Introduction and Lumped Circuit Abstraction

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What is engineering?

- Engineering is the purposeful use of science by Stephen D. Senturia.
- Electrical engineering is one of many engineering.
- Electrical engineering is the purposeful use of Maxwell's Equations (or Abstractions) for electromagnetic phenomena.
- Maxwell's Equations

Gauss's Law $\nabla \cdot \mathbf{D} = \rho$

Gauss's Law $\nabla \cdot \mathbf{B} = 0$

Faraday's Law $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

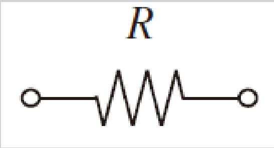
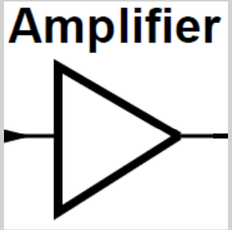
Ampere's Law $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$

Coninuity Eq. $\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$

What is EE2210 about?

- Gainful employment of Maxwell’s equations
- From electrons to digital gates and op-amps.

<table>
<tr><th>Nature as observed in experiments</th><th>Physics laws or “abstractions”</th><th>Lumped circuit abstraction</th><th>Simple amplifier abstraction Electronics (EE 2255)</th></tr>
<tr><td><table><tr><th>V (V)</th><th>3</th><th>6</th><th>9</th><th>12</th></tr><tr><td>I (mA)</td><td>1</td><td>2</td><td>3</td><td>4</td></tr></table></td><td>Maxwell’s Eqs

Ohm’s Law</td><td></td><td></td></tr>
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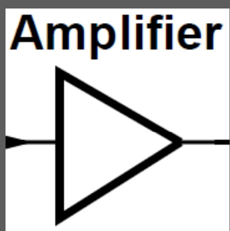
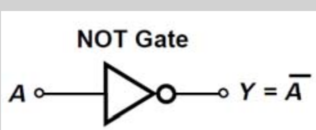
Analog and Digital Abstraction

Analog  	**Analog Integrated Circuits Analysis and Design (EE 3235)** **Electronic Circuit Design (EE 4280)** **Introduction to Integrated Circuit Design (EE 4290)** **Power Electronics (EE 4830)** **Power System (I) (EE 4710)**	**Embedded System Laboratory (EE 2405)** **Special Topic on Implementation (EE 3900)**
Digital  	Logic design (EE 2280) Introduction to Programming (EE 2310) Data Structures (EE 2410) Microprocessor Systems (EE 2401) Computer Architecture (EE 3450)	

Lumped Circuit Abstraction

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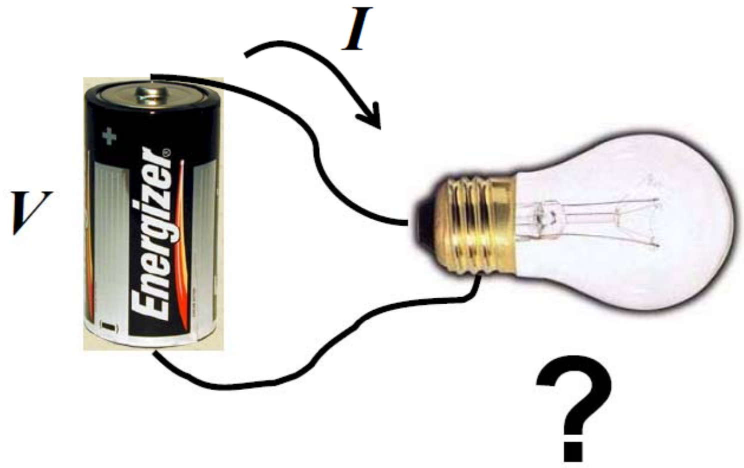
- The Big Jump from physics to EE
- Consider



- Suppose we wish to answer this question:
  What is the current through the bulb?

We could do it the Hard Way…

- Apply Maxwells Equations :

Gauss's Law $\nabla \cdot \mathbf{D} = \rho$ $\oint_S \mathbf{D} \cdot d\mathbf{s} = \oint_V \rho dv$

$\nabla \cdot \mathbf{B} = 0$ $\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$

Faraday's Law $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ $\oint_C \mathbf{E} \cdot d\ell = \int_S \left(-\frac{\partial \mathbf{B}}{\partial t} \right) \cdot d\mathbf{s} = -\frac{\partial \phi_B}{\partial t}$

Ampere's Law $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$ $\oint_C \mathbf{H} \cdot d\ell = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{s}$

Coninuity Eq. $\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$ $\oint_S \mathbf{J} \cdot d\mathbf{s} = \oint_V \left(-\frac{\partial \rho}{\partial t} \right) dv$

Instead, there is an Easy Way…

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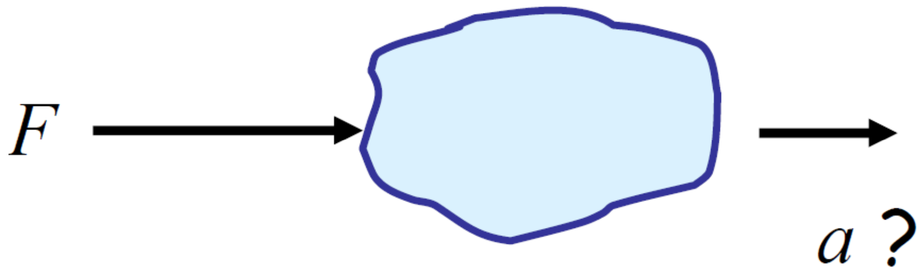
- First, let us build some insight:

Analogy



- I ask you: What is the acceleration?

You may quickly ask me: What is the mass?

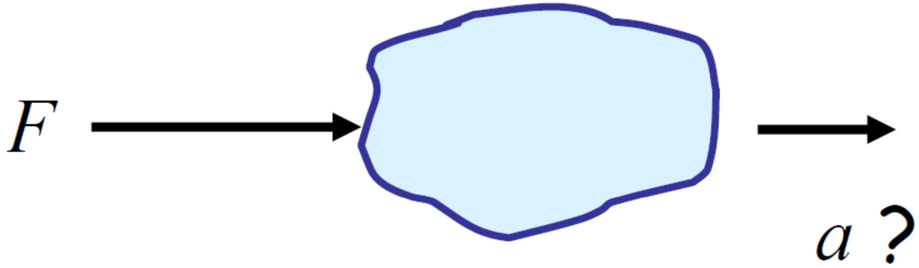
I tell you: m

You respond: $a = F/m$ **Done!!!**

Instead, there is an Easy Way…

- First, let us build some insight:

  Analogy



- In doing so, you ignored
  - The object's shape
  - Its temperature
  - Its color
  - Point of force application
- **Point-mass discretization**

The Easy Way…

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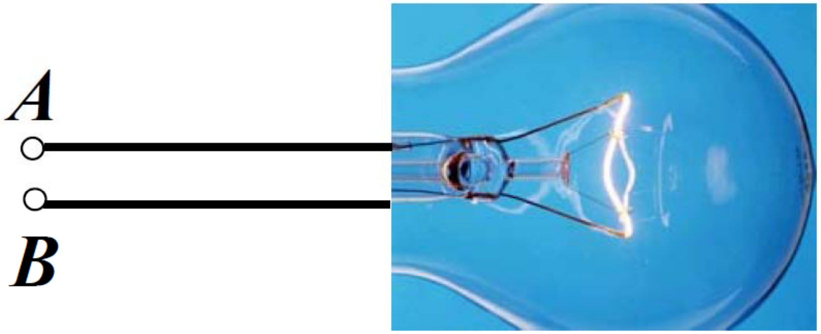
- Consider the filament of the light bulb.



- We do not care about
  - how current flows inside the filament
  - its temperature, shape, orientation, etc.
- Then, we can replace the bulb with a

  discrete resistor

  for the purpose of calculating the current.

The Easy Way…

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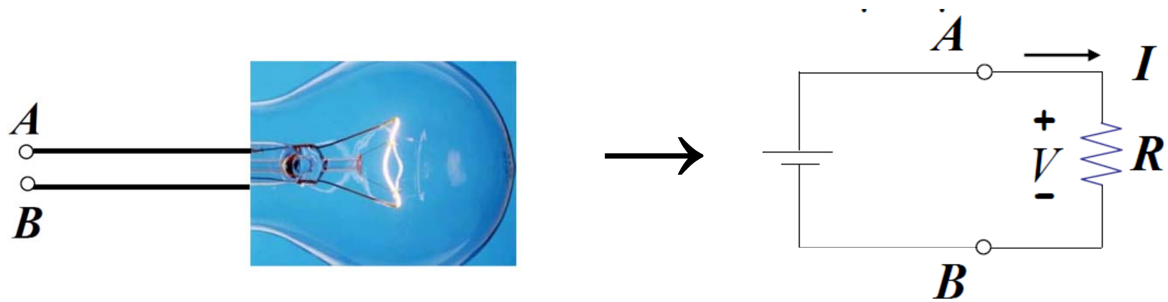
- Replace the bulb with a
  discrete resistor
  for the purpose of calculating the current.



$$I = \frac{V}{R}$$

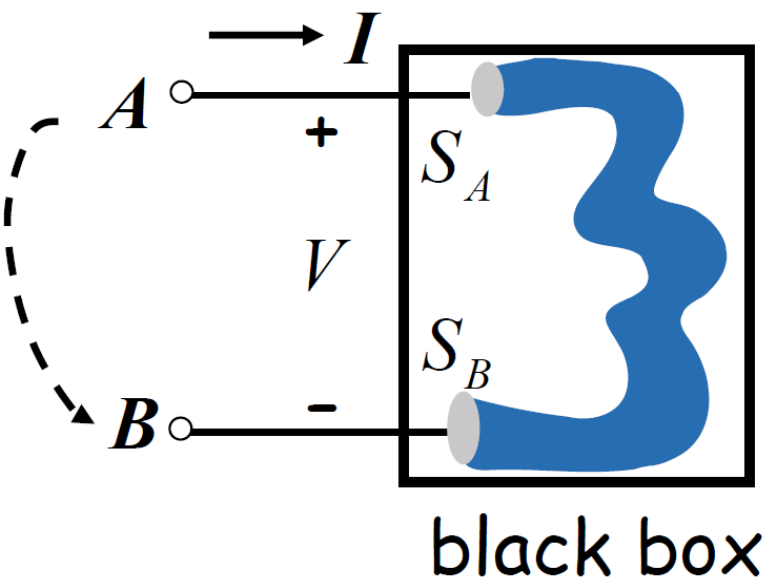
- In EE, we do things the easy way…
  R represents the only property of interest!
  Like with point-mass: replace objects with their mass m
  To find $a = F/m$

R is a lumped element abstraction for the bulb?

- Not so fast, though …
- Although we will take the easy way using lumped abstractions for the rest of this course, we must make sure (at least the first time) that our abstraction is reasonable. In this case, ensuring that

V and ***I***

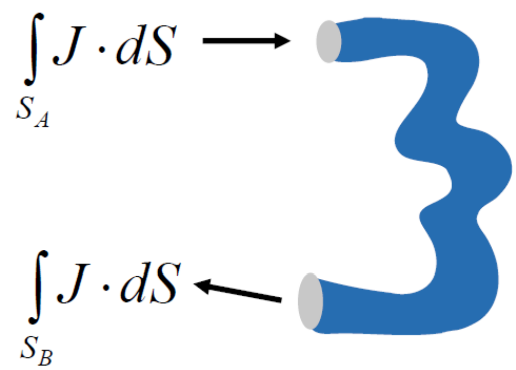
are defined for the element



Current $\underline{I}$ must be defined

- True when

  I into $S_A = I$ out of S_B



- True only when $\frac{\partial q}{\partial t} = 0$ in the filament is zero.

- From Maxwell's Eq, $\oint_S \mathbf{J} \cdot d\mathbf{s} = -\int_{S_A} \mathbf{J} \cdot d\mathbf{s} + \int_{S_B} \mathbf{J} \cdot d\mathbf{s} = \int_V \left(-\frac{\partial \rho}{\partial t} \right) dv$

$$I_A - I_B = \frac{\partial q}{\partial t}$$

⇨ $I_A = I_B$ only if $\frac{\partial q}{\partial t} = 0$ Let us assume this $\frac{\partial q}{\partial t} = 0$

Voltage $\underline{V}$ must be defined

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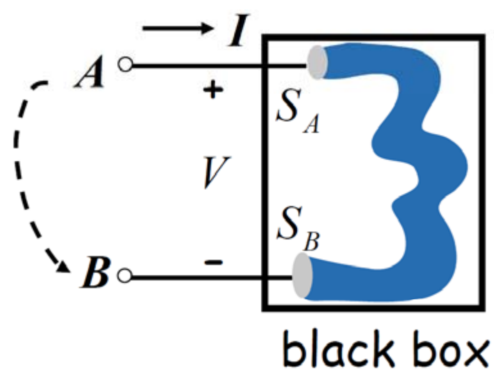
- True when
  V is uniquely defined



- True only when $\dfrac{\partial \phi_B}{\partial t} = 0$ for any closed loop outside is zero.
- From Maxwell's Eq, $\oint_C \mathbf{E} \cdot d\ell = \int_A^B \mathbf{E} \cdot d\ell - V_{AB} = \int_S \left(-\dfrac{\partial \mathbf{B}}{\partial t} \right) \cdot d\mathbf{s} = -\dfrac{\partial \phi_B}{\partial t}$

$$V_{AB} - \int_A^B \mathbf{E} \cdot d\ell = \frac{\partial \phi_B}{\partial t}$$

$\Rightarrow V_{AB} = \int_A^B \mathbf{E} \cdot d\ell$ is defined only if $\dfrac{\partial \phi_B}{\partial t} = 0$ Let us assume this $\dfrac{\partial \phi_B}{\partial t} = 0$

Lumped Matter Discipline (LMD)

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- Or self imposed constraints:
- *Choose lumped element boundaries such that the rate of change of magnetic flux linked with any closed loop outside an element must be zero for all time.*

$$\frac{\partial \phi_B}{\partial t} = 0 \quad \text{through any closed path outside the element.}$$

- *Choose lumped element boundaries so that there is no total time varying charge within the element for all time.*

$$\frac{\partial q}{\partial t} = 0 \quad \text{where q is the total charge inside the element.}$$

- *Operate in the regime in which signal timescales of interest are much larger than the propagation phase delay of electromagnetic waves across the lumped elements.* $kL << 1 \Rightarrow L << \frac{\lambda}{2\pi}$

Two-Terminal Element

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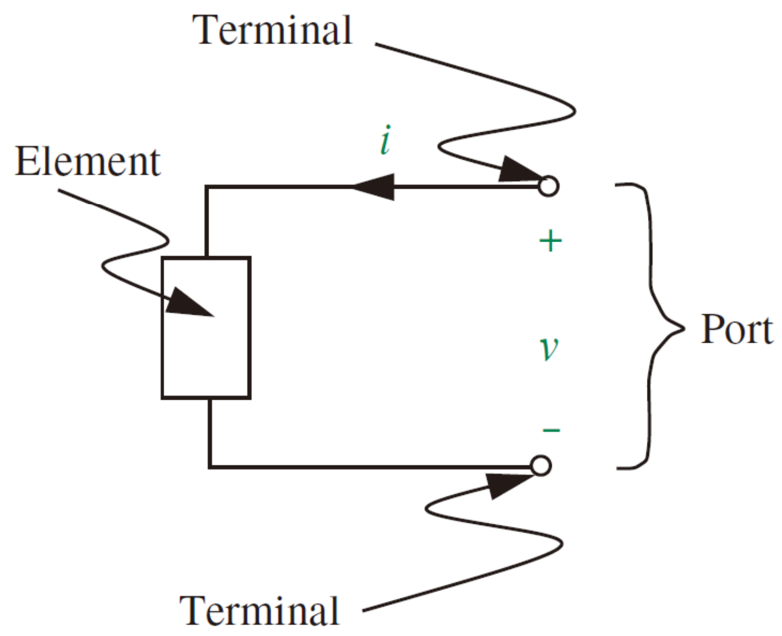
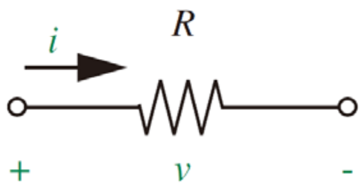
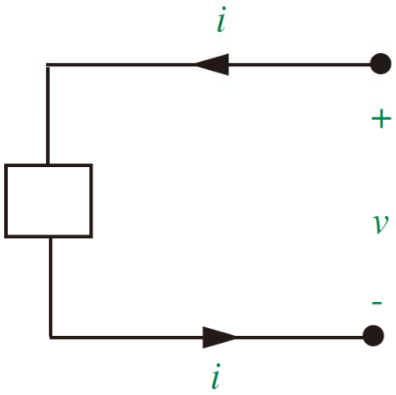
- Electronic access to an element is made through its terminals. At times, terminals are paired together in a natural way to form ports. An example of an arbitrary element with two terminals and one port is shown here. Elements may have three or more terminals, and two or more ports.



Associated Variables Convention

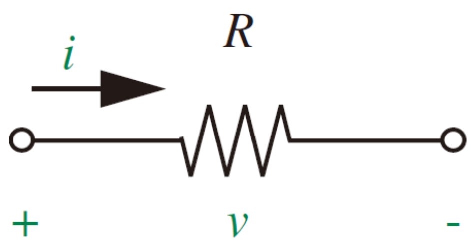
- Associated Variables Convention Define current to flow in at the device terminal assigned to be positive in voltage.



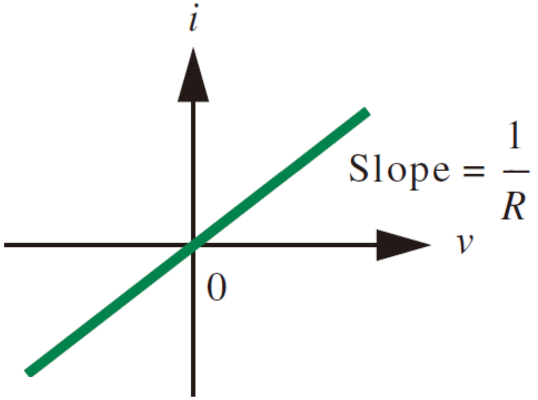


- When the voltage v and current i for an element are defined under the associated variables convention, the power into the element is positive when both v and i are positive, $p = vi$.

Linear Resistor



and $i = \dfrac{v}{R}$



- R represents the only property of interest!

  R relates element v and i

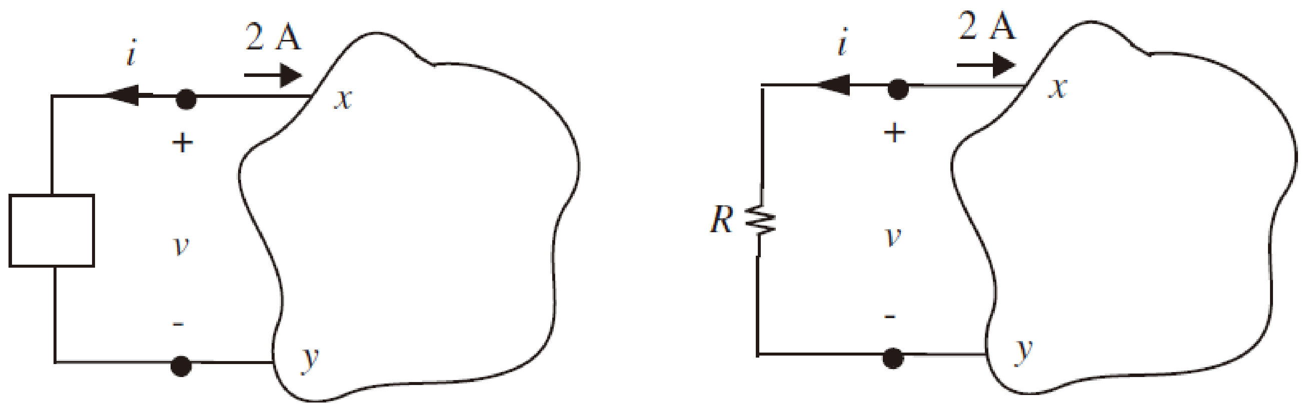
$$i = \frac{v}{R}$$ called element v-i relationship

- R is a lumped element abstraction for the bulb.

- Power consumed by element is vi

Examples

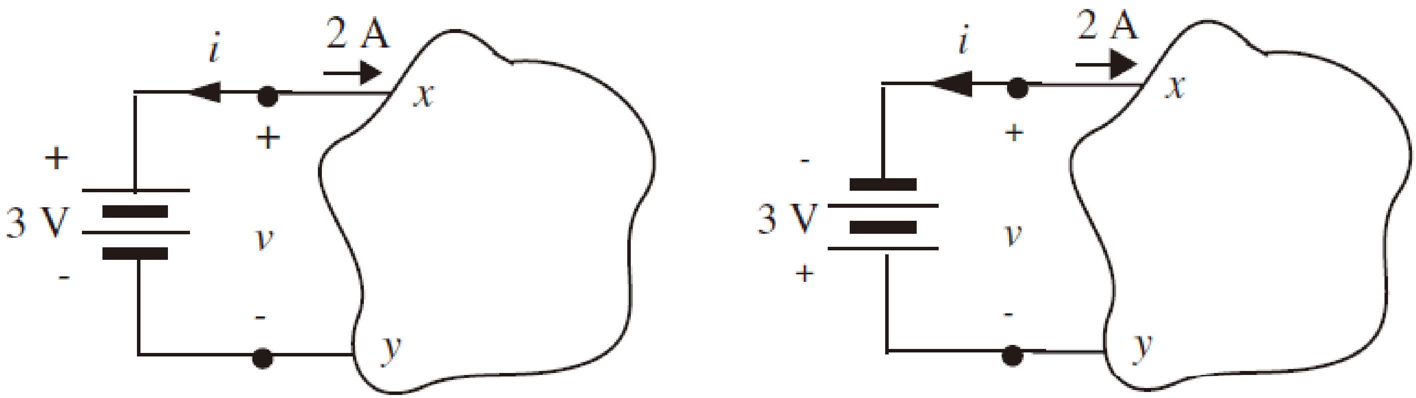
- Suppose that a current of 2 A flows into the circuit terminal marked x. What is the value of terminal variable i?



- $i = -2$ A
- Suppose that the two terminal element is a resistor with resistance R = 10 Ohms. Determine the value of v.
- $v = iR = (-2)10 = -20$ V

Examples

- Suppose that the two terminal element is a 3 V battery with the polarity as shown below left. Determine the values of terminal variables *v* and *i*.



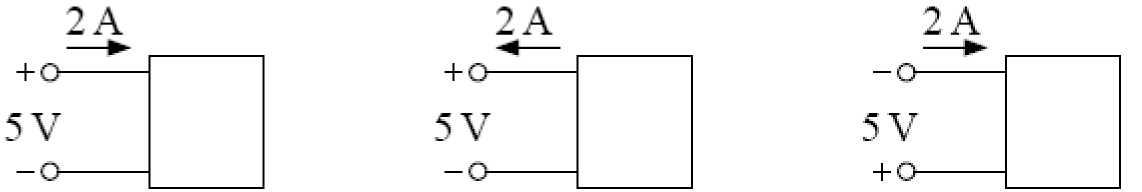
- *i* = −2 A and *v* = 3 V.
- Suppose that the two terminal element is a 3 V battery with the polarity as shown above right. Determine the values of terminal variables *v* and *i*.
- *i* = −2 A and *v* = −3 V

Power and Energy

- Power is the time rate of expending or absorbing energy.

$$P = \frac{dW}{dt} \quad \left(Watt, \; Joule/\text{sec} \right)$$

$$= \frac{dW}{dq} \cdot \frac{dq}{dt} = v \cdot i$$



$5 \times 2 = 10\ (\text{W})$ $\quad$ $5 \times (-2) = -10\ (\text{W})$ $\quad$ $(-5) \times 2 = -10\ (\text{W})$

- Energy is the capacity to do *work.*

$$W = \int_{t_0}^{t_1} P\, dt = \int_{t_0}^{t_1} vi\, dt \quad (Joule)$$

Power of last 3 Examples

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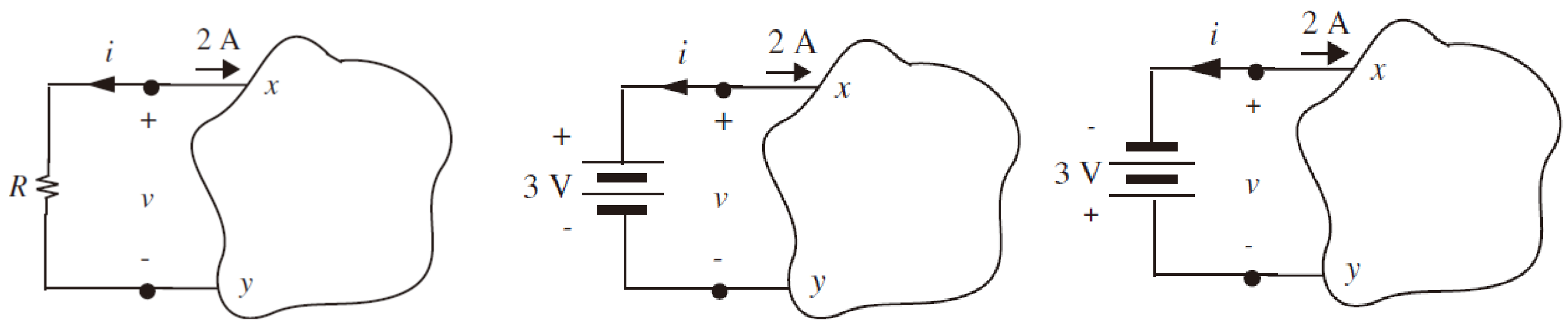
- Determine the power for the resistor and battery of the last 3 examples using the two assignments of terminal variables.



- $p = vi = (-20\text{V})(-2\text{A}) = 40\text{ W}.$
- $p = vi = (3\text{V})(-2\text{A}) = -6\text{ W}.$
- $p = vi = (-3\text{V})(-2\text{A}) = 6\text{ W}.$

Power Rating of Resistor

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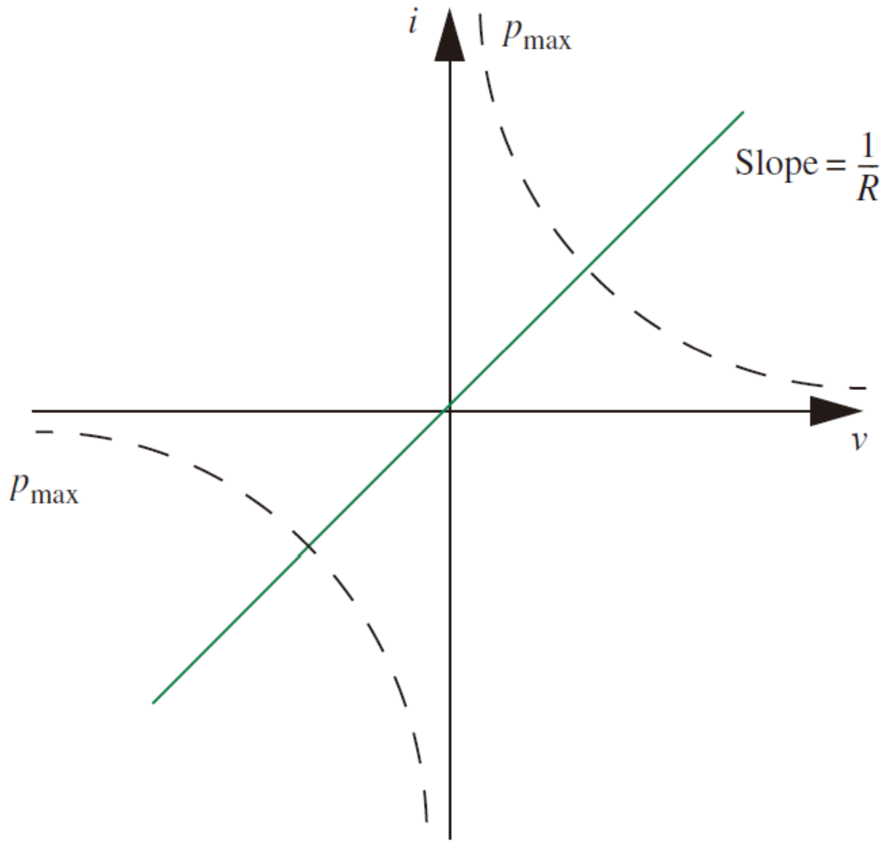
- The power rating of a resistor is the maximum power that a resistor can dissipate.



Ideal two terminal elements

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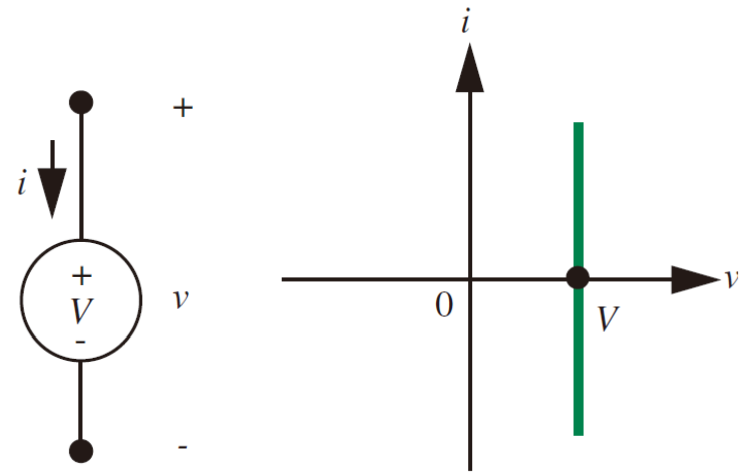
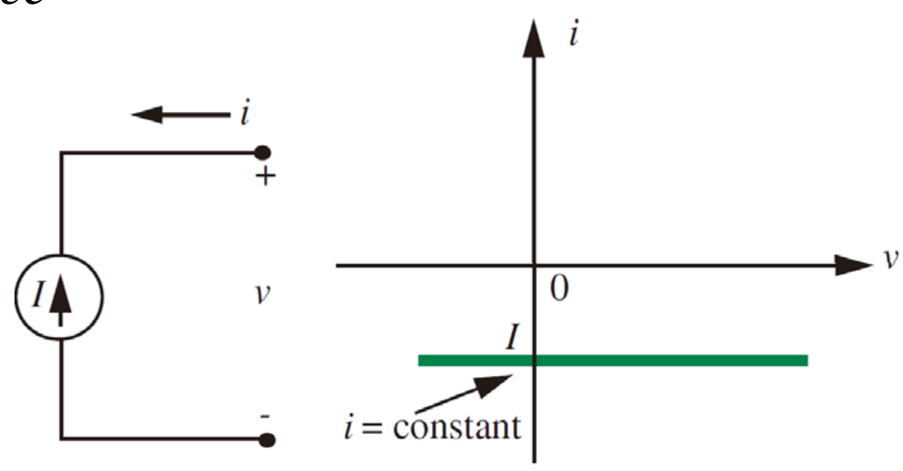
- *Ideal independent voltage source*
  - *Element v-i* relationship

$$v=V$$



- *Ideal independent current source*
  - *Element v-i* relationship

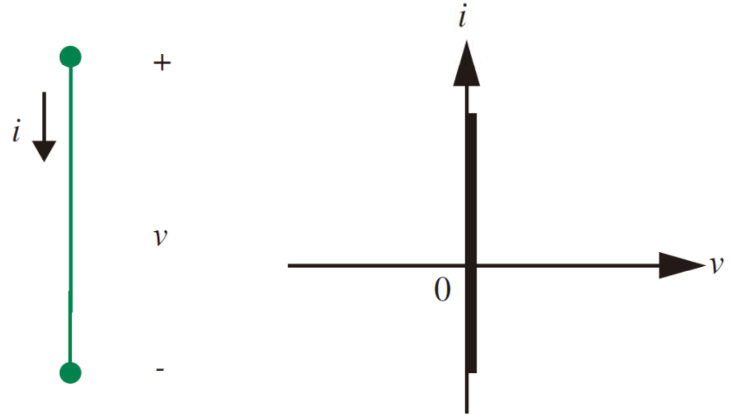
$$i=I$$



Ideal wire and open circuit element

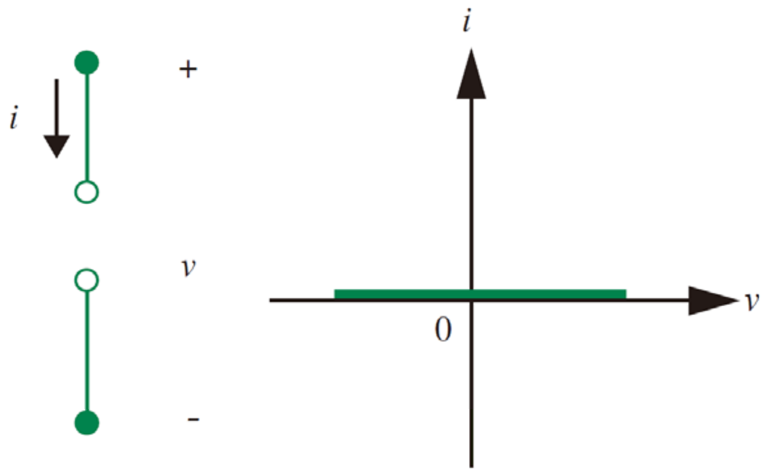
- *Ideal wire*
  - *Element v-i* relationship

$$v=0$$



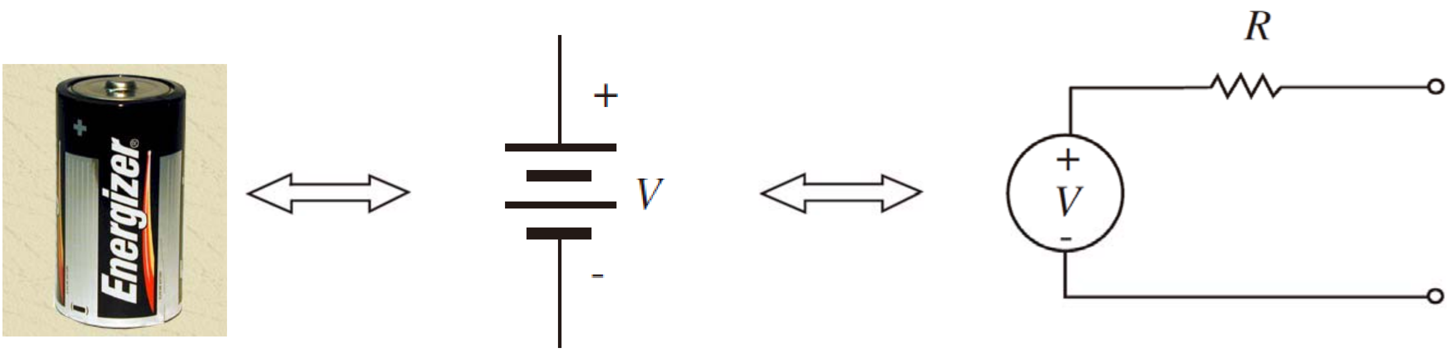
- *Ideal open circuit element*
  - *Element v-i* relationship

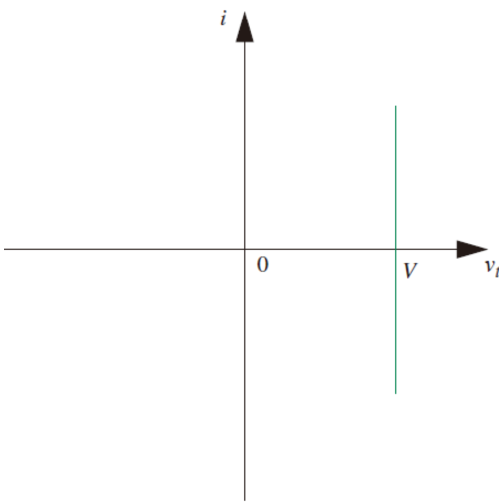
$$i=0$$

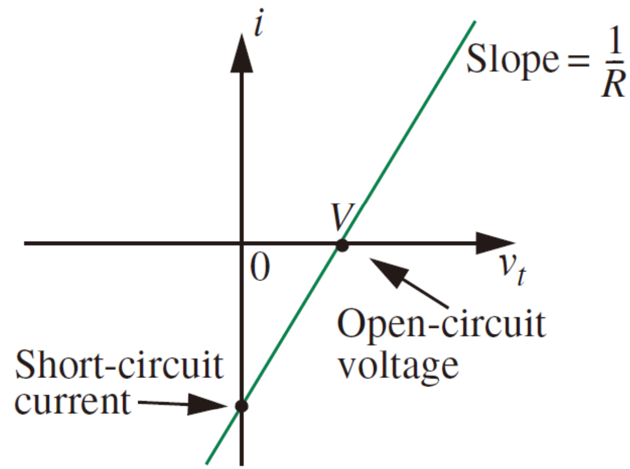


Modeling physical elements

- *Battery*







Lumped circuit abstraction

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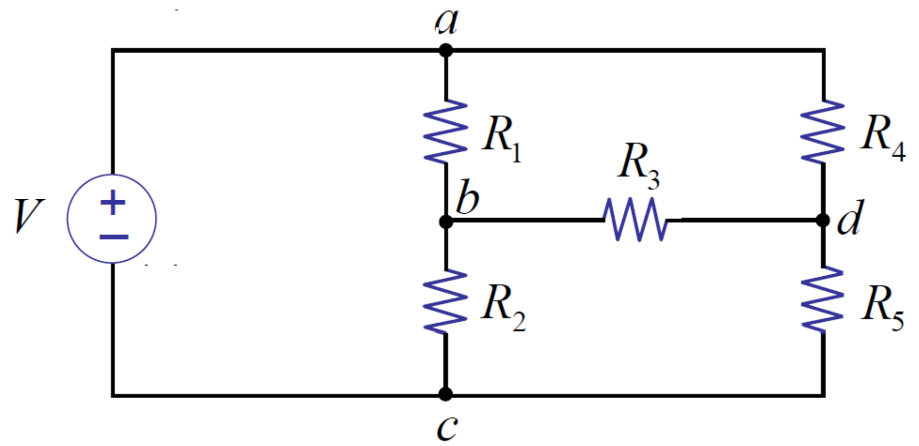
- Capped a set of lumped elements that obey the lumped matter discipline using ideal wires to form an assembly that performs a specific function results in the ***lumped circuit abstraction*.**

So, what does this buy us?

- For example —



- What can we say about voltages in a loop under the lumped matter discipline?

Conclusion

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- Lumped Matter Discipline (LMD)

$$\frac{\partial \phi_B}{\partial t} = 0$$ Outside element

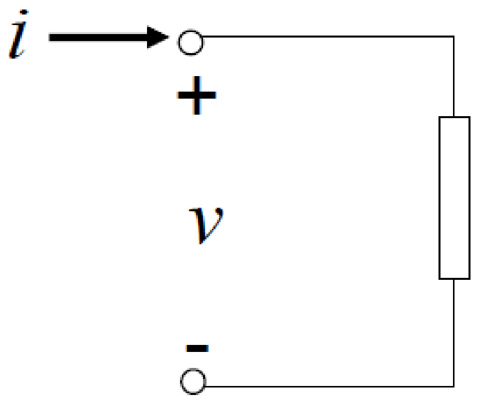
$$\frac{\partial q}{\partial t} = 0$$ Inside element

$$L << \frac{\lambda}{2\pi}$$

- Lumped circuit abstraction
  - Associated Variables Convention
  - Power consumed by element is *vi*
  - Element law
- Linear resistor, ideal wire,

  ideal independent voltage source,

  ideal independent current source.



Lumped circuit element