Resistive Networks

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Terminology

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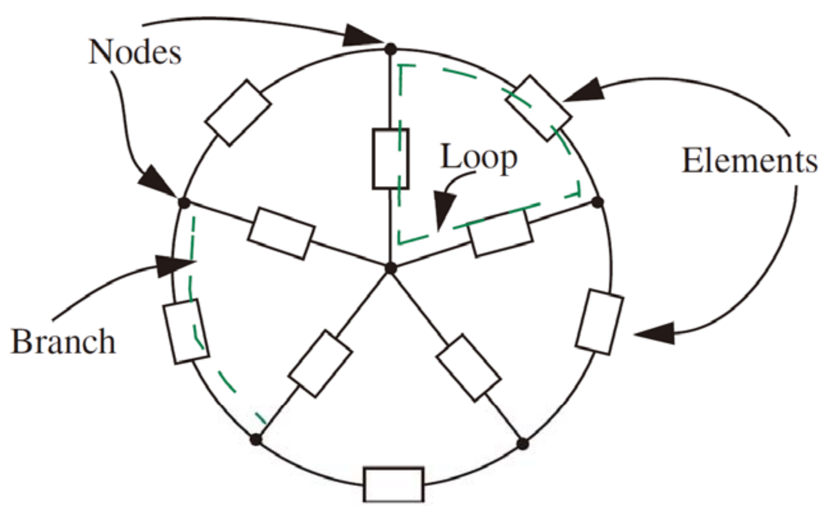
- A ***circuit*** is constructed by connecting together a collection of separate elements at their terminals.
- The junction points at which the terminals of two or more elements are connected are referred to as the ***nodes*** of a circuit.
- The connections between the nodes are referred to as the ***edges*** or ***branches*** of a circuit.
- Circuit ***loops*** are defined to be closed paths through a circuit along its branches.

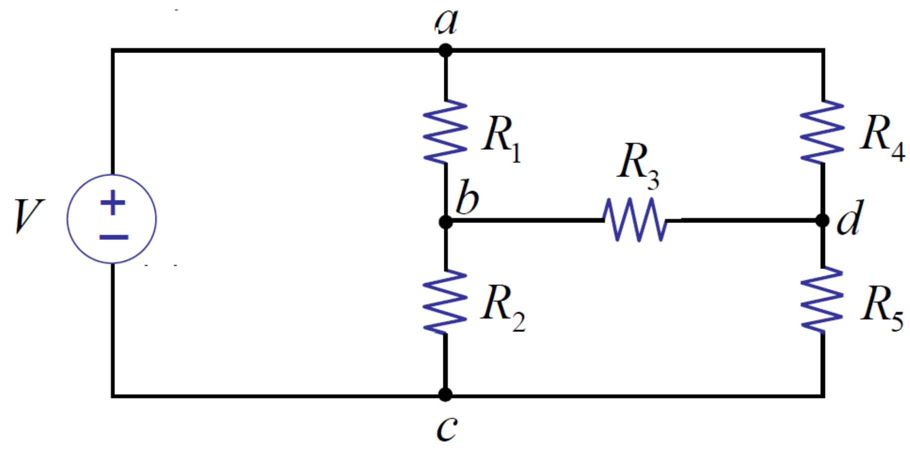


Lumped circuit abstraction

- Capped a set of lumped elements that obey the lumped matter discipline using ideal wires to form an assembly that performs a specific function results in the ***lumped circuit abstraction*.**

So, what does this buy us?

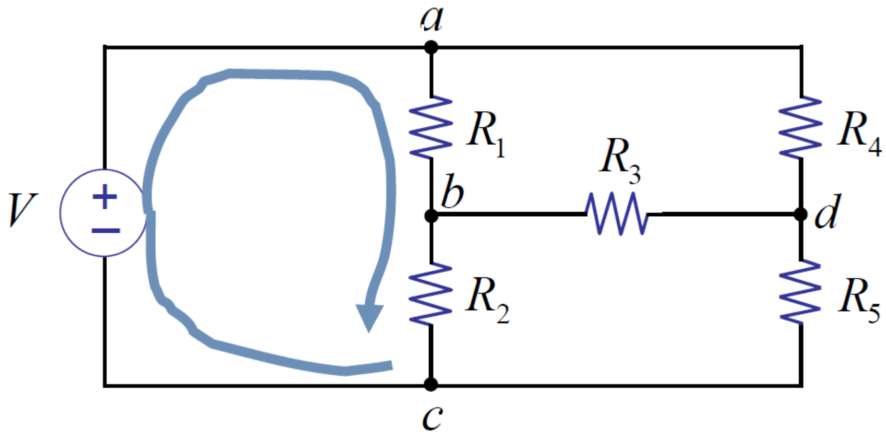
- For example —



- What can we say about voltages in a loop under the lumped matter discipline?

Kirchhoff's Voltage Law (KVL)

- What can we say about voltages in a loop under the lumped matter discipline?



$$\oint_C \mathbf{E}\cdot d\ell = -\frac{\partial \phi_B}{\partial t} = 0$$

$$\int_a^b \mathbf{E}\cdot d\ell + \int_b^c \mathbf{E}\cdot d\ell + \int_c^a \mathbf{E}\cdot d\ell = 0$$

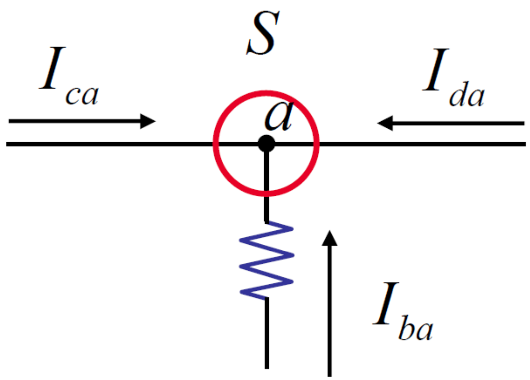
$$V_{ab} + V_{bc} + V_{ca} = 0$$

- **Kirchhoff's Voltage Law (KVL):**

  The sum of the voltages in a loop is 0.

Kirchhoff's Current Law (KCL)

- What can we say about currents at a node under the lumped matter discipline?
- Consider —



$$\oint_S \mathbf{J} \cdot d\mathbf{s} = -\frac{\partial q}{\partial t} = 0 \text{ under LMD}$$

$$I_{ba} + I_{ca} + I_{da} = 0$$

- **Kirchhoff's Current Law (KCL):**

  The sum of the currents into a node is 0.

- This is simply the conservation of charges.

KVL and KCL Summary

- Maxwell's equations simplify to algebraic KVL and KCL under LMD!
- KVL:

$$\sum_j v_j = 0 \text{ for Loop}$$

- KCL:

$$\sum_j i_j = 0 \text{ for Node}$$

Method 1:Basic KVL, KCL method of Circuit analysis

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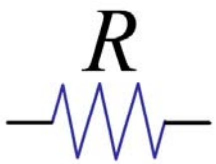
- Goal: Find all element *v*'s and *i*'s.
- Steps

  1. Write element *v*-*i* relationships (from lumped circuit abstraction).
  2. Write KCL for all nodes.
  3. Write KVL for all loops.

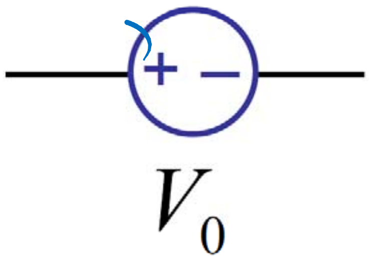
Lots of unknowns, lots of equations to be solved.

Element *v-i* relationships

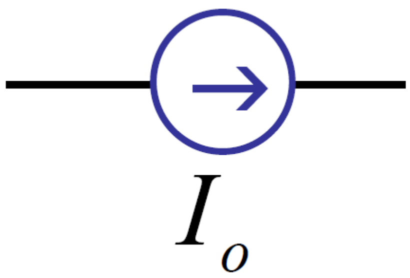
- For resistor R $$i = \frac{v}{R}$$



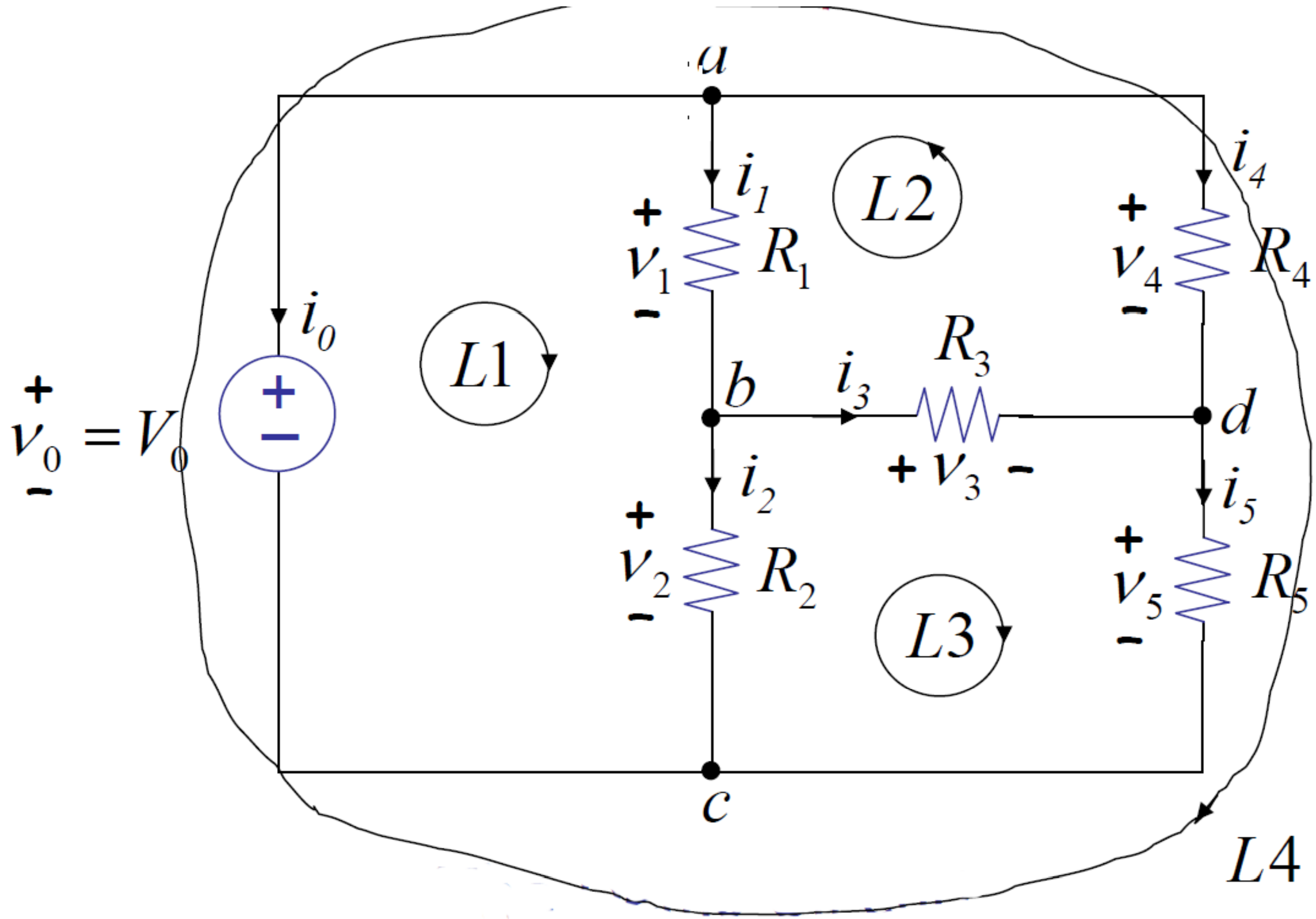
- For voltage source $$v = V_0$$



- For current source $$i = I_0$$



The circuit



Analysis

- 12 unknowns: $v_0, \ldots, v_5, i_0, \ldots, i_5$

1. Element relationships: 6 equations.

$$v_0 = V_0 \qquad v_3 = i_3 R_3$$
$$v_1 = i_1 R_1 \qquad v_4 = i_4 R_4$$
$$v_2 = i_2 R_2 \qquad v_5 = i_5 R_5$$

2. KCL at Nodes: 3 equations

$$\mathrm{a}: -i_0 - i_1 - i_4 = 0$$
$$\mathrm{b}: i_1 - i_2 - i_3 = 0$$
$$\mathrm{d}: i_3 + i_4 - i_5 = 0$$
$$\mathrm{c}: i_0 + i_2 + i_5 = 0 \quad \text{redundant}$$

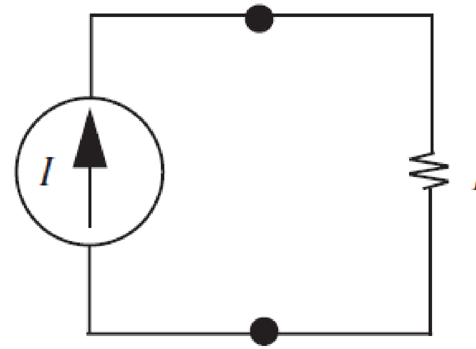
3. KVL for loops: 3 equations

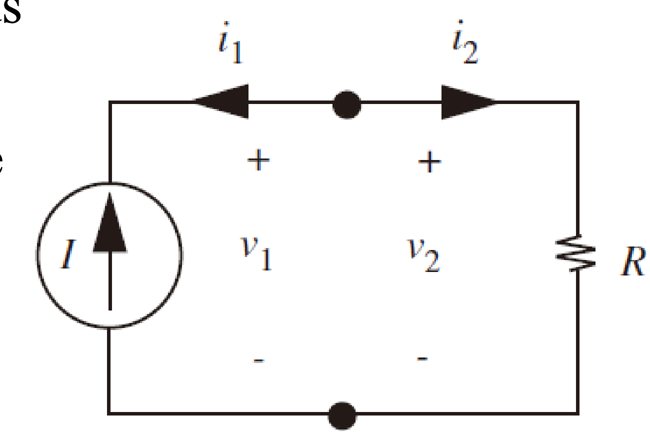
$$\mathrm{L1}: -v_0 + v_1 + v_2 = 0$$
$$\mathrm{L2}: v_1 + v_3 - v_4 = 0$$
$$\mathrm{L3}: v_3 + v_5 - v_2 = 0$$
$$\mathrm{L4}: -v_0 + v_4 + v_5 = 0 \quad \text{redundant}$$

12 unknowns with 12 equations

A very simple example

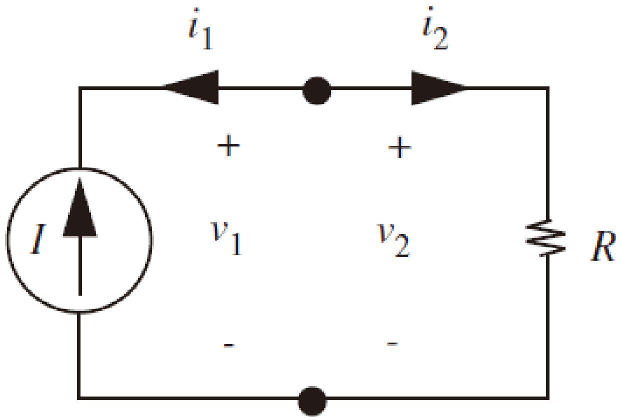
- The circuit has two branches, one independent source and one resistor, each with a current and a voltage.
- The goal of our circuit analysis is to find these branch variables.
- Two element laws:
- $i_1 = -I$ and $v_2 = Ri_2$
- The application of KCL at either node yields
- $i_1 + i_2 = 0$
- The application of KVL at either node yields
- $-v_1 + v_2 = 0$
- we have assigned a − polarity to v_1 since we first encounter the − sign when traversing the v_1 branch.
- Solve jointly to determine all four branch variables: $-i_1 = i_2 = I$ and $v_1 = v_2 = RI$





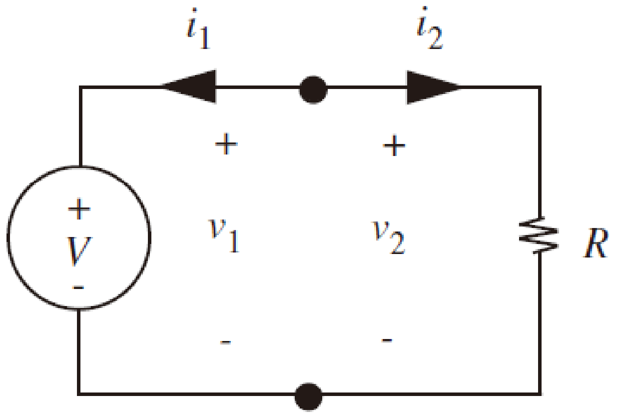
A very simple example

- The circuit has two branches, one independent source and one resistor, each with a current and a voltage.
- The goal of our circuit analysis is to find these branch variables.
- Intuitive approach:
- $i_2 = I$ and $v_2 = Ri_2$
- $v_2 = Ri_2 = RI$
- The important message here is that it is not necessary to first assemble all the circuit equations, and then solve them all at once.
- Rather, using a little intuition, it is likely to be much faster to approach the analysis in a different manner.



A very simple example

- The circuit has two branches, one independent source and one resistor, each with a current and a voltage.
- The goal of our circuit analysis is to find these branch variables.
- Intuitive approach:
- $v_2 = V$ and $i_2 = v_2/R$
- $i_2 = V/R$



- It is also important to realize that the physical results of the analysis of the circuit, cannot depend on the polarities of the definitions of the branch variables.

Voltage divider

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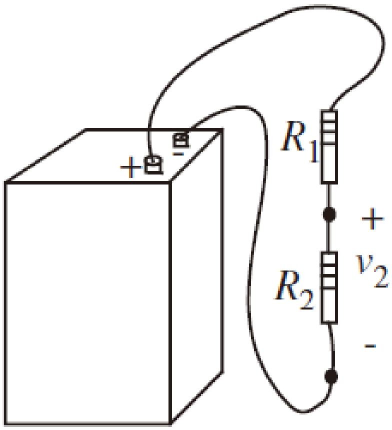
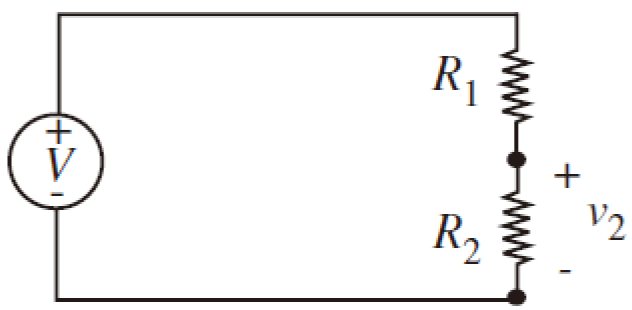
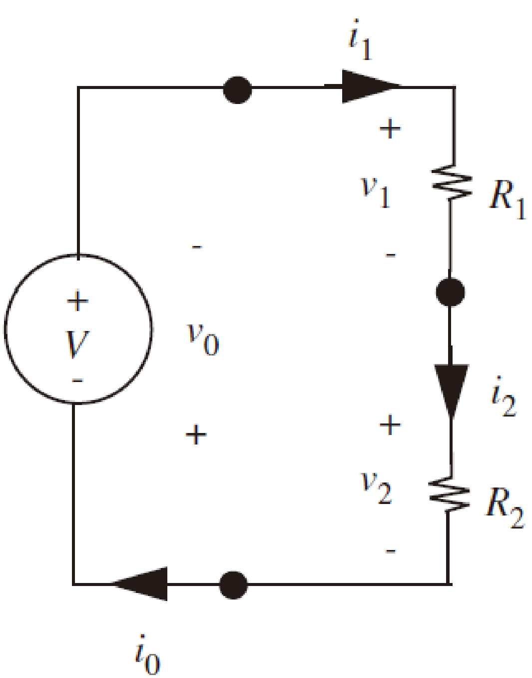
- A voltage divider is an isolated loop that contains two or more resistors and a voltage source in series.
- Here, we have connected two resistors in series, and connected the pair by some wires to a battery.
- We wish to obtain some arbitrary fraction, say 10%, of the battery voltage at the terminals marked v_2.
- Want to find the relation between v_2 and the battery voltage V and resistor values, R_1 and R_2.

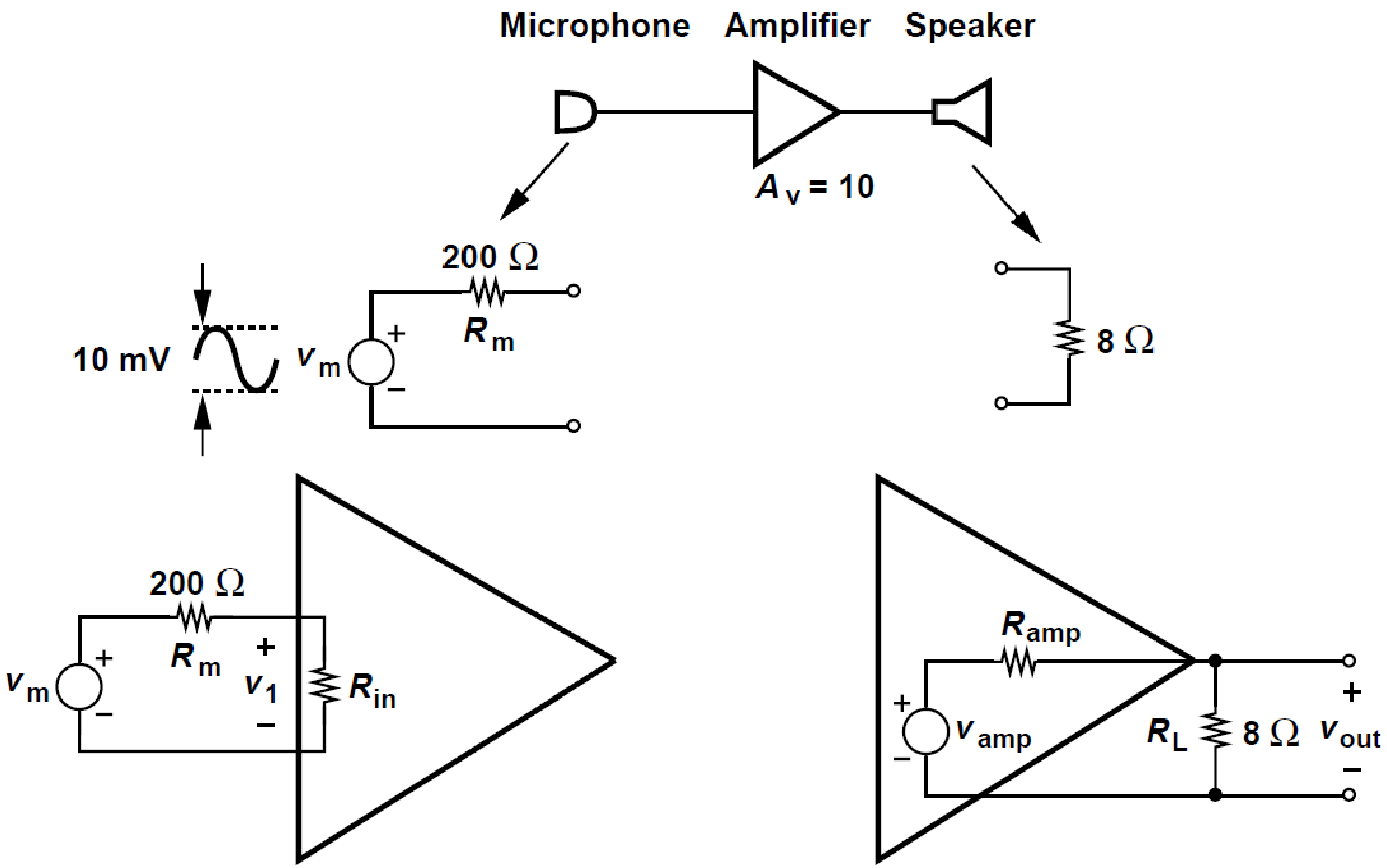






Voltage divider

- The role of input and output resistance of an amplifier.



Voltage divider

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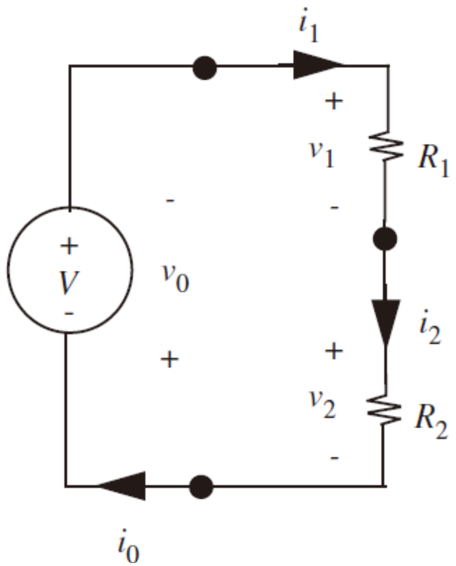
- 6 unknowns: $v_0, v_1, v_2, i_0, i_1, i_2$

1. Element relationships: 3 equations.

$$v_0 = -V$$
$$v_1 = i_1 R_1$$
$$v_2 = i_2 R_2$$



2. KCL at Nodes: 2 equations

$$i_0 = i_1$$
$$i_1 = i_2$$

3. KVL for loops: 1 equation

$$v_0 + v_1 + v_2 = 0$$

$$\Rightarrow$$

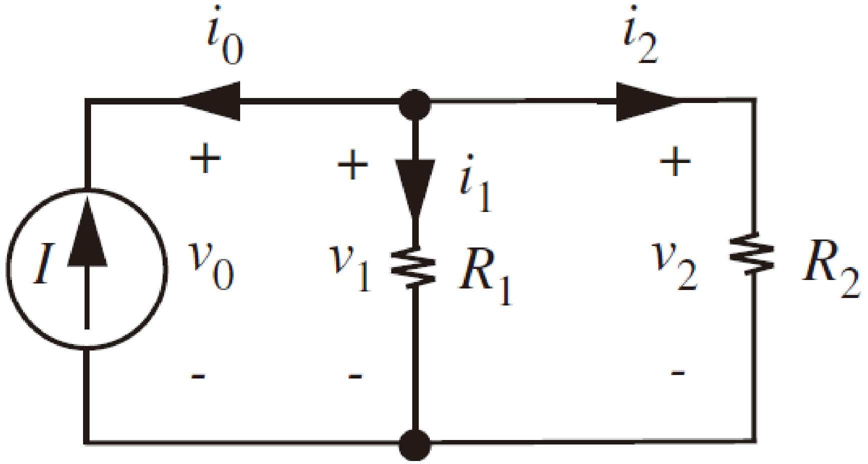
$$i_0 = i_1 = i_2 = \frac{1}{R_1 + R_2} V$$

$$v_1 = \frac{R_1}{R_1 + R_2} V$$

$$v_2 = \frac{R_2}{R_1 + R_2} V$$

Current divider

- A current divider is a circuit with two nodes joining two or more parallel resistors and a current source.
- Here, a current divider with two resistors is shown.
- In this circuits, the resistors share, or divide, the current from the source in proportion to its conductance.
- Want to find the relation between i_2 (or i_1) and the current I and resistor values, R_1 and R_2.



Current divider

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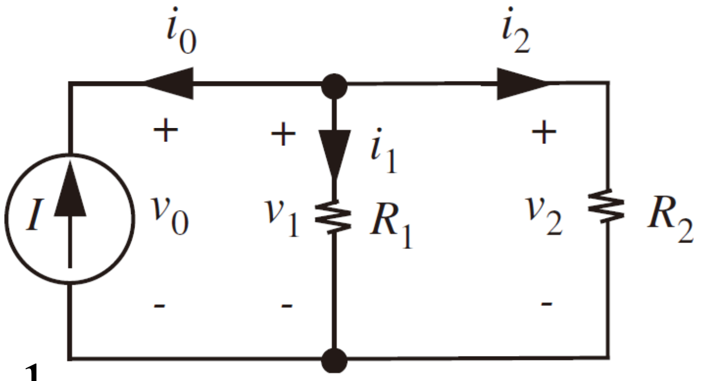
- 6 unknowns: $v_0, v_1, v_2, i_0, i_1, i_2$



1. Element relationships: 3 equations.

$$i_0 = -I$$

$$v_1 = i_1 R_1 \quad \text{or} \quad i_1 = v_1 G_1$$

$$v_2 = i_2 R_2 \quad \text{or} \quad i_2 = v_2 G_2 \quad \text{where} \quad G_i = \frac{1}{R_i}$$

2. KCL at Nodes: 1 equation

$$i_0 + i_1 + i_2 = 0$$

3. KVL for loops: 2 equations

$$v_0 = v_1$$

$$v_1 = v_2$$

$$\Rightarrow$$

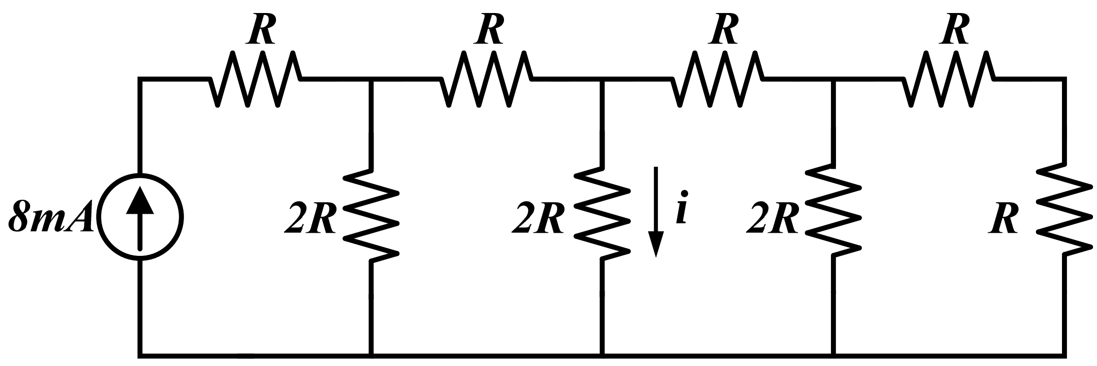
$$v_0 = v_1 = v_2 = \frac{I}{G_1 + G_2}$$

$$i_1 = \frac{G_1}{G_1 + G_2} I$$

$$i_2 = \frac{G_2}{G_1 + G_2} I$$

Example for Current divider

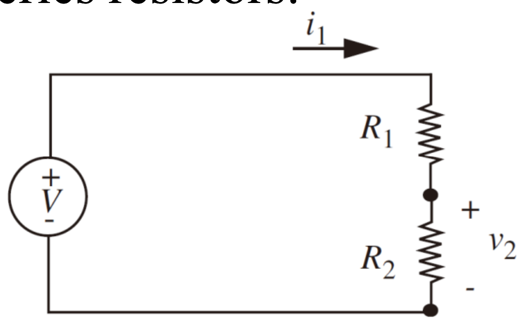
- Determine the indicated branch current *i*. i.e. *i* = ?



- This circuit is used in a D/A convertor.

Equivalent Resistances

- Series resistors:



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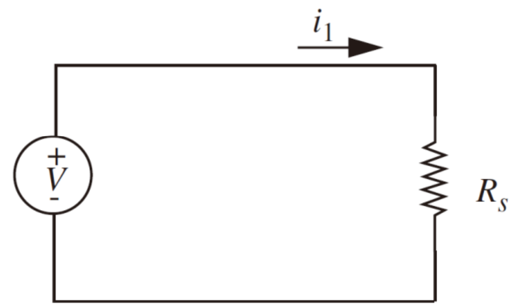
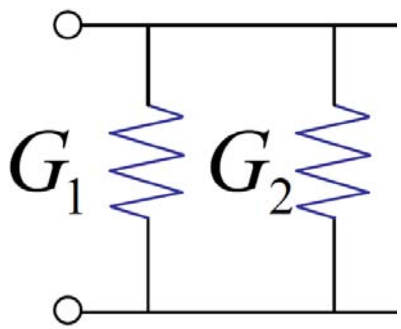


$$i_1 = \frac{V}{R_1 + R_2}$$

$$i_1 = \frac{V}{R_S} \quad \Rightarrow \quad R_S = R_1 + R_2$$

- Parallel resistors:



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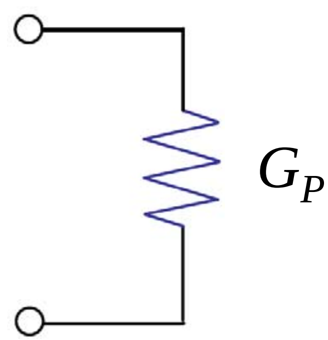
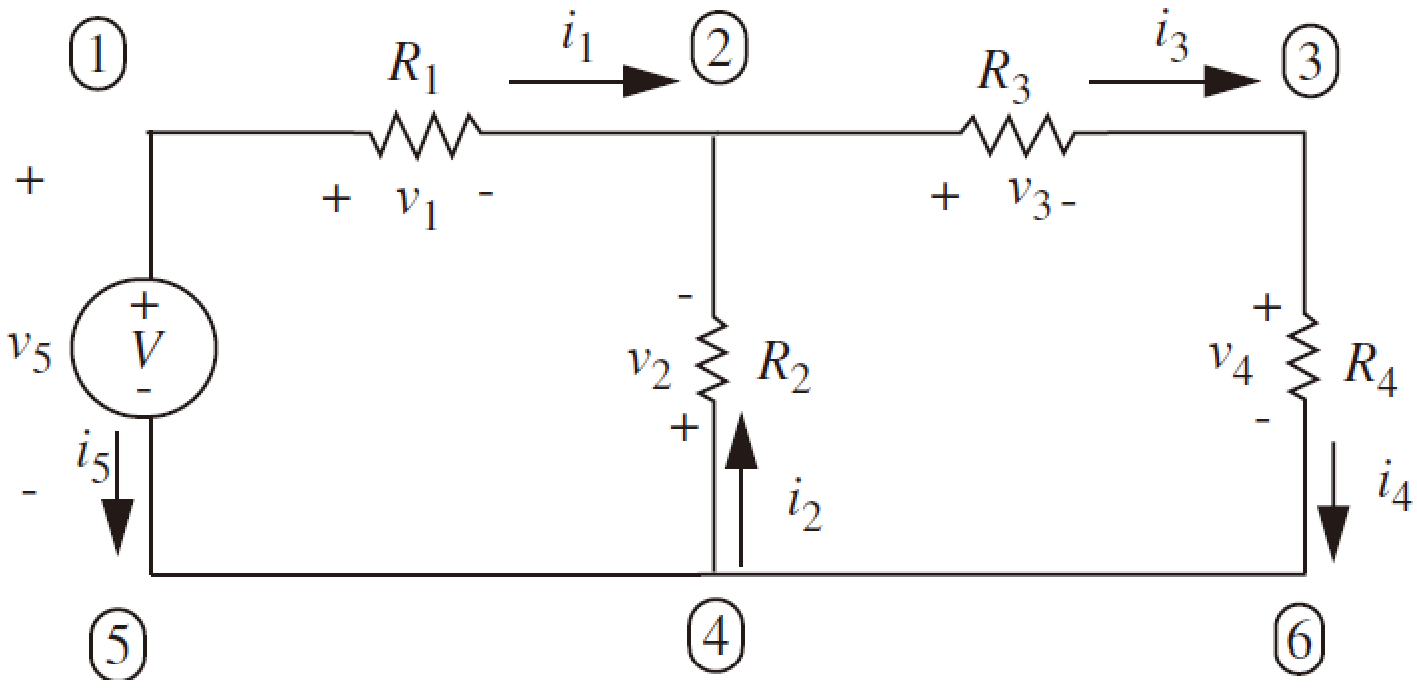


$$G_P = G_1 + G_2$$

A More Complex circuit



Analysis

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- 10 unknowns: $v_1,\ldots,v_5, i_1,\ldots,i_5$

1. Element relationships: 5 equations.

$$v_1 = i_1 R_1 \quad v_4 = i_4 R_4$$
$$v_2 = i_2 R_2 \quad v_5 = V$$
$$v_3 = i_3 R_3$$

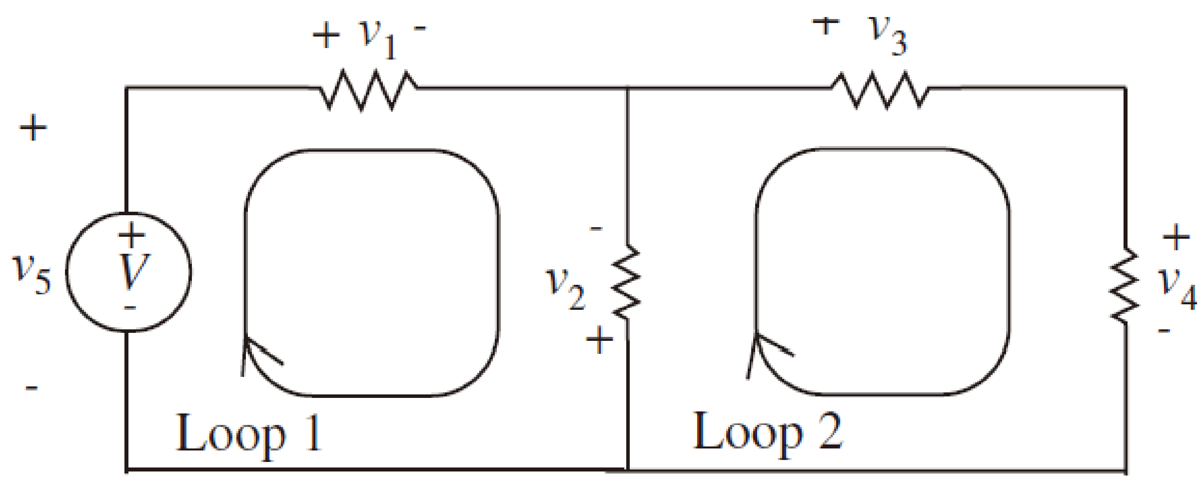
2. KCL at Nodes: 3 equations

$$1: -i_5 - i_1 = 0$$
$$2: i_1 + i_2 - i_3 = 0$$
$$3: i_3 - i_4 = 0$$

3. KVL for loops: 2 equations

$$\mathrm{L1}: -v_5 + v_1 - v_2 = 0$$
$$\mathrm{L2}: v_2 + v_3 + v_4 = 0$$

10 unknowns with 10 equations



Solutions

$$-i_5 = i_1 = \frac{R_2 + R_3 + R_4}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)} V$$

$$i_2 = -\frac{R_3 + R_4}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)} V$$

$$i_3 = i_4 = \frac{R_2}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)} V$$

Solutions

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$$v_1 = \frac{R_1(R_2+R_3+R_4)}{R_1(R_2+R_3+R_4)+R_2(R_3+R_4)}V$$

$$v_2 = -\frac{R_2(R_3+R_4)}{R_1(R_2+R_3+R_4)+R_2(R_3+R_4)}V$$

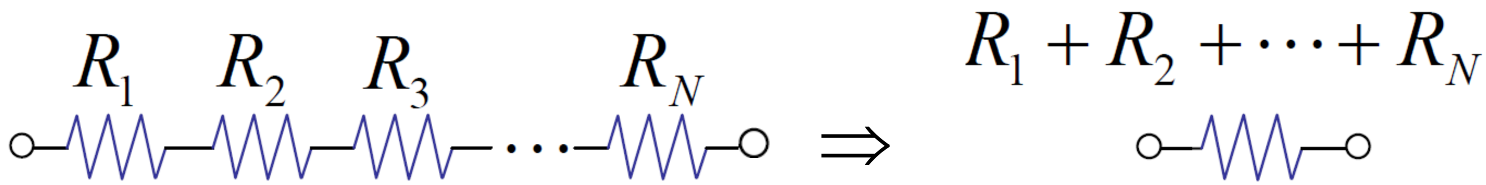
$$v_3 = \frac{R_2R_3}{R_1(R_2+R_3+R_4)+R_2(R_3+R_4)}V$$

$$v_4 = \frac{R_2R_4}{R_1(R_2+R_3+R_4)+R_2(R_3+R_4)}V.$$

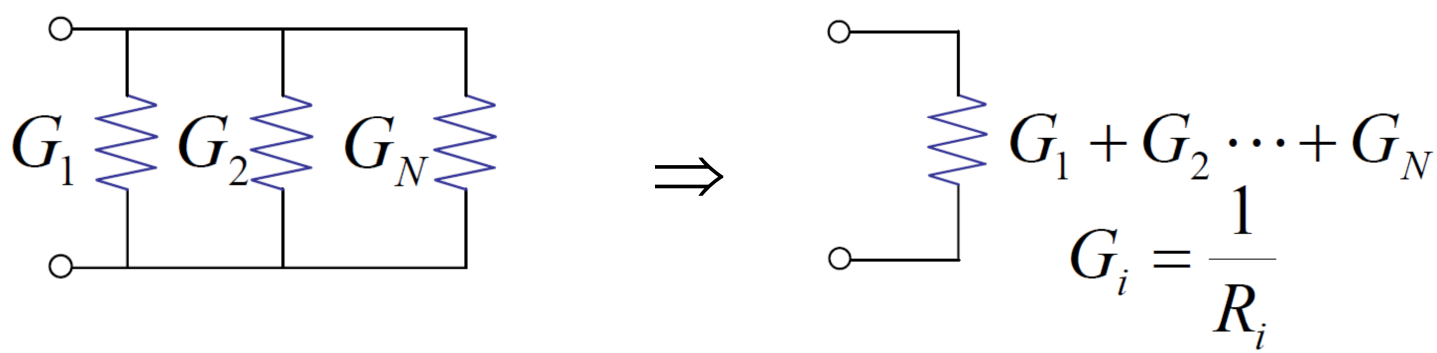
$$v_5 = V$$

Method 2: Intuitive method of circuit analysis: Element combination rules

- By applying KCL and KVL, we have
- Series resistors

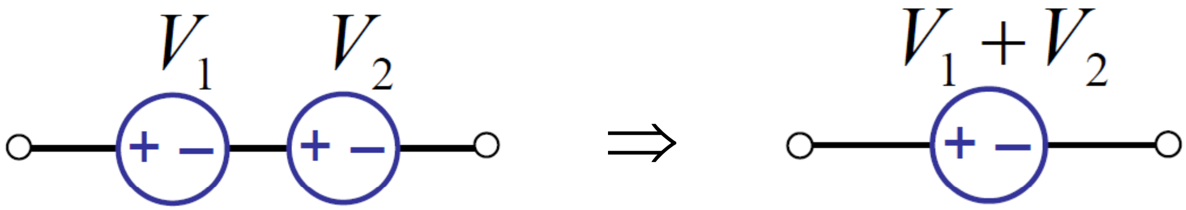


- Parallel resistors

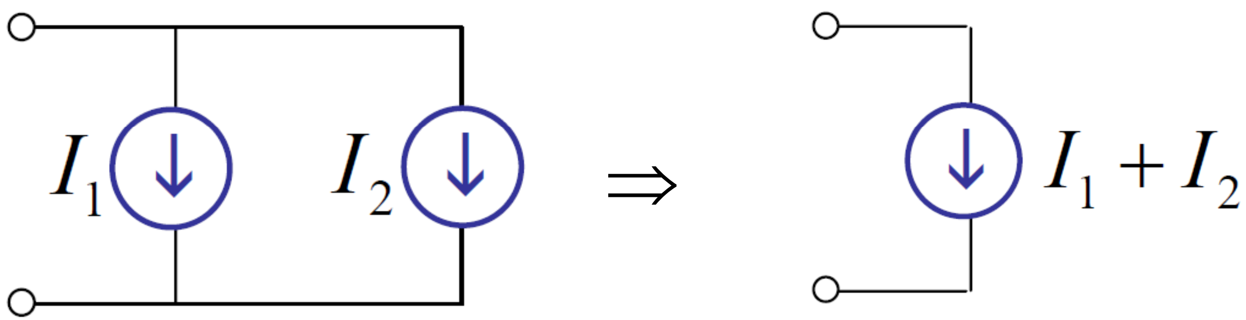


Element combination rules

- Voltage sources

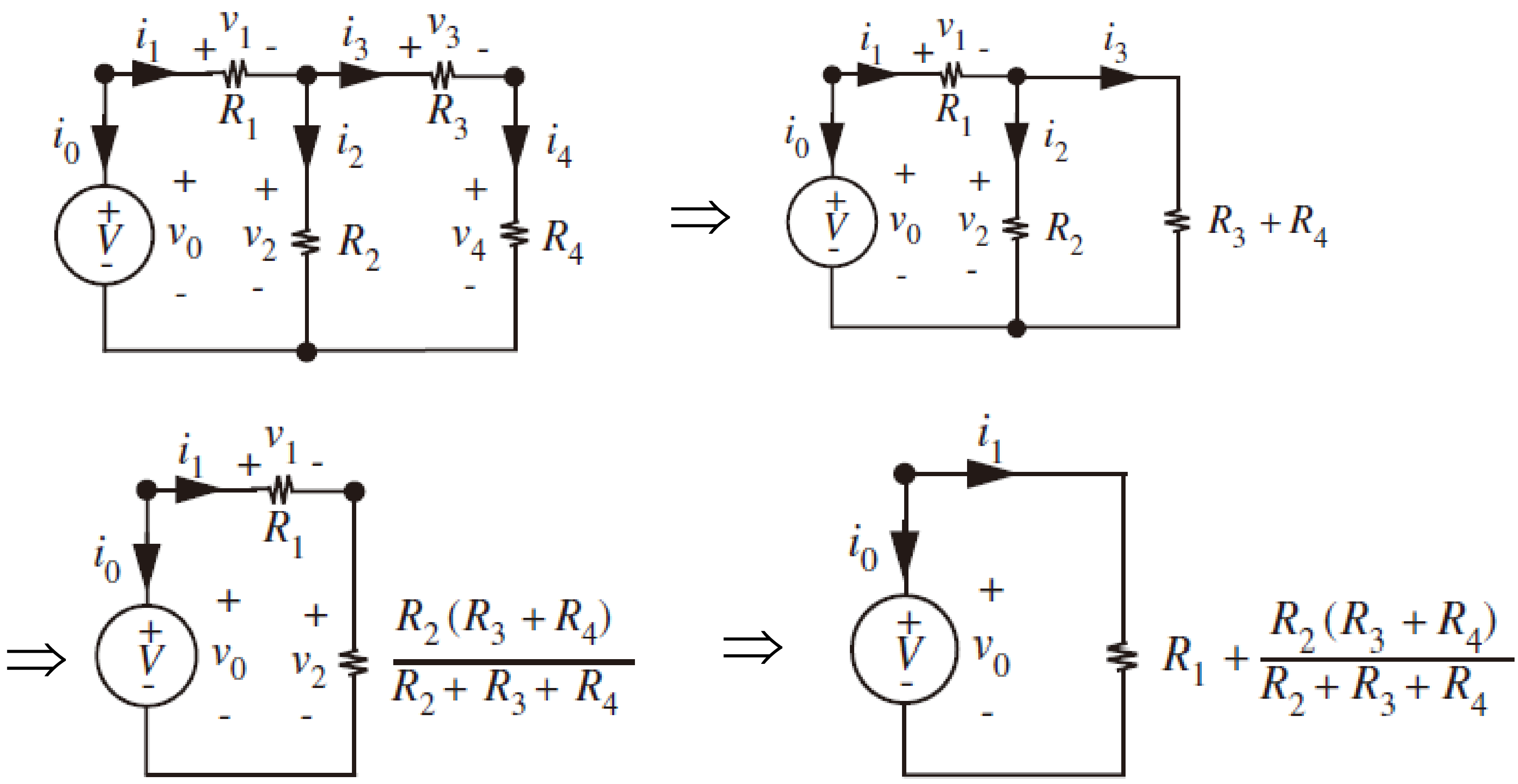


- Current sources



Collapse then Expand Method

- Collapse（簡化）



Collapse then Expand Method

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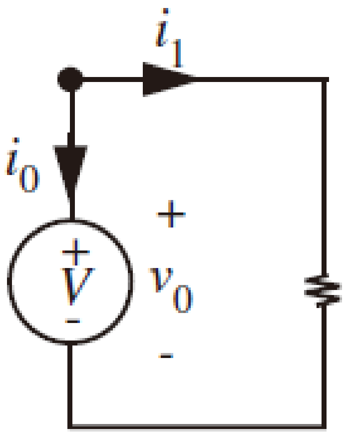
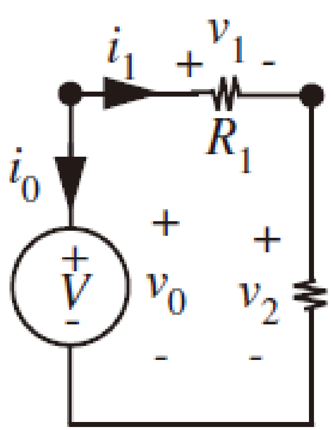
- Expand（展開）
- Find i_1.



$$R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}$$

$$i_1 = \frac{V}{R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}}$$

- Find v_2.



$$\frac{R_2(R_3+R_4)}{R_2+R_3+R_4}$$

$$v_2 = \frac{\frac{R_2(R_3+R_4)}{R_2+R_3+R_4}}{R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}} V$$

Collapse then Expand Method

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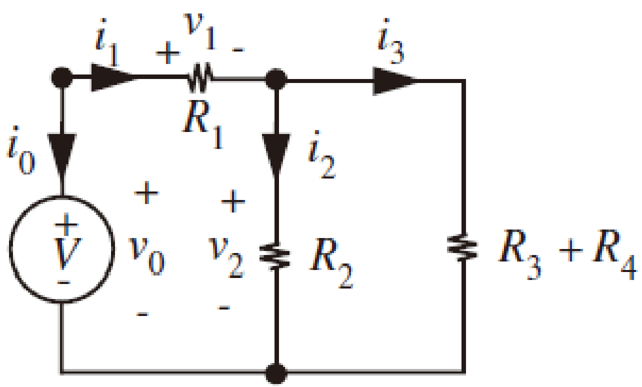
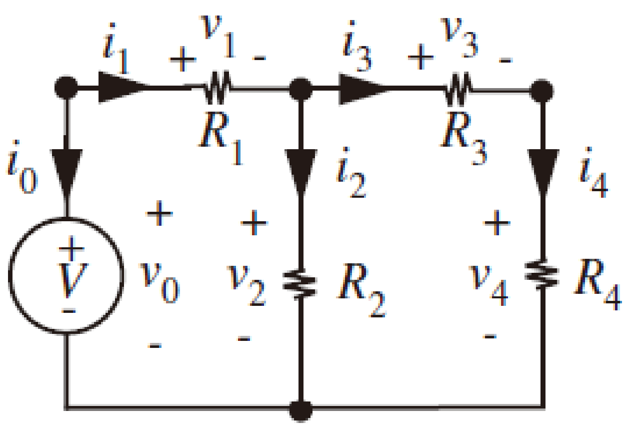
- Expand（展開）
- Find i_2 and i_3.



$$i_2 = \frac{v_2}{R_2}$$

$$i_3 = \frac{v_2}{R_3 + R_4}$$

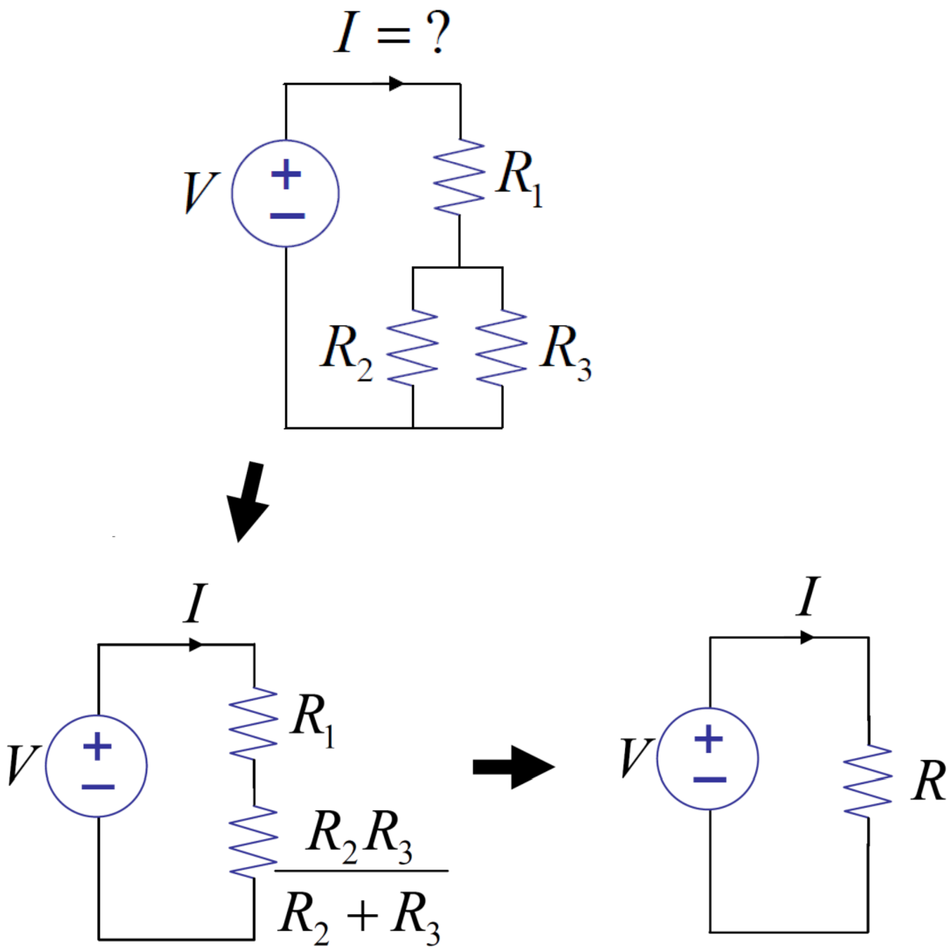
- Find v_3 and v_4.



$$v_3 = i_3 R_3$$

$$v_4 = i_3 R_4$$

Example

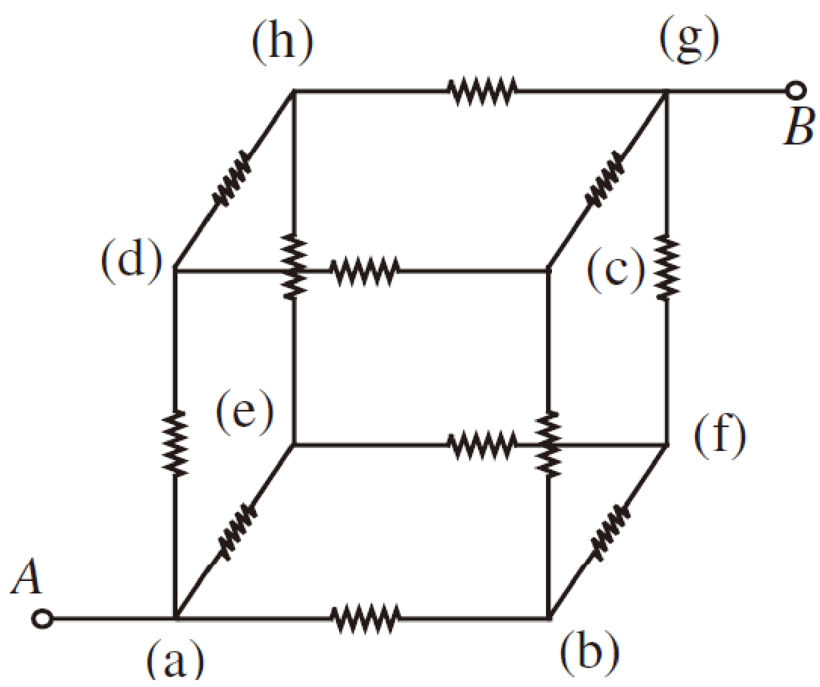


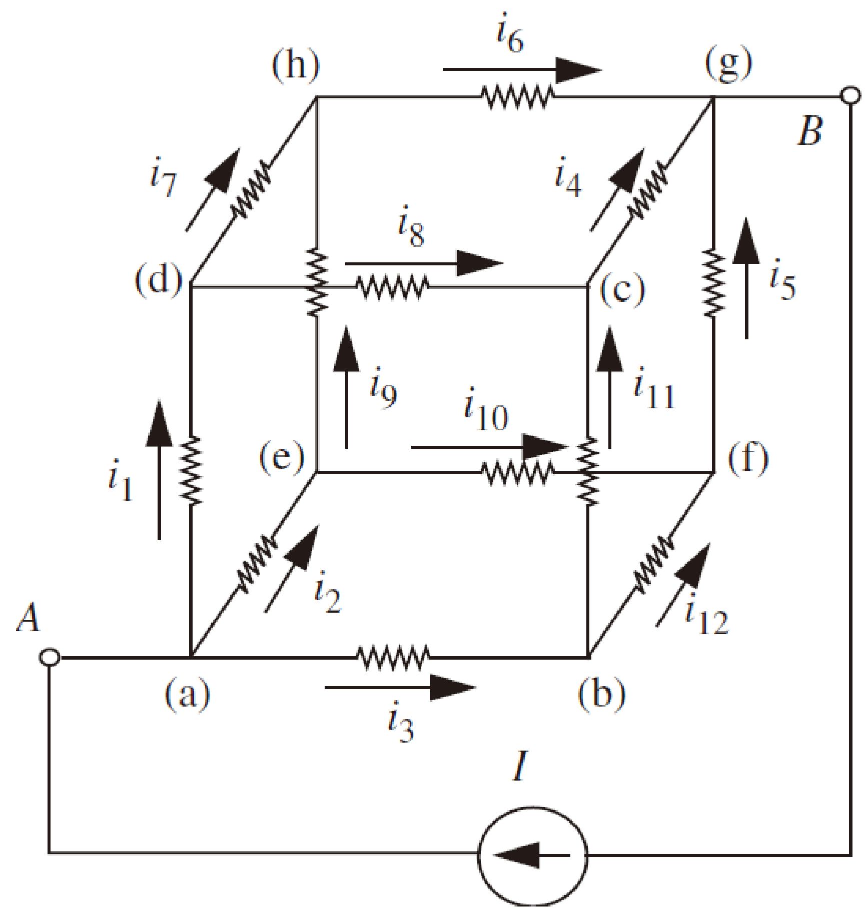
$$\Rightarrow \quad I = \frac{V}{R}$$

$$R = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

Example

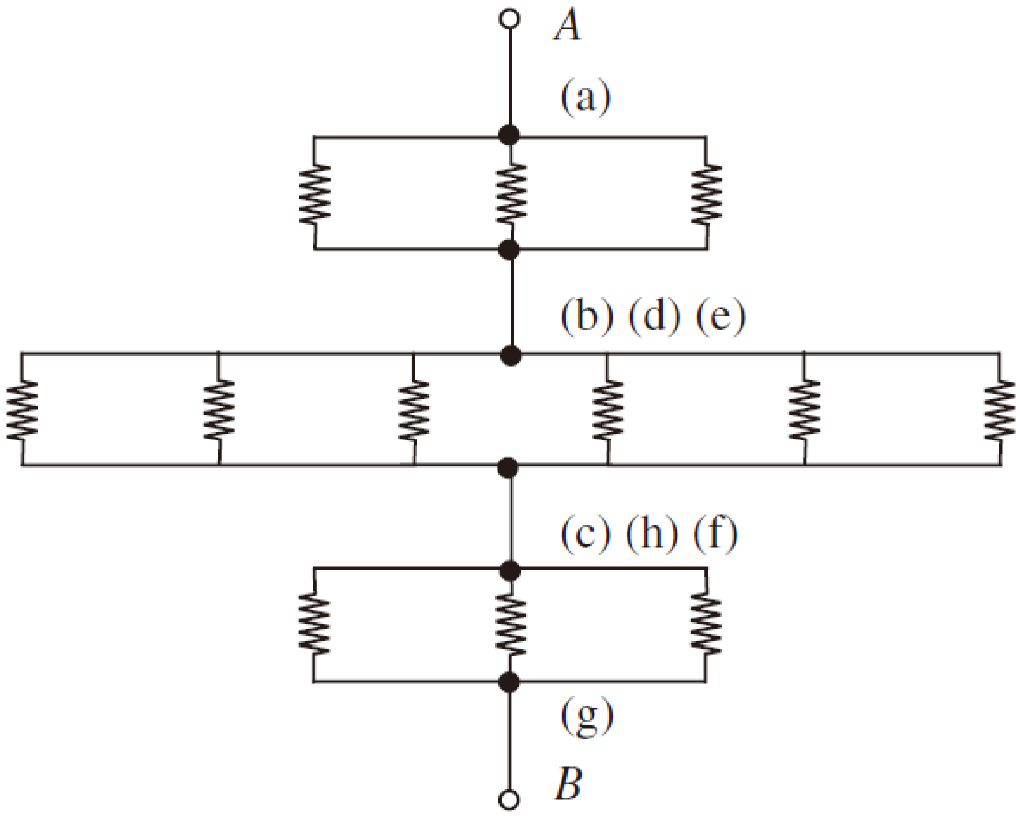
- Determine the network equivalent resistance between terminal A and B.
- We can inject a current I into the network and find (or measure) the voltage between A and B.

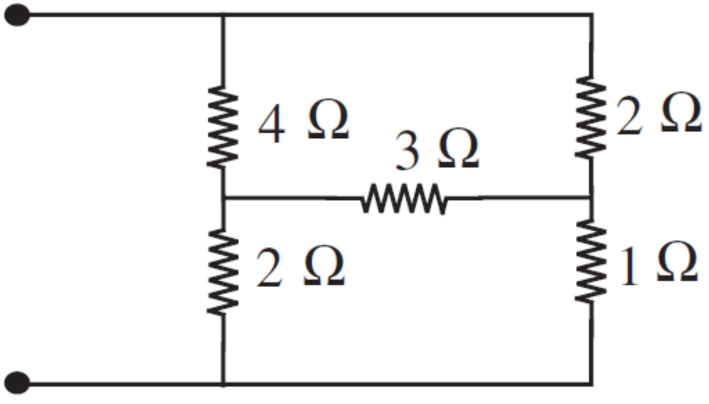




Example

- Determine the network equivalent resistance between terminal A and B.





Δ-Y conversion

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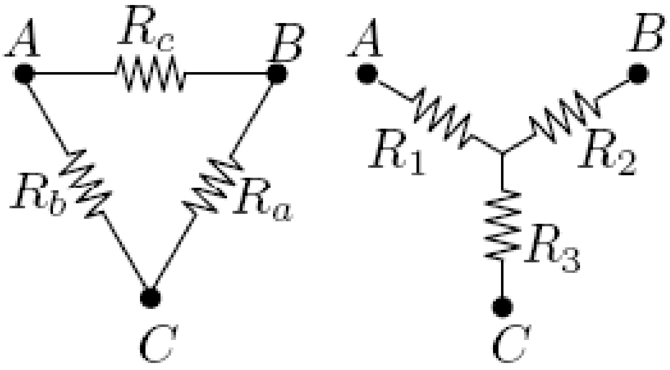
Delta-Wye Conversion



$$R_{AB} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = R_1 + R_2$$

$$R_{BC} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} = R_2 + R_3$$

$$R_{CA} = \frac{R_b(R_c + R_a)}{R_a + R_b + R_c} = R_3 + R_1$$

$$R_1 = \frac{R_c R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

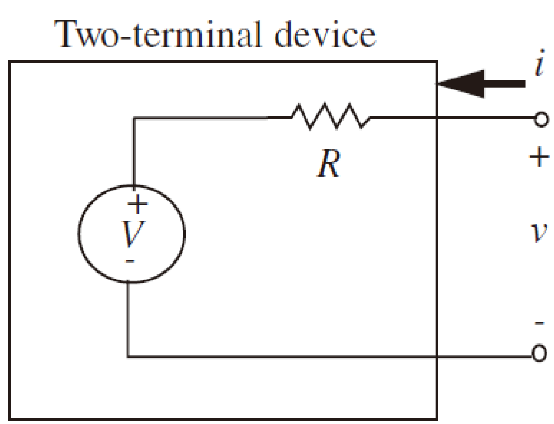
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

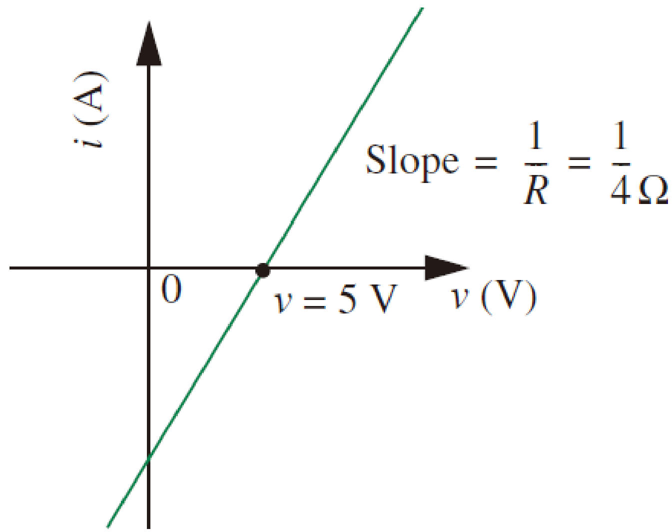
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

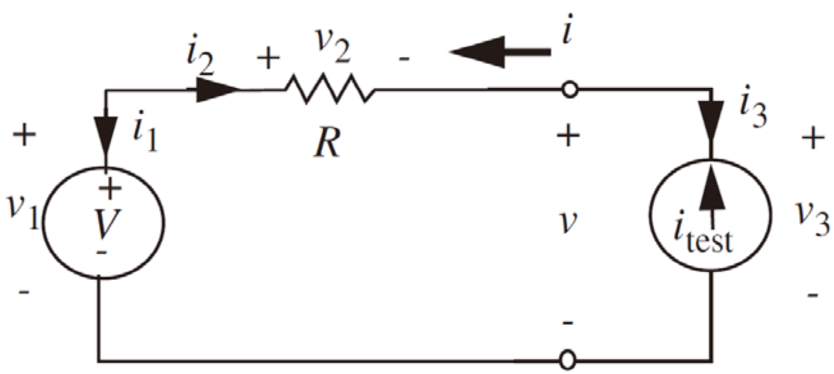
Determine the *i-v* relationship for the two-terminal device

- The two-terminal devices under test





- Test circuit



$$v = V + i_{test} R$$

$$i = \frac{v - V}{R}$$

Conclusion

- KVL and KCL

$$\sum_j v_j = 0 \text{ for Loop}$$

$$\sum_j i_j = 0 \text{ for Node}$$

- KVL, KCL method of circuit analysis
- Element combination rules