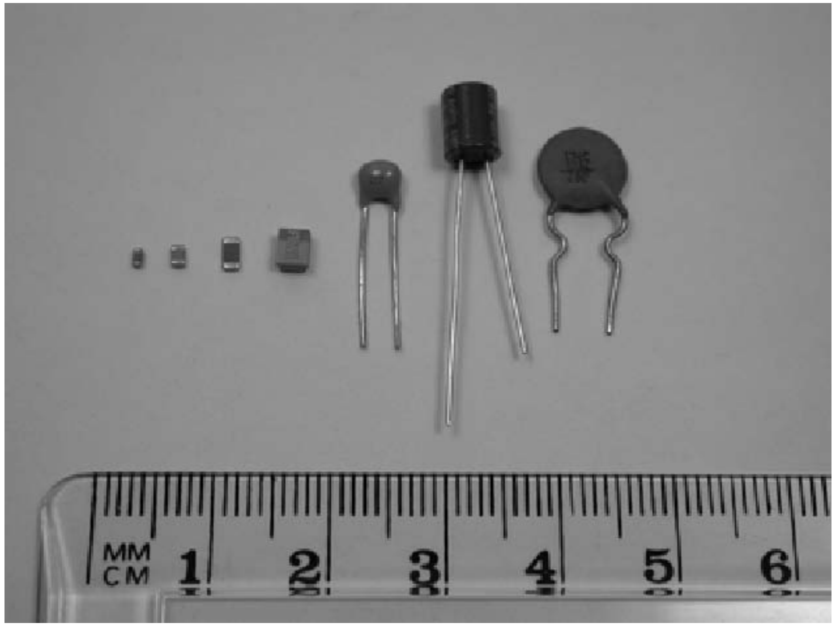
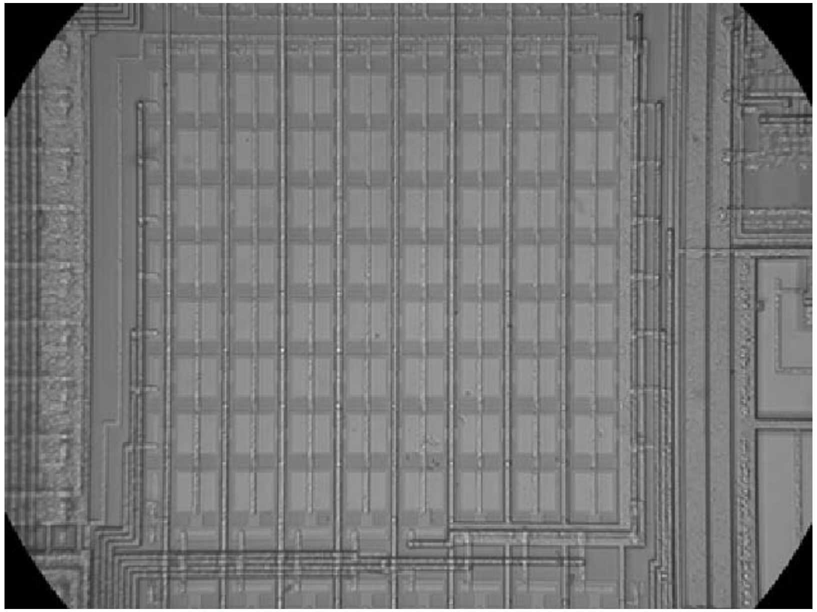


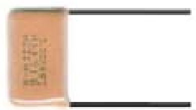
The Gate to Source Capacitor



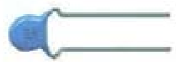
Capacitors, C







Polyester capacitor



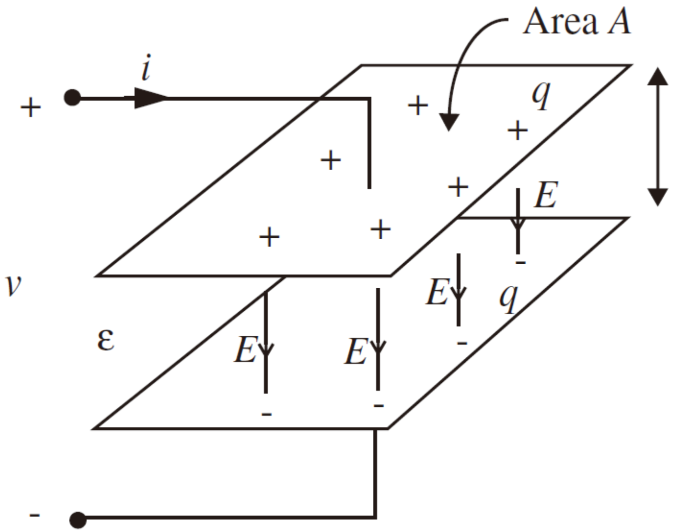
Ceramic capacitor

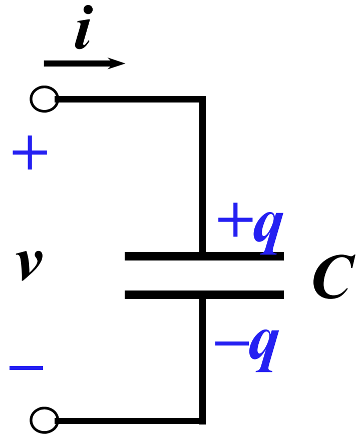
Electrolytic capacitor

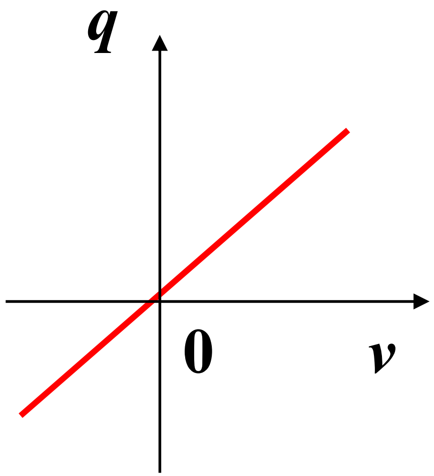
Tantalum capacitor

AC Motor capacitors

Capacitor







- Ideal linear capacitor

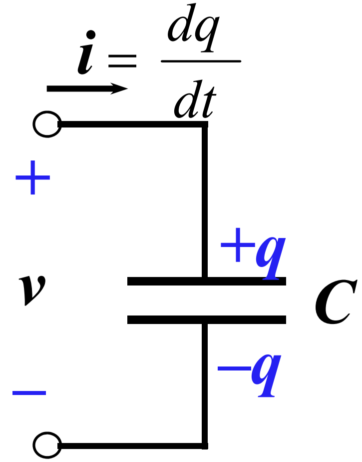
$$E = \frac{1}{\varepsilon} D = \frac{1}{\varepsilon}\frac{q}{A} \text{ and } v = lE \Rightarrow q = \varepsilon \frac{A}{l} v \Rightarrow q = Cv$$

$$q = Cv$$

$$C = \varepsilon \frac{A}{l}$$

- The unit of capacitance is Coulombs/Volt, or Farads (F). Name after Michael Faraday (1781-1867), an English physics and chemist.

Element law for a capacitor



- The capacitance is defined as

$$C = \frac{dq}{dv} \quad \text{or} \quad dq = C dv$$

- The element law of a capacitor can be found as:

$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

- The branch voltage of a capacitor depends on the entire past history of its branch current, which is the essence of ***memory***.

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i d\tau = \frac{1}{C}\int_{-\infty}^{t_0} i d\tau + \frac{1}{C}\int_{t_0}^{t} i d\tau$$

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i d\tau$$

$$q(t) = q(t_0) + \int_{t_0}^{t} i d\tau$$

Switch and Initial Condition

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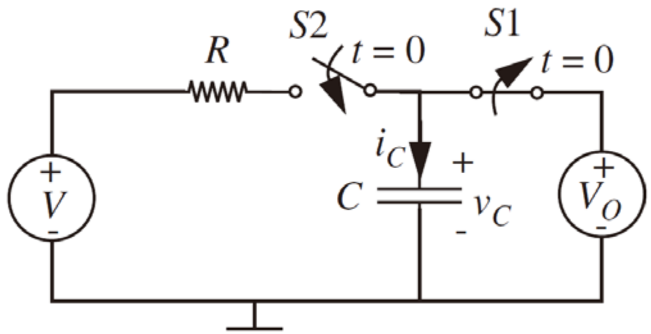
- $t = 0^-$ and $t = 0^+$
- Switching at $t = 0$



$$v_C(0^-) = \lim_{\substack{t \to 0 \\ t < o}} v_C(t) \qquad v_C(0^+) = \lim_{\substack{t \to 0 \\ t > o}} v_C(t)$$

- $v_C(0^+)$ is the initial condition for v_C.

$$v_C(t) = v_C(0^-) + \frac{1}{C}\int_{0^-}^{t} i_C(\tau)\mathrm{d}\tau$$

$$v_C(0^+) = v_C(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i_C(\tau)\mathrm{d}\tau$$

- When $i_C(\tau)$ is finite,

$$v_C(0^+) = v_C(0^-) \qquad q_C(0^+) = q_C(0^-)$$

Electric Energy Storage

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- Associated with the ability to exhibit memory is the property of energy storage, which is often exploited by circuits that process energy.
- The energy is stored in the form of electric field .
- Electric energy w_E stored in a capacitor

$$P_E = iv \;\Rightarrow\; \frac{dw_E}{dt} = iv \;\Rightarrow\; \frac{dw_E}{dt} = v\frac{dq}{dt} \;\Rightarrow\; dw_E = vdq$$

$$w_E = \int_0^q vdq' = \int_0^q \frac{q'}{C}dq' = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}Cv^2$$

- Unlike a resistor, a capacitor ***stores energy*** rather than dissipates it.

$$w_E = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}Cv^2$$

i-v Behaviors of Capacitor

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- Its current depends on the *changing rate* of voltage *v*.
- Steady state characteristics
  - The capacitor is an ***open circuit to DC*** at steady state.

$$i_{c_{dc}} = C\frac{dv_{c_{dc}}}{dt} = 0$$

  - The capacitor is a ***short circuit to high frequency*** signals at steady state.

    Assume $v_c = V\sin(\omega t)$,

$$i_c = C\frac{dv_c}{dt} = \omega CV\cos(\omega t)$$

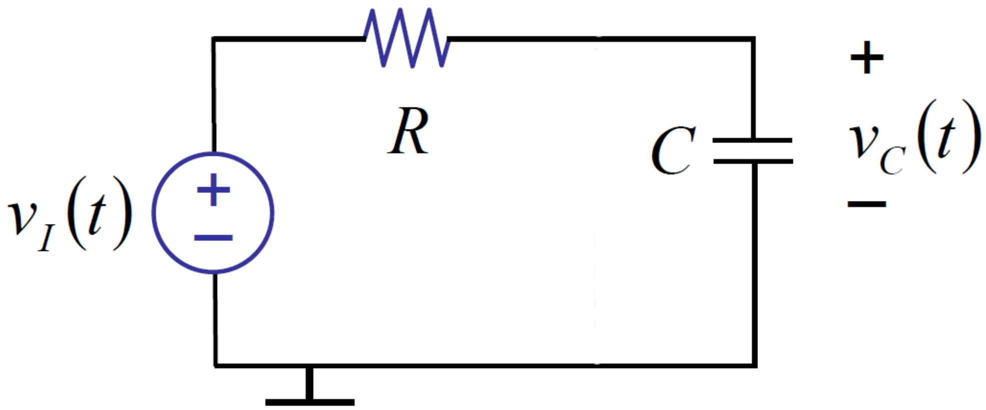
As $\omega \to \infty$, $i_c \to \infty$, similar to a short-circuit.

- ***The voltage on a capacitor does not change abruptly.*** Discontinuous change in the capacitor voltage requires an infinite current.
- For finite current, $v_C(0^+) = v_C(0^-)$

The Inverter Chain

- Apply node method:

$$\frac{v_C - v_I}{R} + C\frac{dv_C}{dt} = 0$$



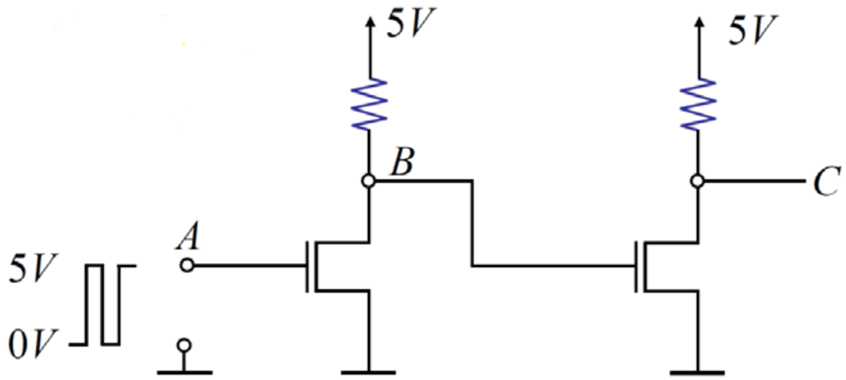
Given:

$$v_I(t) = V_I = 5\text{ V for } t \geq 0$$

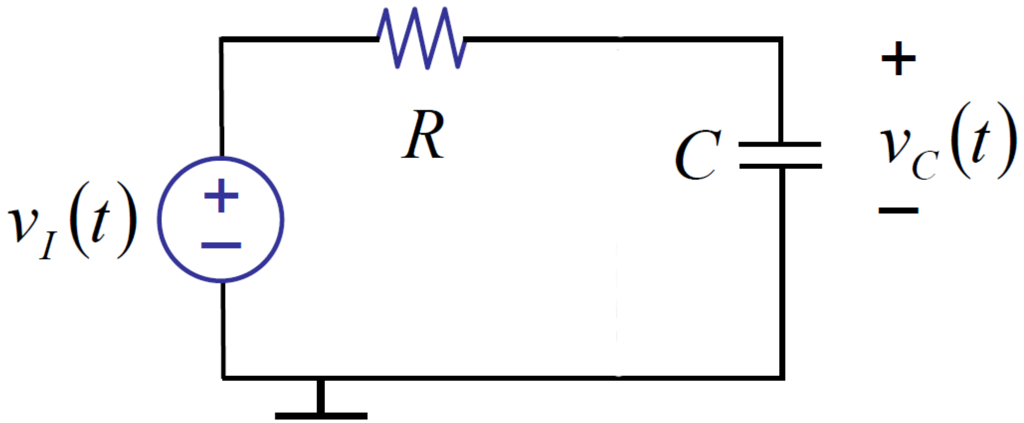
$$v_C(0) = V_0 = 0\text{ V}$$ is the given initial state value of the capacitor.

$$RC\frac{dv_C}{dt} + v_C = V_I = 5\text{ V}$$ The input drive switch from 0 to 5V for $t \geq 0$

$$RC\frac{dv_C}{dt} + v_C = V_I = 5\text{ V for } t \geq 0$$

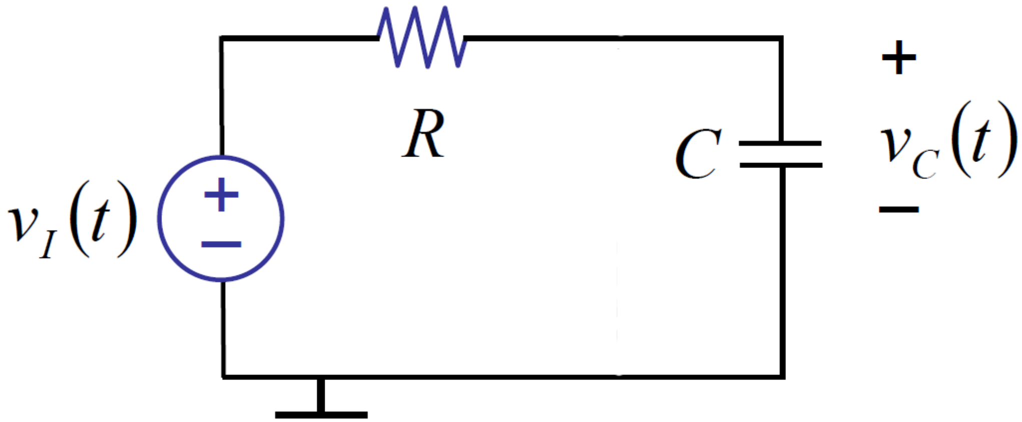


Method of homogeneous and particular solutions



- Find the particular solution, v_{CP}.
- Find the homogeneous solution , v_{CH}.
- The total solution is the sum of the particular and homogeneous solutions , $v_C = v_{CP} + v_{CH}$.
- Use the initial conditions to solve for the remaining constants.

The Particular solution v_{CP}



- Find the particular solution, v_{CP}.
- v_{CP} :any solution that satisfies the original equation $RC\frac{dv_{CP}}{dt}+v_{CP}=5\text{ V}$
- Use trial and error : Try $v_{CP}=5\text{ V}$,

$$RC\frac{dv_{CP}}{dt}+v_{CP}=5\text{ V}\Rightarrow RC\frac{d5}{dt}+5=5\ \text{ Worked!!!}$$

The Homogeneous Solution

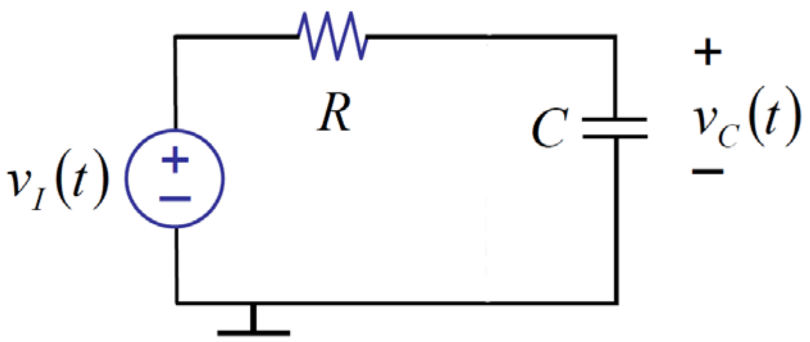
- Find the homogeneous solution , v_{CH}.
- v_{CH} :solution to the homogeneous equation by setting the input drive v_I to 0. $RC\frac{dv_{CH}}{dt}+v_{CH}=0$
- Assume solution is of this form : $v_{CH}=Ae^{st}$

$$RC\frac{dAe^{st}}{dt}+Ae^{st}=0 \Rightarrow RCsAe^{st}+Ae^{st}=0 \Rightarrow RCs+1=0$$

- Characteristic equation $RCs+1=0$ $\Rightarrow s=-\frac{1}{RC}=-\frac{1}{\tau}$
- The homogeneous solution , v_{CH}:

$$v_{CH}=Ae^{-\frac{t}{RC}}$$

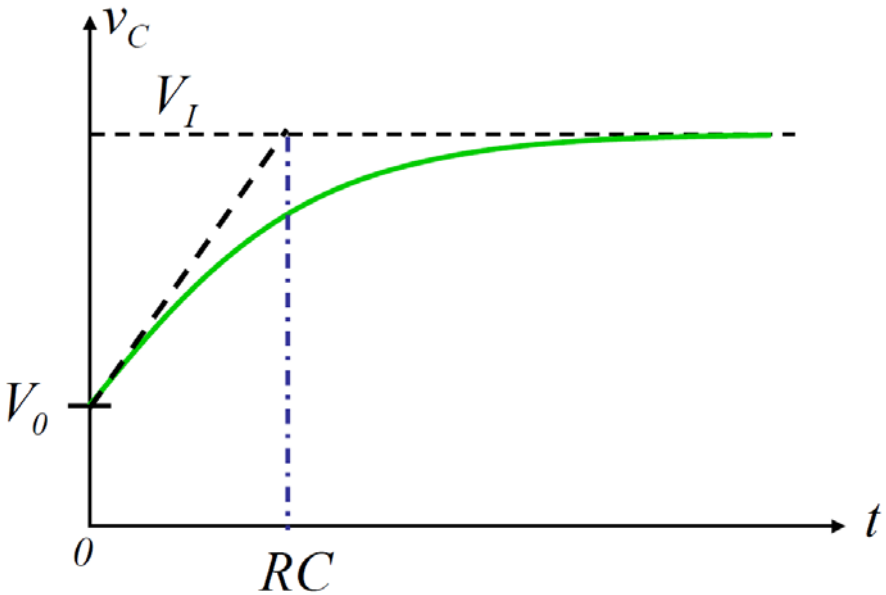
- RC is called time constant τ.



The Total solution

- The total solution is the sum of the particular and homogeneous solutions:

$$v_C = v_{CP} + v_{CH} = 5 + Ae^{-\frac{t}{RC}}$$



- Use the initial conditions:

$$v_C(0) = V_0 = 0\ \text{V}$$

to solve for the remaining constants.

$$V_0 = 0 = V_I + Ae^{-\frac{0}{RC}} = 5 + Ae^{-\frac{0}{RC}}$$

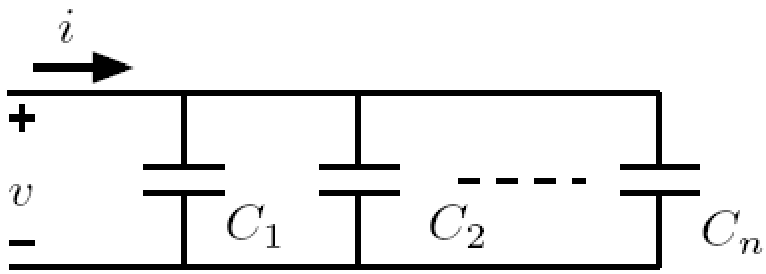
$$\Rightarrow \quad A = V_0 - V_I = 0 - 5 = -5\ \text{V}$$

- The total solution v_C :

$$\boxed{v_C = V_I + (V_0 - V_I)e^{-\frac{t}{RC}}} = 5 + (0-5)e^{-\frac{t}{RC}} = 5\left(1 - e^{-\frac{t}{RC}}\right)$$

Capacitors, C

- Parallel connection

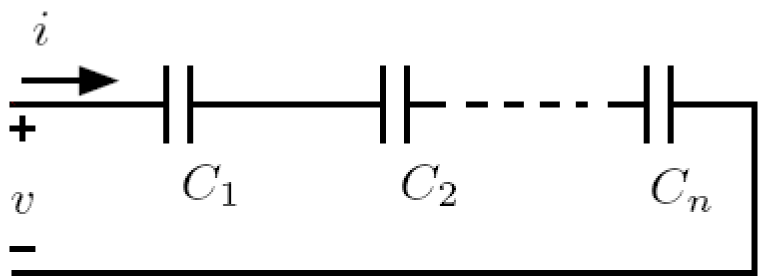


$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots + C_n\frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{n} C_k\right)\frac{dv}{dt}$$

$$\rightarrow C_{eq} = \sum_{k=1}^{n} C_k$$

- Series connection

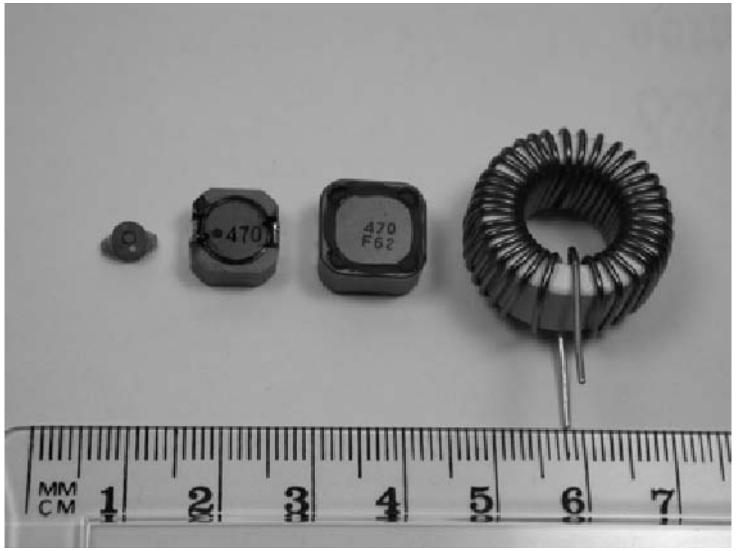


$$v = \frac{1}{C_1}\int_0^t i\,dt + \cdots + \frac{1}{C_n}\int_0^t i\,dt$$

$$= \left(\sum_{k=1}^{n}\frac{1}{C_k}\right)\int_0^t i\,dt$$

$$\rightarrow \frac{1}{C_{eq}} = \sum_{k=1}^{n}\frac{1}{C_k}$$

Inductor, L




Toroid inductor

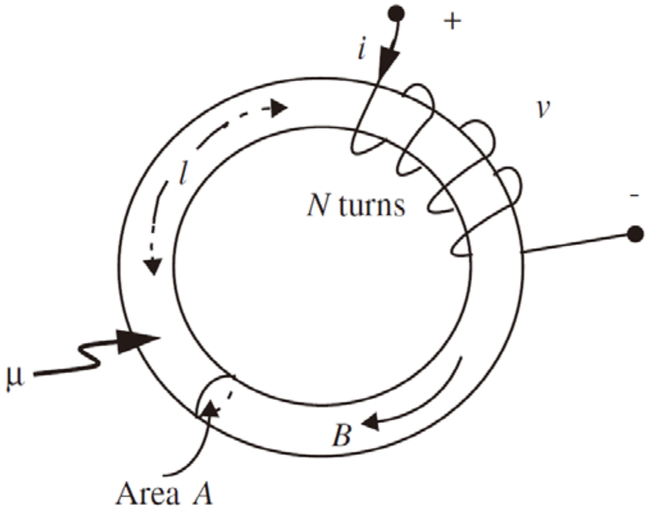

Wound inductor

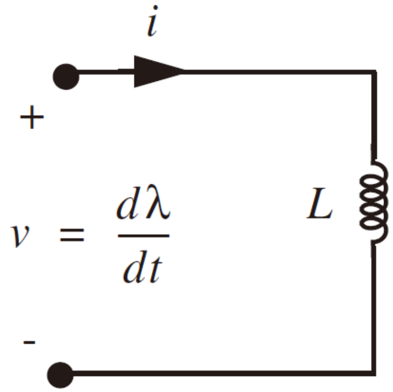

Transformer

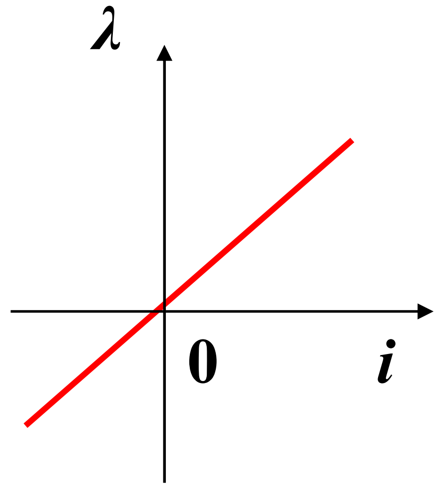

AC line inductor


AC shunt inductor

Inductor







- Total Flux linkage λ

$$\lambda = N\Phi = NAB = NA\mu\frac{Ni}{l} = \mu\frac{N^2A}{l}i$$

$$\lambda = Li$$

$$L = \mu\frac{N^2A}{l}$$

- L has the units of Webers/Ampere, or Henrys (H). Name after Joseph Henry (1797-1878), an American physics.

Element law for an inductor

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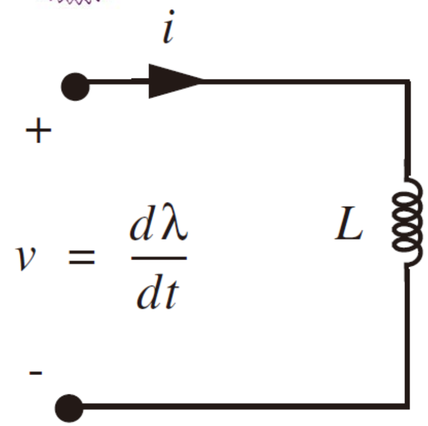


- The inductance is defined as

$$L = \frac{d\lambda}{di} \quad \text{or} \quad d\lambda = L di$$

- The element law of a inductor can be found as:

$$v = \frac{d\lambda}{dt} = L\frac{di}{dt}$$

- The branch current of an inductor depends on the entire past history of its branch voltage, which is the essence of ***memory***.

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v \mathrm{d}\tau = \frac{1}{L}\int_{-\infty}^{t_0} v \mathrm{d}\tau + \frac{1}{L}\int_{t_0}^{t} v \mathrm{d}\tau$$

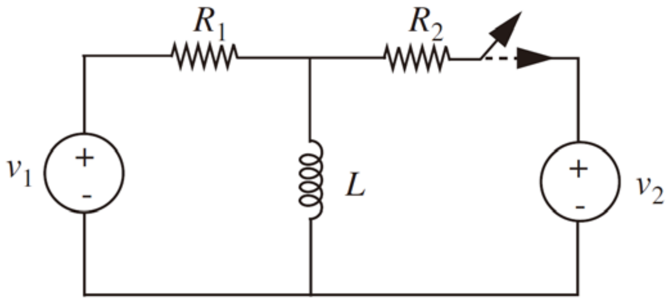
$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v \mathrm{d}\tau$$

$$\lambda(t) = \lambda(t_0) + \int_{t_0}^{t} v \mathrm{d}\tau$$

Switch and Initial Condition

- $t = 0^-$ and $t = 0^+$
- Switching at $t = 0$

$$i_L(0^-) = \lim_{\substack{t\to 0 \\ t<o}} i_L(t) \qquad i_L(0^+) = \lim_{\substack{t\to 0 \\ t>o}} i_L(t)$$



- $v_C(0^+)$ is the initial condition for v_C.

$$i_L(t) = i_L(0^-) + \frac{1}{C}\int_{0^-}^{t} v_L(\tau)\mathrm{d}\tau$$

$$i_L(0^+) = i_L(0^-) + \frac{1}{C}\int_{0^-}^{0^+} v_L(\tau)\mathrm{d}\tau$$

- When $i_C(\tau)$ is finite,

$$i_L(0^+) = i_L(0^-) \qquad\qquad \lambda_L(0^+) = \lambda_L(0^-)$$

Magnetic Energy Storage

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- Associated with the ability to exhibit memory is the property of energy storage, which is often exploited by circuits that process energy.
- The energy is stored in the form of magnetic field .
- Magnetic energy w_M stored in an inductor

$$P_M = iv \;\Rightarrow\; \frac{dw_M}{dt} = iv \;\Rightarrow\; \frac{dw_M}{dt} = i\frac{d\lambda}{dt} \;\Rightarrow\; dw_M = id\lambda$$

$$w_M = \int_0^{\lambda} id\lambda' = \int_0^{q} \frac{\lambda'}{L} dq' = \frac{1}{2}\frac{\lambda^2}{L} = \frac{1}{2}Li^2$$

- Unlike a resistor, an ***inductor stores energy*** rather than dissipates it.

$$w_M = \frac{1}{2}\frac{\lambda^2}{L} = \frac{1}{2}L\lambda^2$$

i-v Behaviors of Inductor

- Its voltage depends on the *changing rate* of current *i*.
- Steady state characteristics
  - The inductor is ***a short circuit to DC*** at steady state .

$$v_{L_{dc}} = L\frac{di_{L_{dc}}}{dt} = 0$$

  - The inductor is an ***open circuit to high frequency*** signals at steady state.

Assume $i_L = I\sin(\omega t)$,

$$v_L = L\frac{di_L}{dt} = \omega LI\cos(\omega t)$$

As $\omega \rightarrow \infty, v_L \rightarrow \infty$ similar to an open-circuit.

- ***The current through an inductor does not change abruptly.*** A discontinuous change of the inductor current requires an infinite voltage.
- For finite voltage, $i_L(0^+) = i_L(0^-)$

Inductors, L

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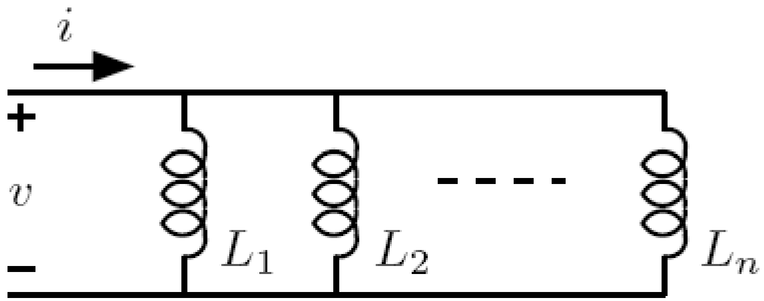
- Parallel connection

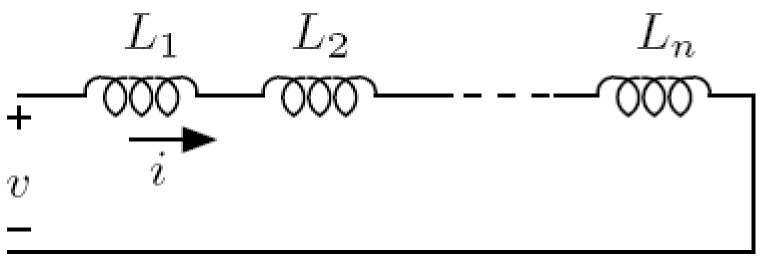


$$i = \frac{1}{L_1}\int_0^t vdt + \cdots + \frac{1}{L_n}\int_0^t vdt$$

$$= \left(\sum_{k=1}^{n} \frac{1}{L_k}\right)\int_0^t vdt$$

$$\rightarrow \frac{1}{L_{eq}} = \sum_{k=1}^{n} \frac{1}{L_k}$$

- Series connection



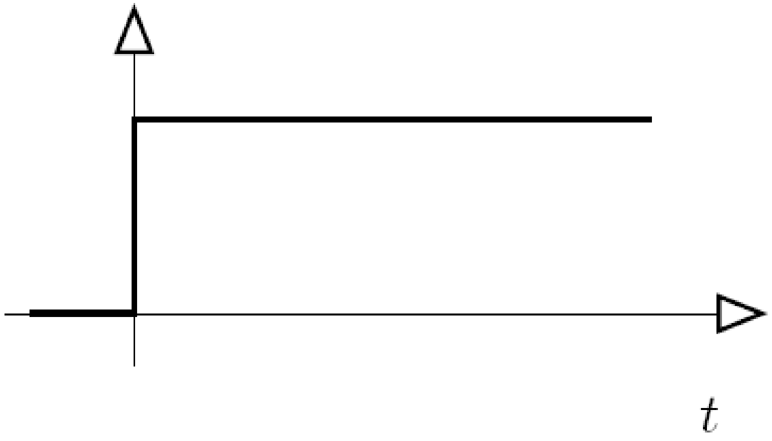
$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + \cdots + L_n\frac{di}{dt}$$

$$= \left(\sum_{k=1}^{n} L_k\right)\frac{di}{dt}$$

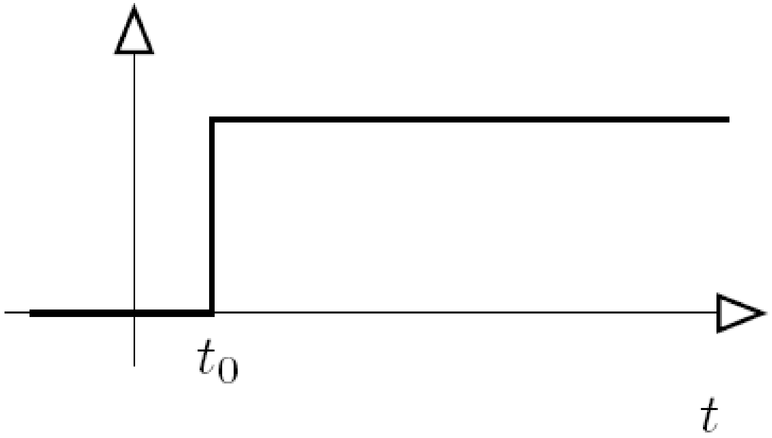
$$\rightarrow L_{eq} = \sum_{k=1}^{n} L_k$$

Excitations

- Step function;



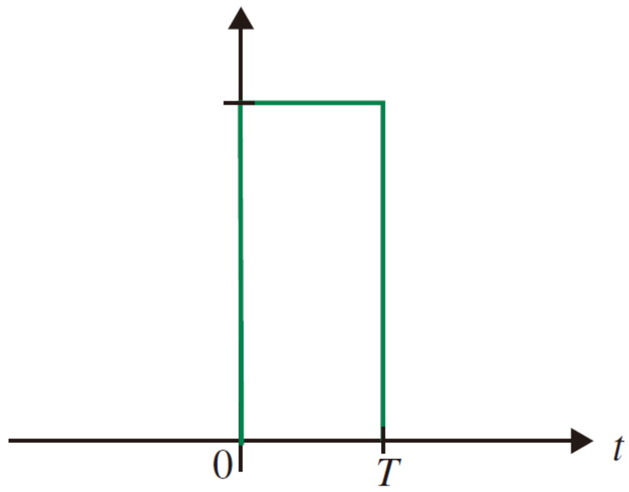
$$u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$



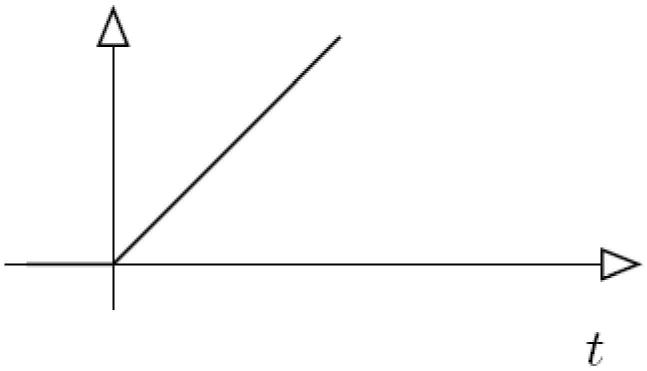
$$u(t - t_0) = \begin{cases} 0 & \text{if } t < t_0 \\ 1 & \text{if } t \geq t_0 \end{cases}$$

Excitations

- Pulse



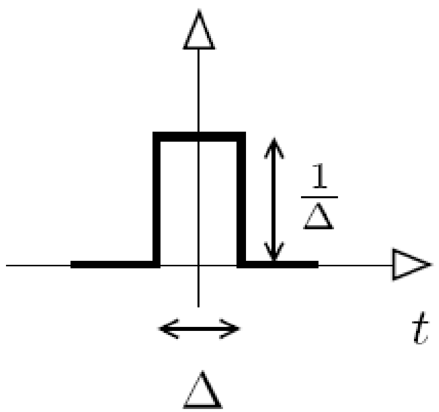
- Ramp function



$$r(t) = \int u(t)dt = \begin{cases} 0 & \text{if } t < 0 \\ t & \text{if } t \geq 0 \end{cases}$$

Excitations

- Impulse;

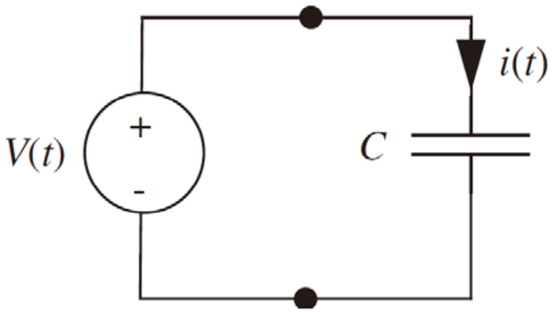


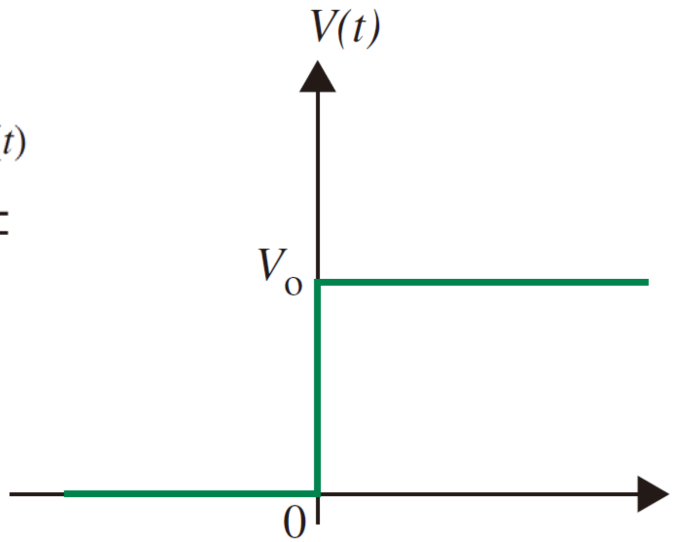
$$\delta(t) = \frac{du(t)}{dt} = \begin{cases} 0 & t > \frac{\Delta}{2},\ t < -\frac{\Delta}{2},\ \Delta \to 0. \\ \frac{1}{\Delta} & -\frac{\Delta}{2} < t < \frac{\Delta}{2}, \Delta \to 0. \end{cases}$$

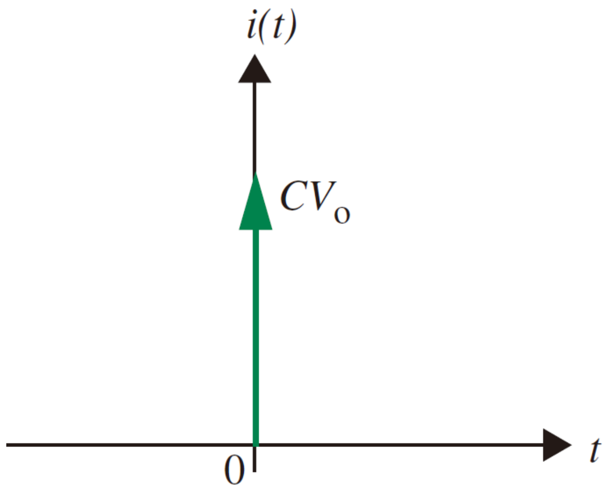
$$\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \delta(t)dt = 1$$

- Since $i_C(0) = \infty$

$$v_C(0^+) - v_C(0^-) = V_0$$

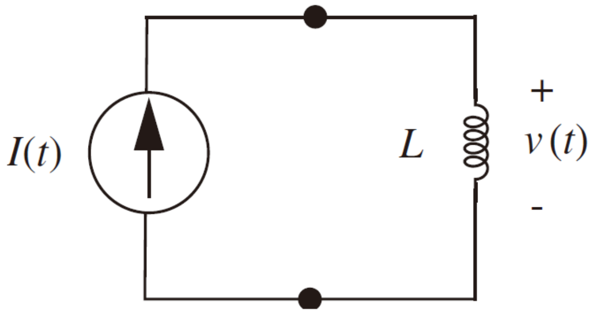


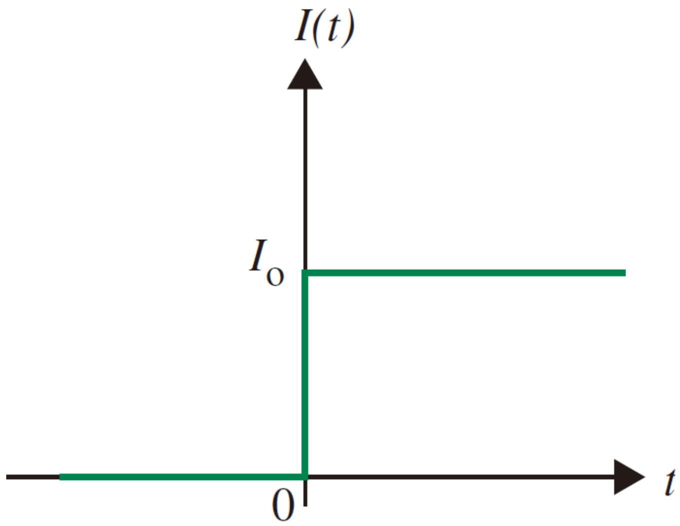




Impulse

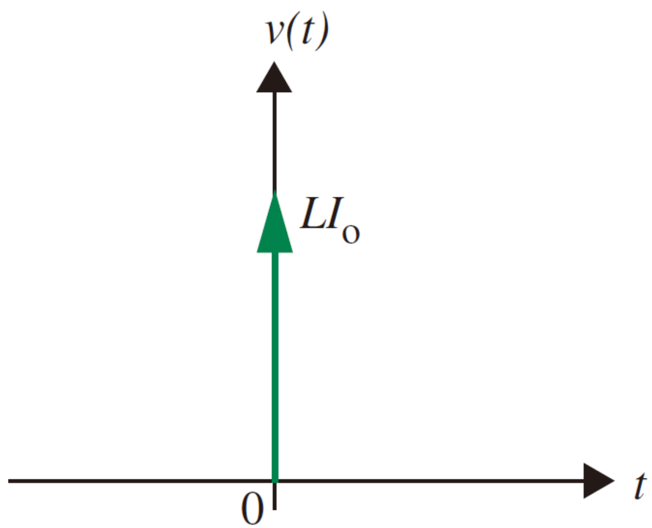
- Inductor



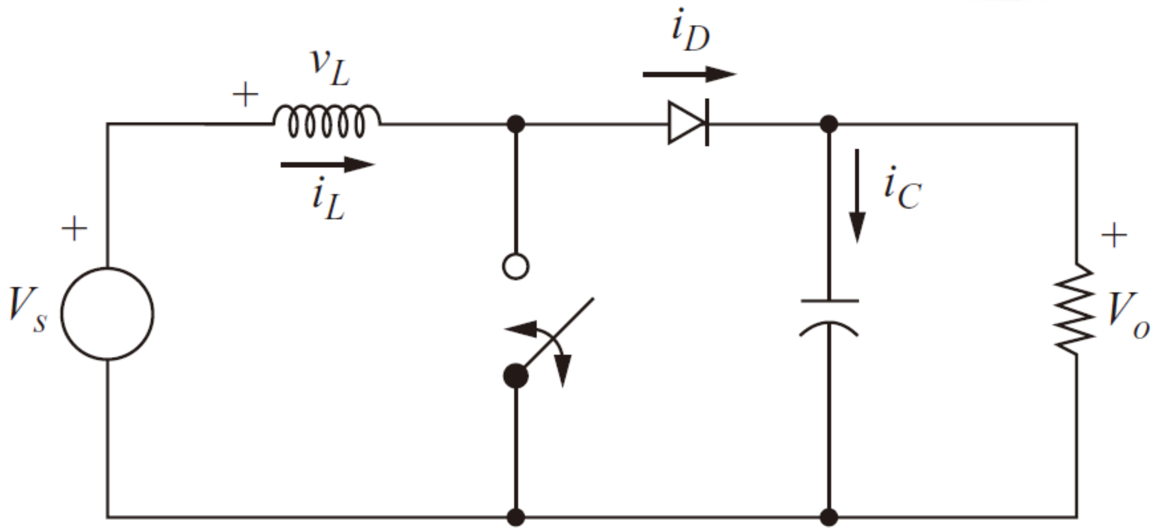


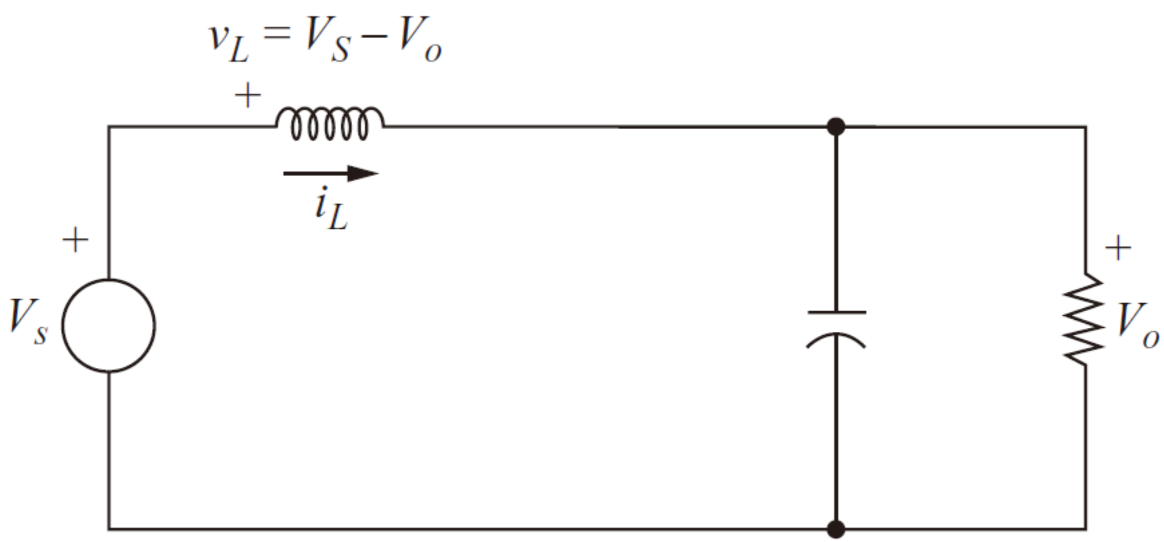
- Since $v_L(0) = \infty$

$$i_L(0^+) - i_L(0^-) = I_0$$



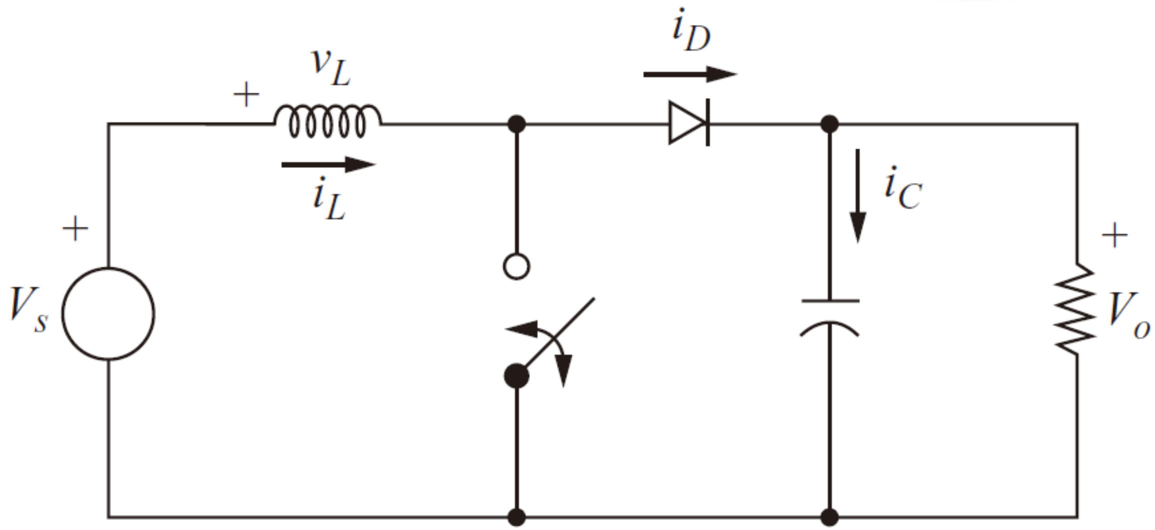
Boost converter



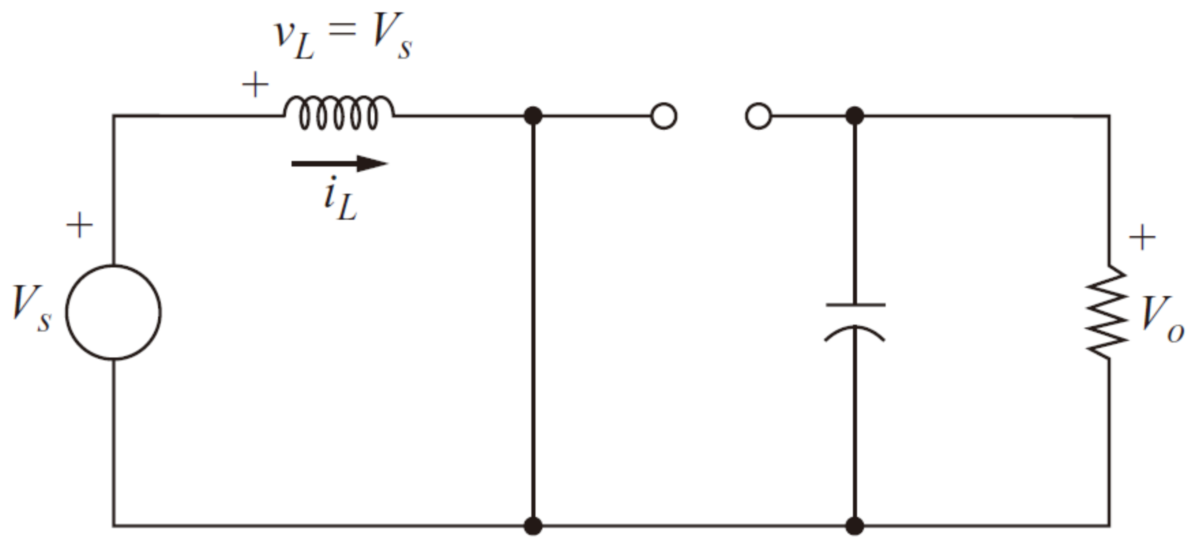


Switch Open

Boost converter



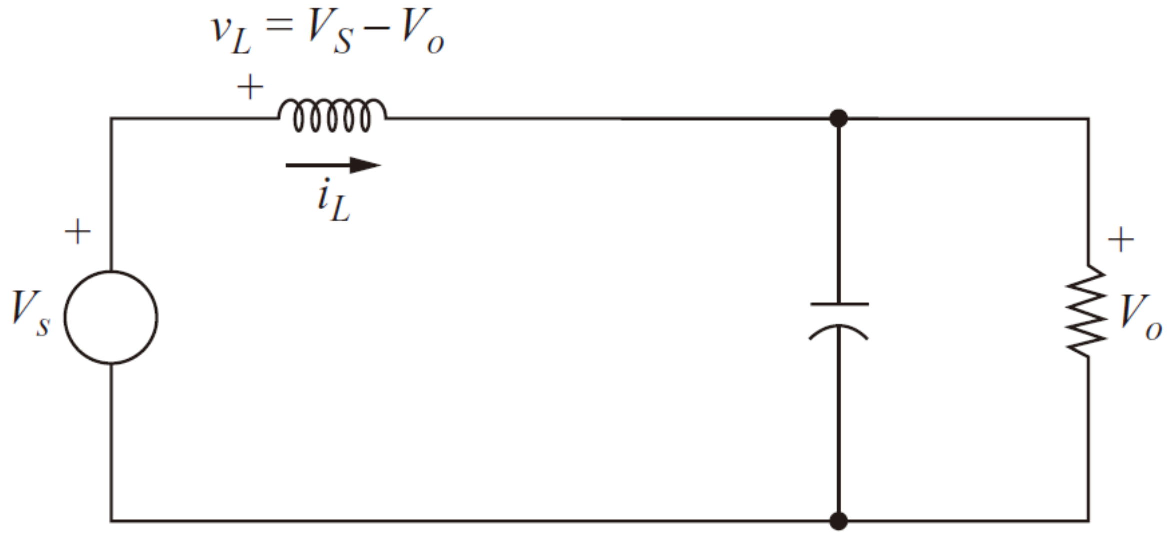
Switch Closed



Boost converter

- The analysis assumes the following:
- The switching period is T, and the switch is closed for time DT and open for $(1-D)T$.
- The inductor current is continuous (always positive).
- The capacitor is very large, and the output voltage is held constant at voltage V_o.
- Steady-state conditions exist.
- The components are ideal.

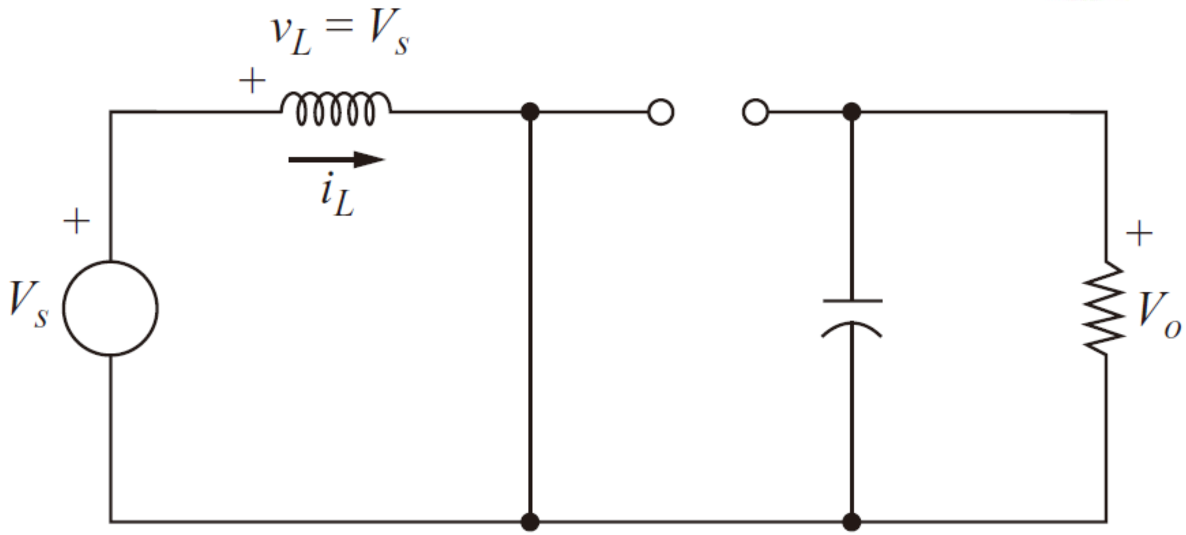
When Switch is Open



$$v_L = V_s - V_o = L\frac{di_L}{dt} \qquad \frac{di_L}{dt} = \frac{V_s - V_o}{L} \qquad \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_s - V_o}{L}$$

$$(\Delta i_L)_{open} = \frac{(V_s - V_o)(1-D)T}{L}$$

When Switch is Closed



$$v_L = V_s = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_s}{L}$$

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s}{L}$$

$$(\Delta i_L)_{closed} = \frac{V_s DT}{L}$$

Boost converter

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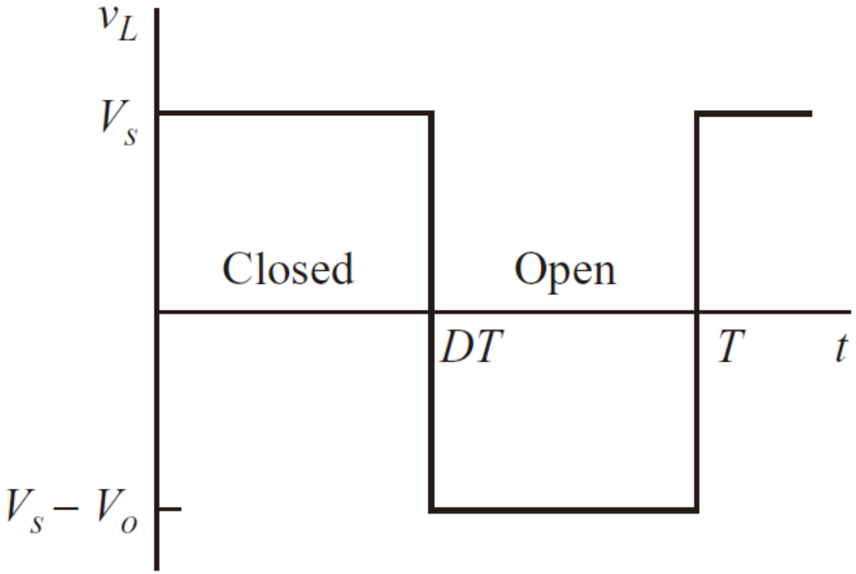

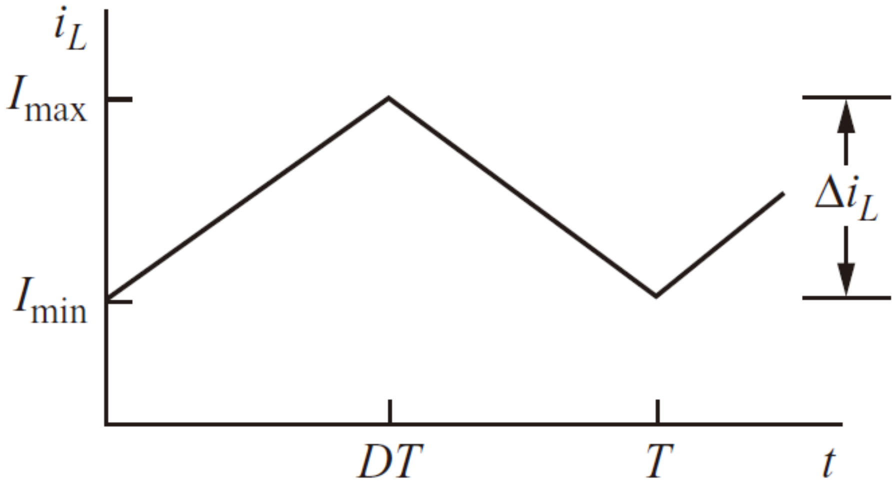

$$(\Delta i_L)_{open} + (\Delta i_L)_{open} = 0$$

$$\frac{(V_s - V_o)(1-D)T}{L} + \frac{V_s DT}{L} = 0$$

$$V_o = \frac{V_s}{1-D}$$

Summary

	R	C	L
$v - i$	$v_R = i_R R$	$v_C = \frac{1}{C}\int_{t_0}^{t} i_C dt$	$v_L = L\frac{di_L}{dt}$
$i - v$	$i = \frac{v_R}{R}$	$i_C = C\frac{dv_c}{dt}$	$i_L = \frac{1}{L}\int_{t_0}^{t} v_L dt$
Power, Energy	$p_R = i_R^2 R = \frac{v_R^2}{R}$	$W_C = \frac{1}{2}Cv_C^2$	$W_L = \frac{1}{2}Li_L^2$
Series	$R_{eq} = \Sigma R_k$	$\frac{1}{C_{eq}} = \Sigma\frac{1}{C_k}$	$L_{eq} = \Sigma L_k$
Parallel	$\frac{1}{R_{eq}} = \frac{1}{R_k}$	$C_{eq} = \Sigma C_k$	$\frac{1}{L_{eq}} = \frac{1}{L_k}$
DC steady state	(same)	open-circuit	short-circuit
Continuity	(no restriction)	v_C	i_L

Fundamental Circuit Variables and Elements

- ***4 fundamental circuit variables***: current, *i*; voltage,*v*; charge, *q*; magnetic flux linkage, λ (φ instead λ of is adopted in this slide).
- ***6 mathematical relations*** (or Elements) might be construed to connect pairs of these 4 fundamental circuit variables.