Sinusoidal Steady State
Filters and Resonance

Chenhsin Lien and Po-Tai Cheng
CENTER FOR ADVANCED POWER TECHNOLOGIES
Dept. of Electrical Engineering
National Tsing Hua University
Hsinchu, TAIWAN

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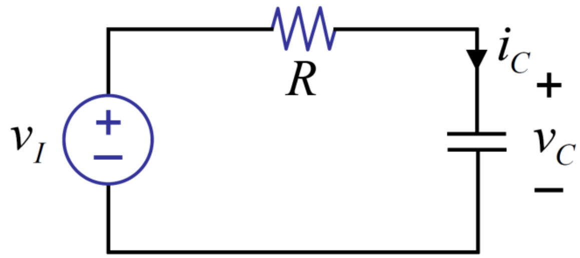
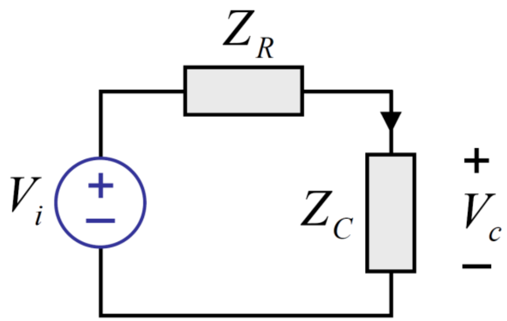
Back to RC Network

- Circuit:



- Impedance model:



- To find V_P

$$\mathbf{V_c} = \mathbf{V_i}\frac{Z_C}{Z_C + R} = \mathbf{V_i}\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \quad \Rightarrow \quad \mathbf{V_c} = \frac{1}{1 + j\omega RC}\mathbf{V_i}$$

Transfer Function

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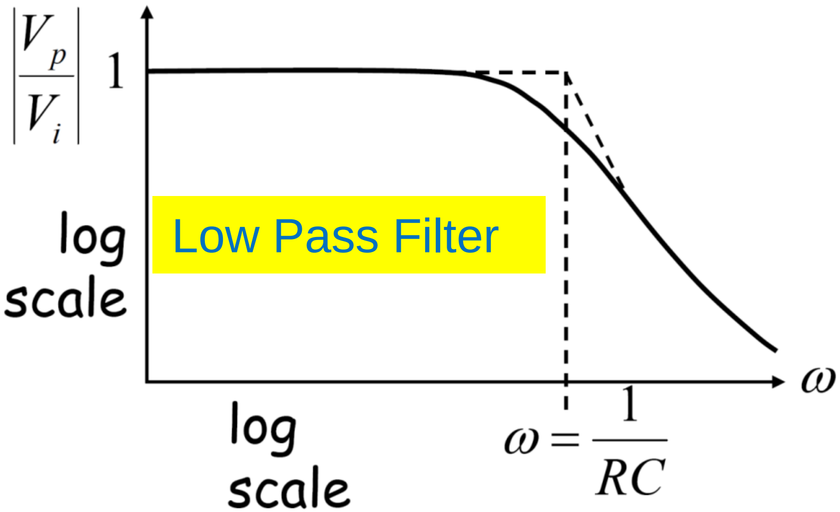
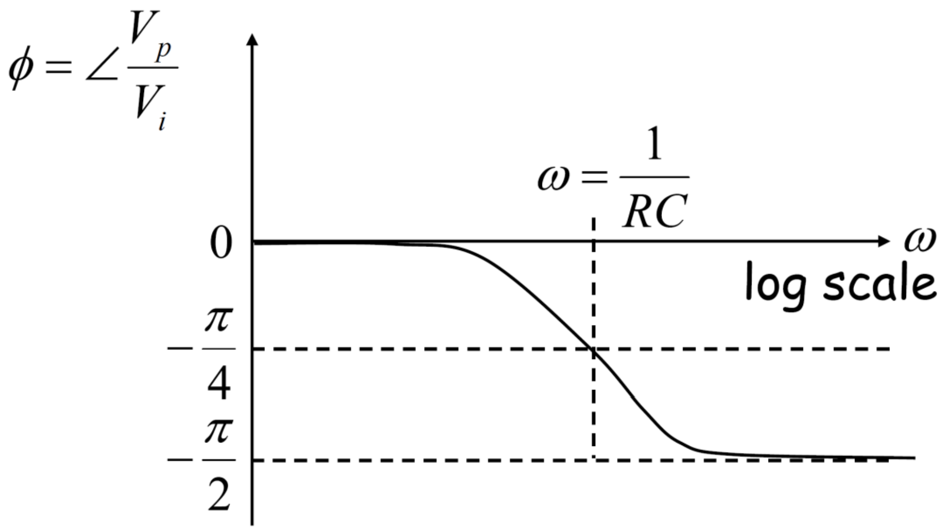
- **Transfer function** $H(j\omega) = \dfrac{\mathbf{V_c}}{\mathbf{V_i}} = \dfrac{1}{1 + j\omega RC}$
- Magnitude and Phase plots, **A low pass filter**.





Magnitude: $\left|\dfrac{\mathbf{V_P}}{\mathbf{V_i}}\right| = \dfrac{V_P}{V_i} = \dfrac{1}{\sqrt{1+\omega^2 R^2 C^2}}$

Phase ϕ: $\phi = \angle \dfrac{\mathbf{V_P}}{\mathbf{V_i}} = -\tan^{-1} \omega RC$

Bode Plot

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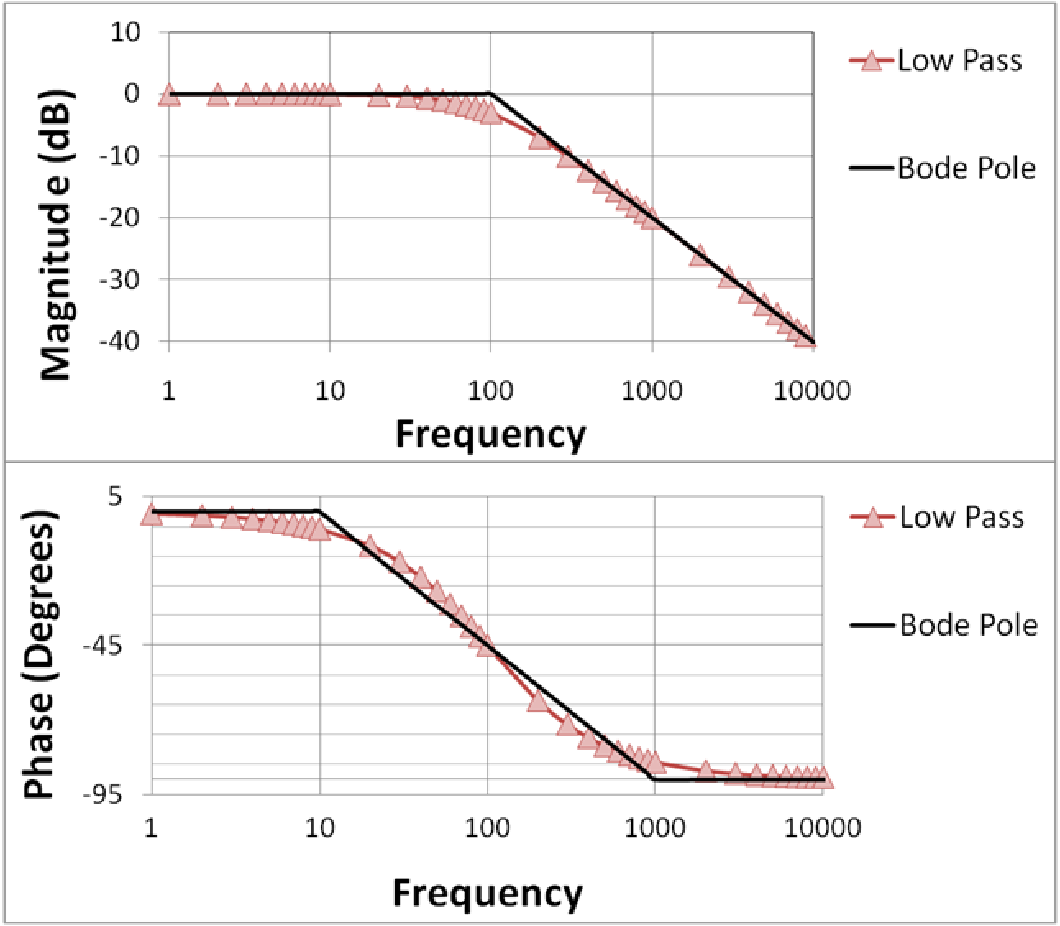
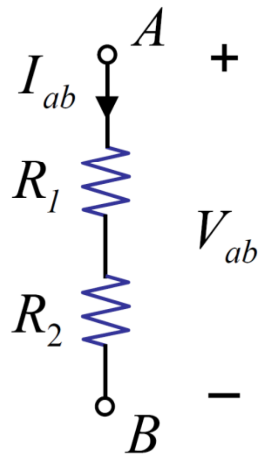
- **Transfer function** $H(j\omega)=\dfrac{\mathbf{V_c}}{\mathbf{V_i}}=\dfrac{1}{1+j\omega RC}$
- **Bode Plot** for a transfer function assuming RC = 2π×100 rad/sec.



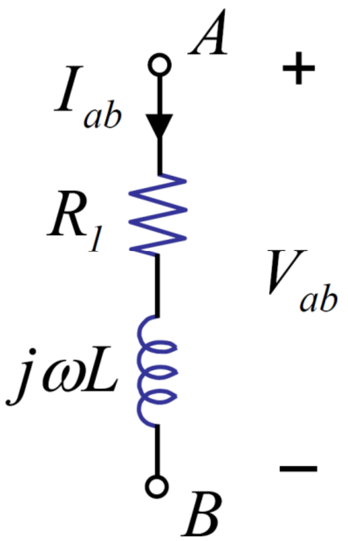
Quick Review of Impedances

- Example1:



$$R_{ab} = \frac{\mathbf{V_{ab}}}{\mathbf{I_{ab}}} = R_1 + R_2$$

- Example2 :



$$Z_{ab} = \frac{\mathbf{V_{ab}}}{\mathbf{I_{ab}}} = R_1 + j\omega L_i$$

Quick Review of Impedances

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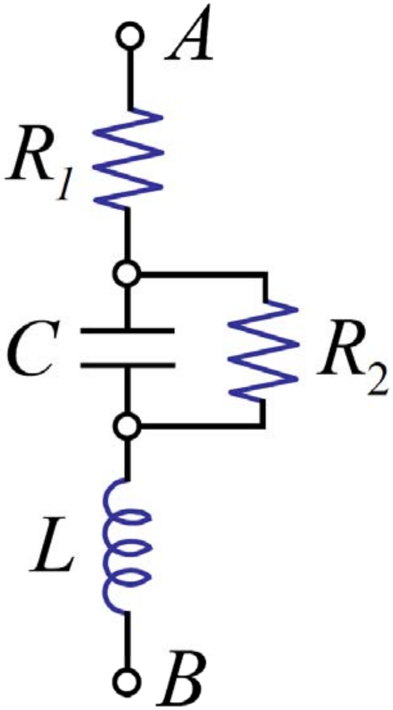
- Example3:



$$Z_{ab} = \frac{\mathbf{V_{ab}}}{\mathbf{I_{ab}}} = R_1 + Z_C \| R_2 + Z_L$$

$$= R_1 + \frac{Z_C R_2}{Z_C + R_2} + Z_L$$

$$= R_1 + \frac{R_2}{1 + j\omega C R_2} + Z_L$$

RCL Impedances

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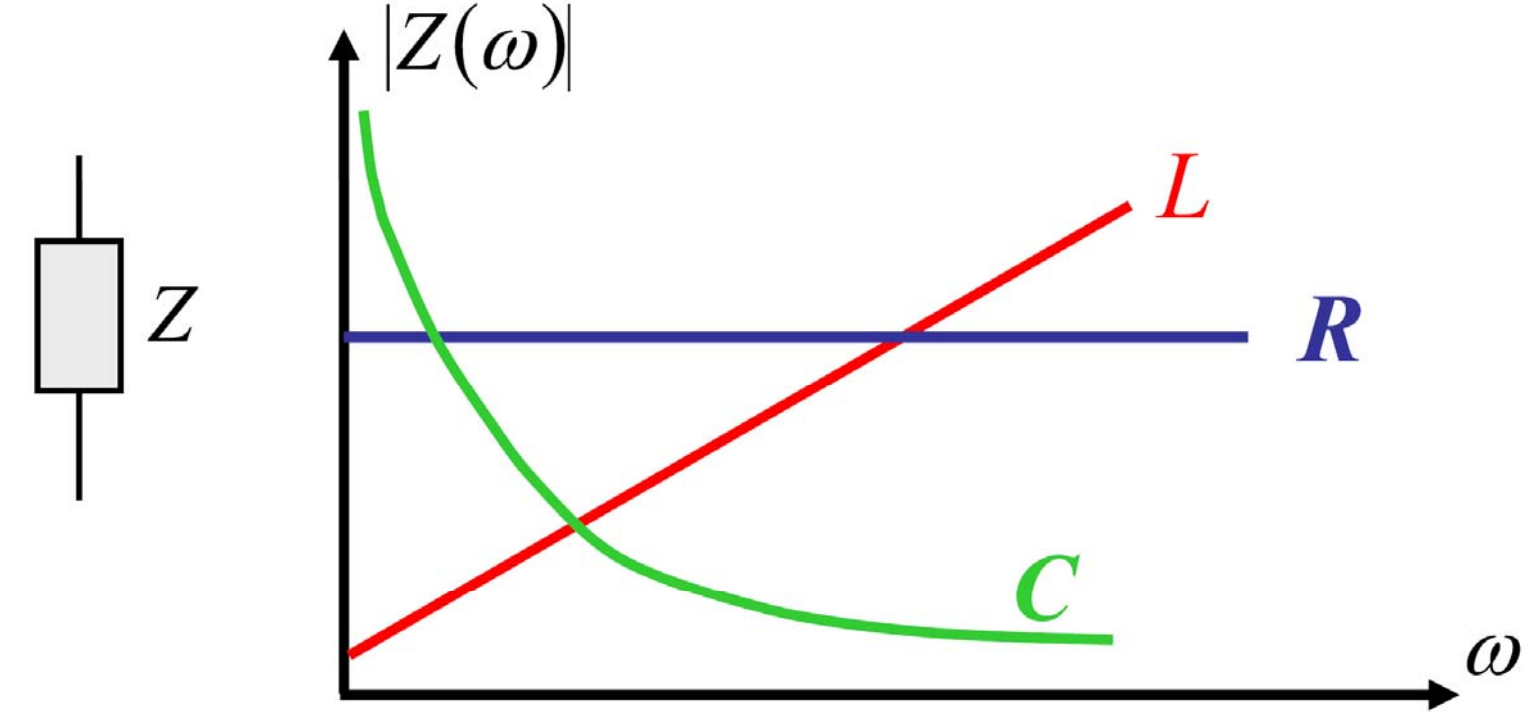
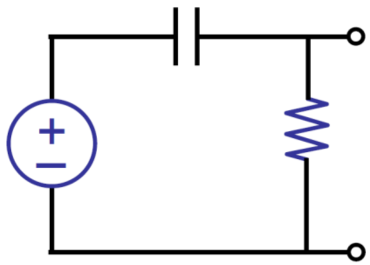
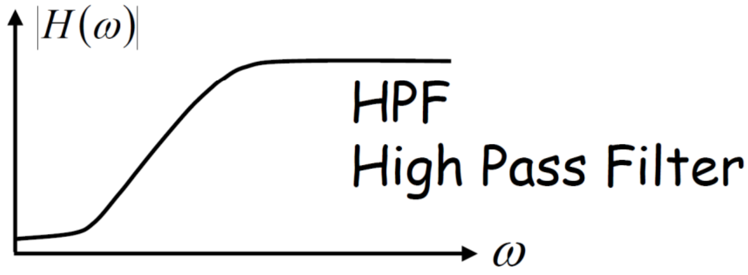
- RLC impedances as a function of frequency:

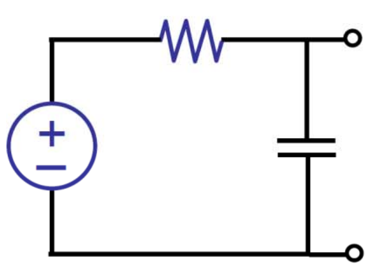


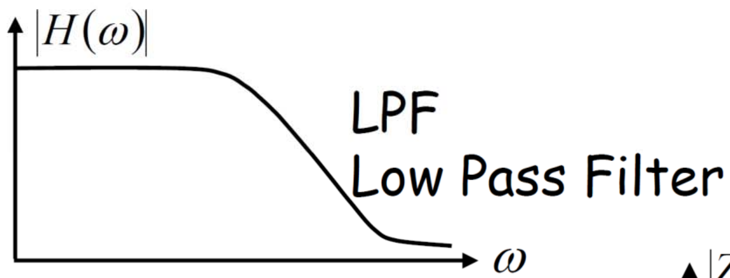
Filters built by Combining Impedances

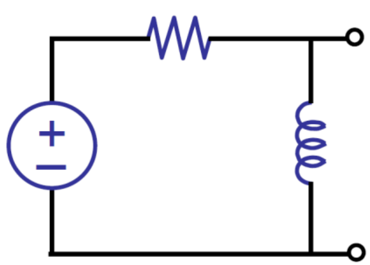
- Filters can be built by Combining Impedances :

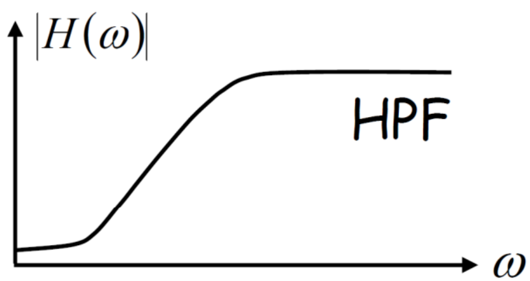


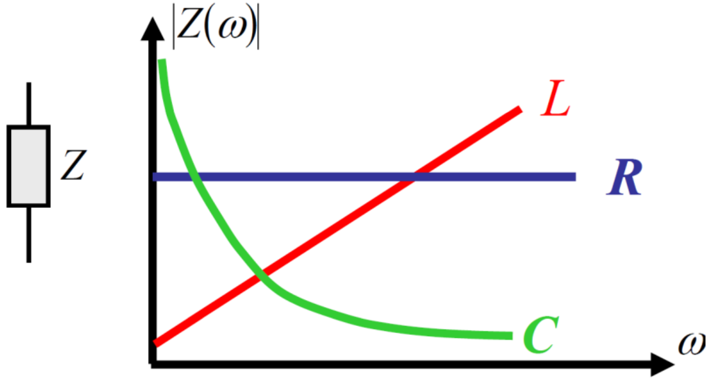




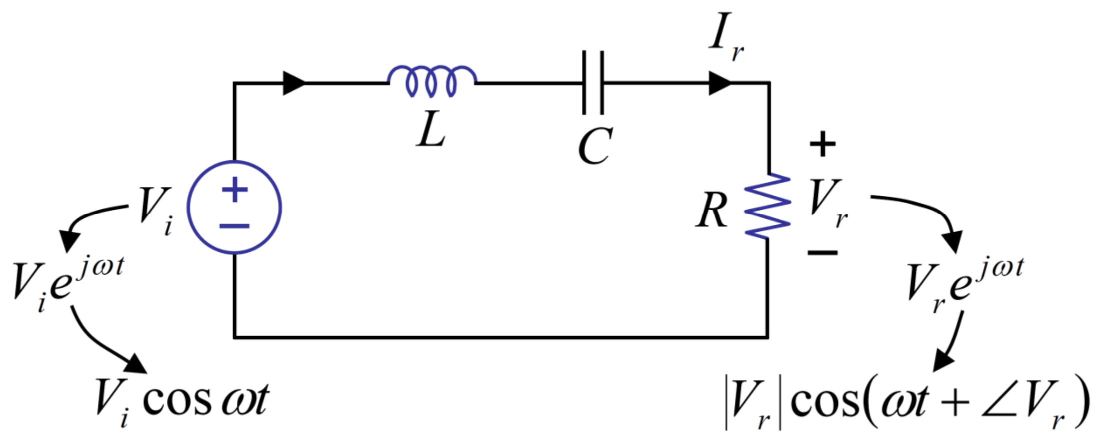








Series RLC



- To find V_r

$$\mathbf{V_r} = \mathbf{V_i}\frac{Z_R}{Z_L + Z_C + Z_R}$$

$$\mathbf{V_r} = \mathbf{V_i}\frac{R}{j\omega L + \dfrac{1}{j\omega C} + R}$$

$$\mathbf{V_r} = \mathbf{V_i}\frac{j\omega CR}{-\omega^2 LC + 1 + j\omega CR}$$

Transfer Function

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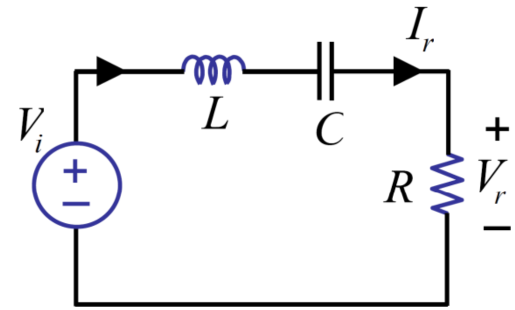
- Transfer function

$$\frac{\mathbf{V_r}}{\mathbf{V_i}} = \frac{j\omega CR}{-\omega^2 LC + 1 + j\omega CR}$$



- Let's study this transfer function

$$\frac{\mathbf{V_r}}{\mathbf{V_i}} = \frac{j\omega CR}{\left(1-\omega^2 LC\right) + j\omega CR} = \frac{j\omega CR}{\left(1-\omega^2 LC\right) + j\omega CR} \cdot \frac{\left(1-\omega^2 LC\right) - j\omega CR}{\left(1-\omega^2 LC\right) - j\omega CR}$$

$$\left|\frac{\mathbf{V_r}}{\mathbf{V_i}}\right| = \frac{V_r}{V_i} = \frac{\omega CR}{\sqrt{\left(1-\omega^2 LC\right)^2 + \left(\omega CR\right)^2}}$$

$$\text{Low } \omega: \quad \frac{V_r}{V_i} \approx \omega CR$$

$$\text{High } \omega: \quad \frac{V_r}{V_i} \approx \frac{R}{\omega L}$$

$$\omega\sqrt{LC} = 1: \quad \frac{V_r}{V_i} = 1$$

Graphically

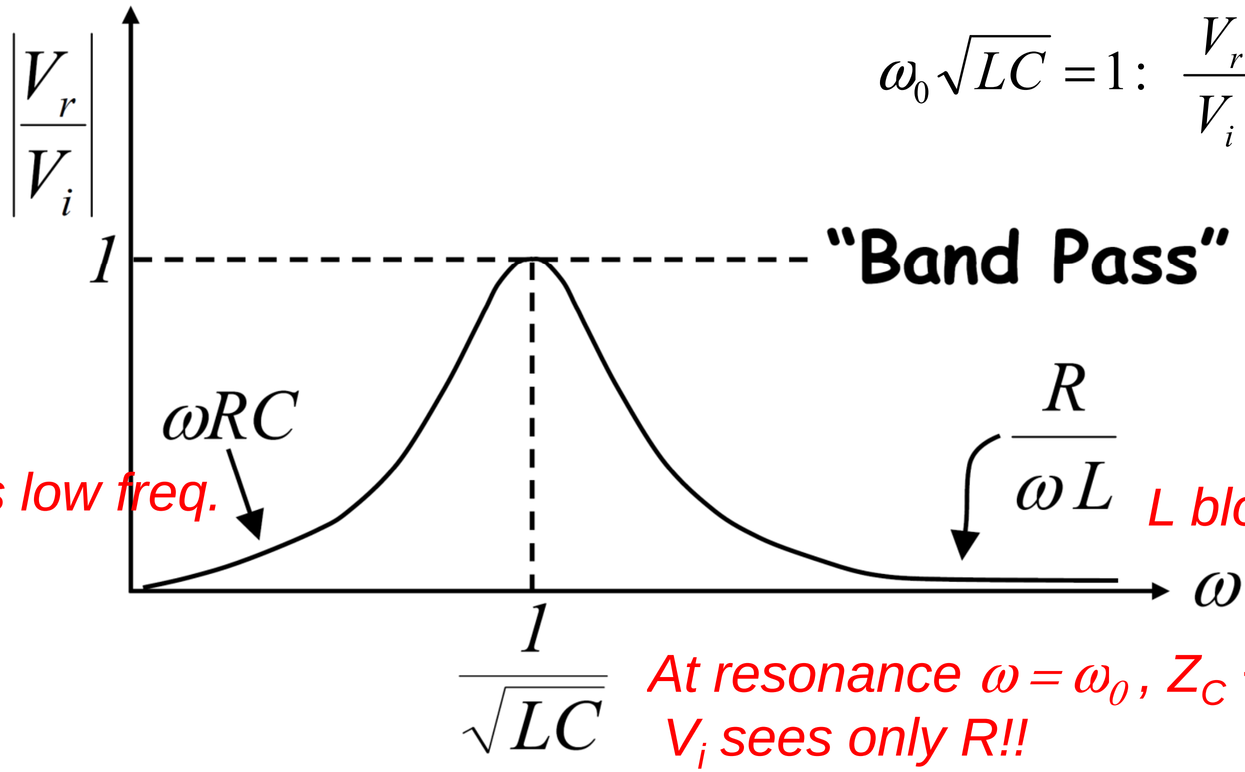
- Transfer function

$$\left|\frac{\mathbf{V_r}}{\mathbf{V_i}}\right| = \frac{V_r}{V_i} = \frac{\omega CR}{\sqrt{\left(1-\omega^2 LC\right)^2 + \left(\omega CR\right)^2}}$$

Low ω: $\quad \dfrac{V_r}{V_i} \approx \omega CR$

High ω: $\quad \dfrac{V_r}{V_i} \approx \dfrac{R}{\omega L}$

$\omega_0\sqrt{LC} = 1$: $\quad \dfrac{V_r}{V_i} = 1$



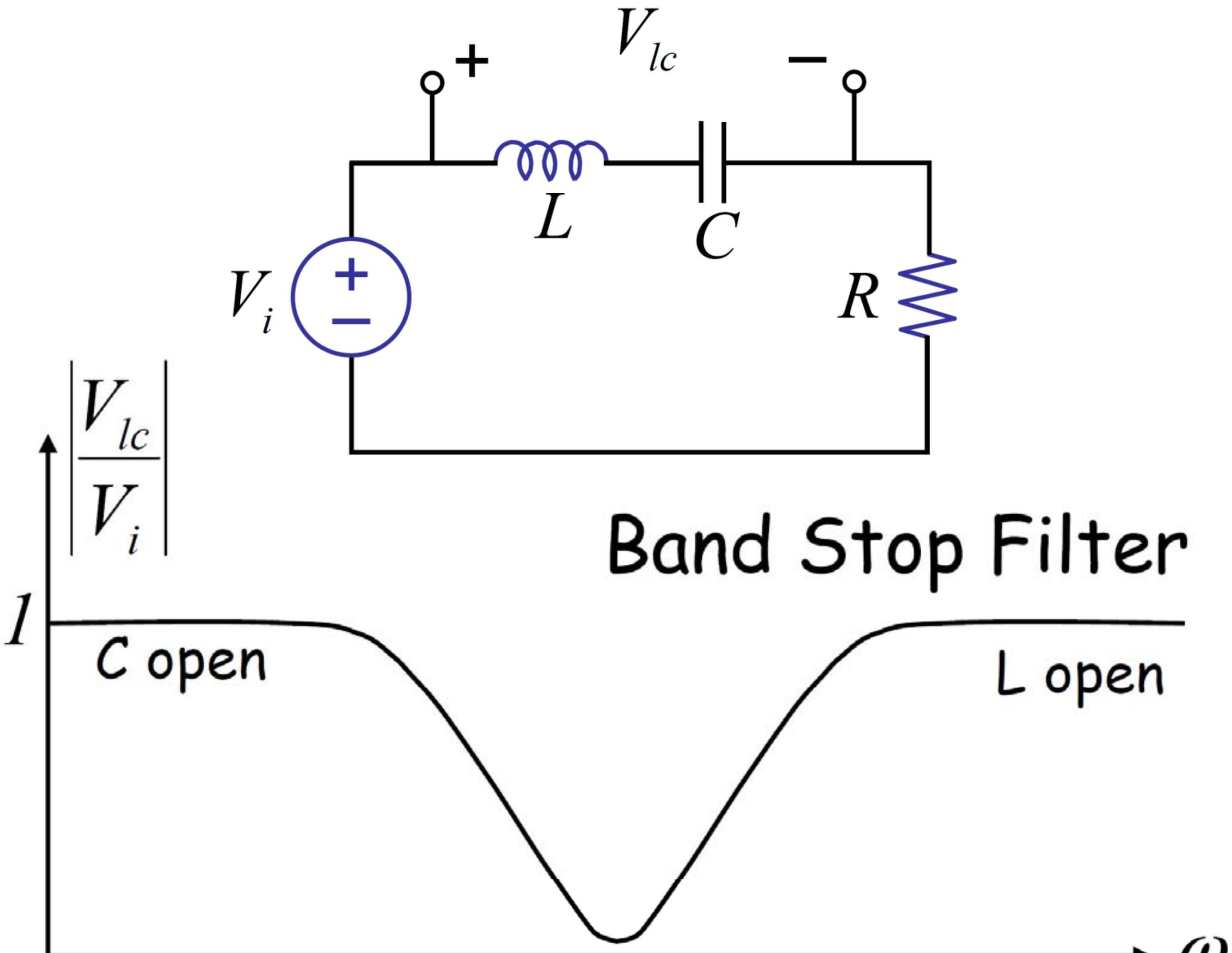
C blocks low freq.

L blocks high freq.

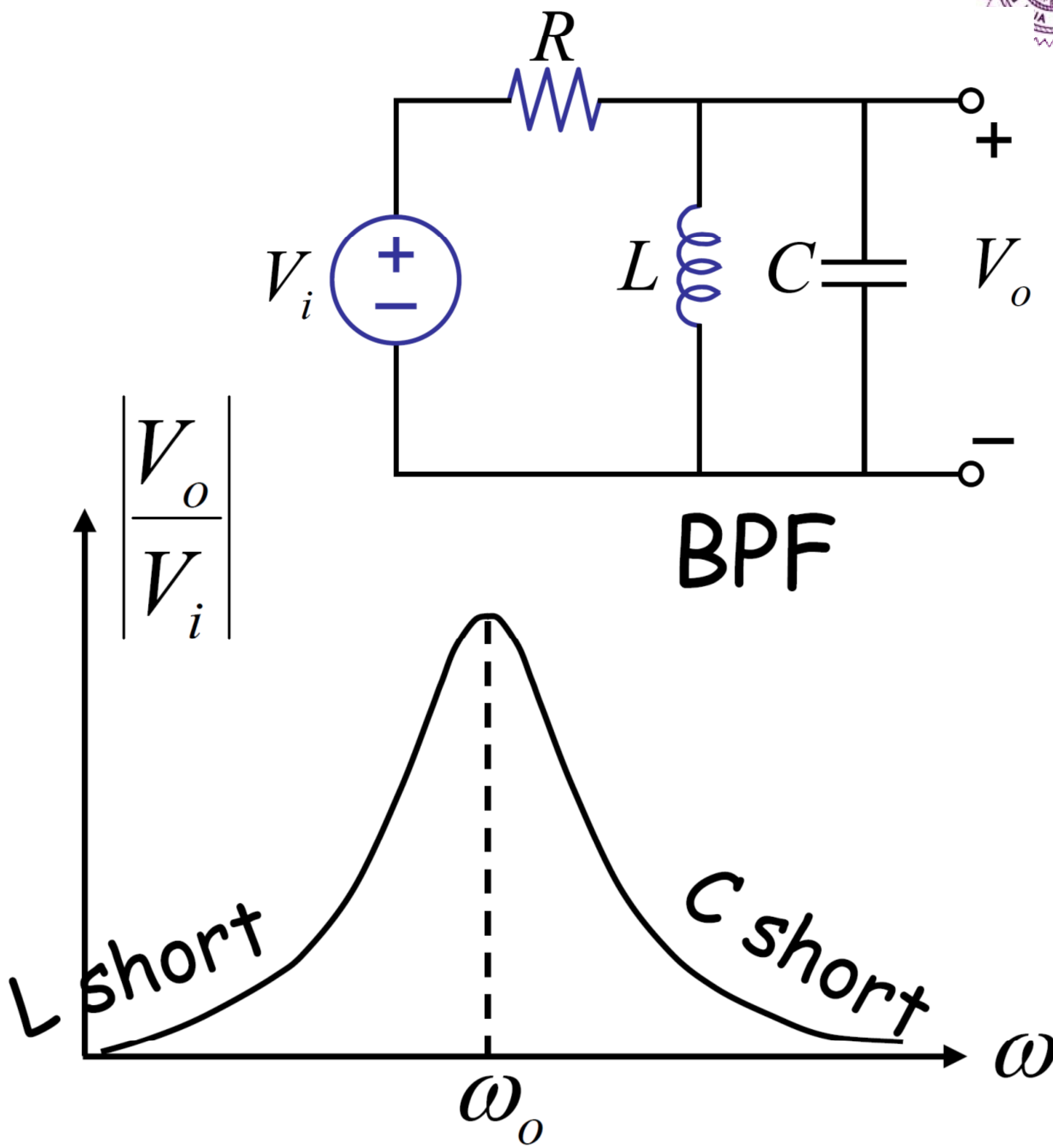
At resonance $\omega = \omega_0$, $Z_C + Z_L = 0$.
V_i sees only R!!

What about?

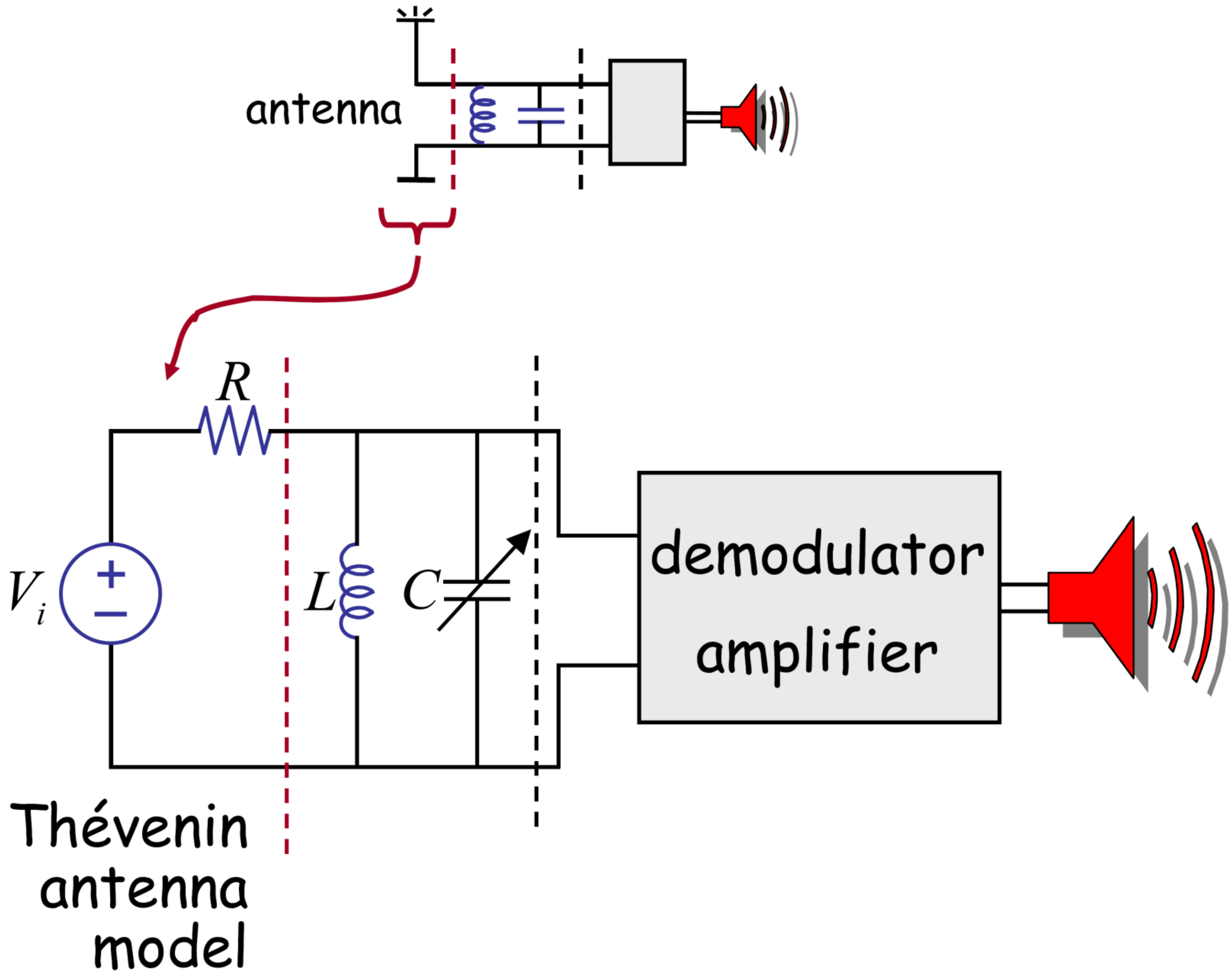
- What about taking V_{lc} as output?



Another example:



AM Radio Receiver



AM Radio Receiver

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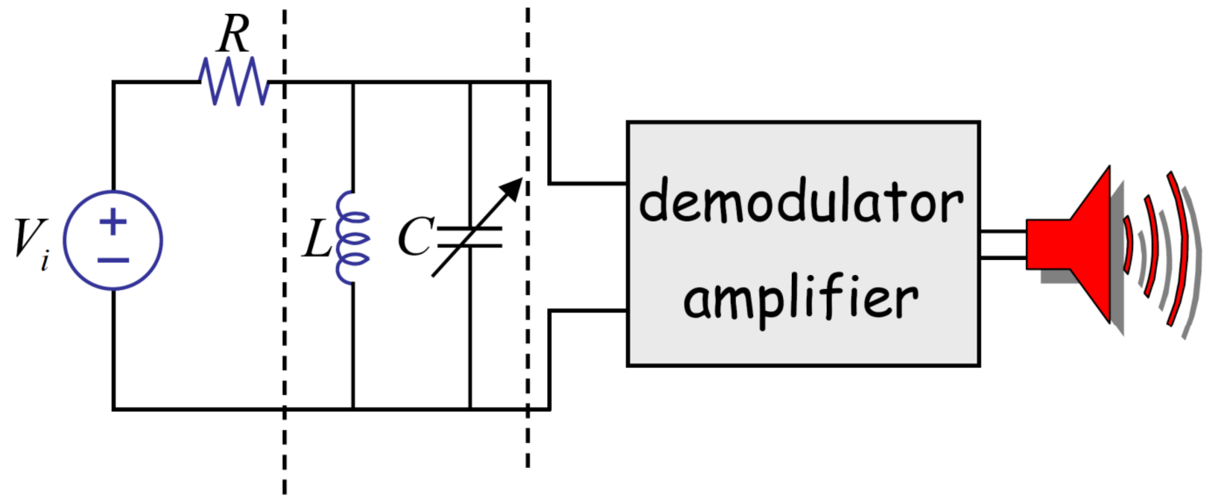
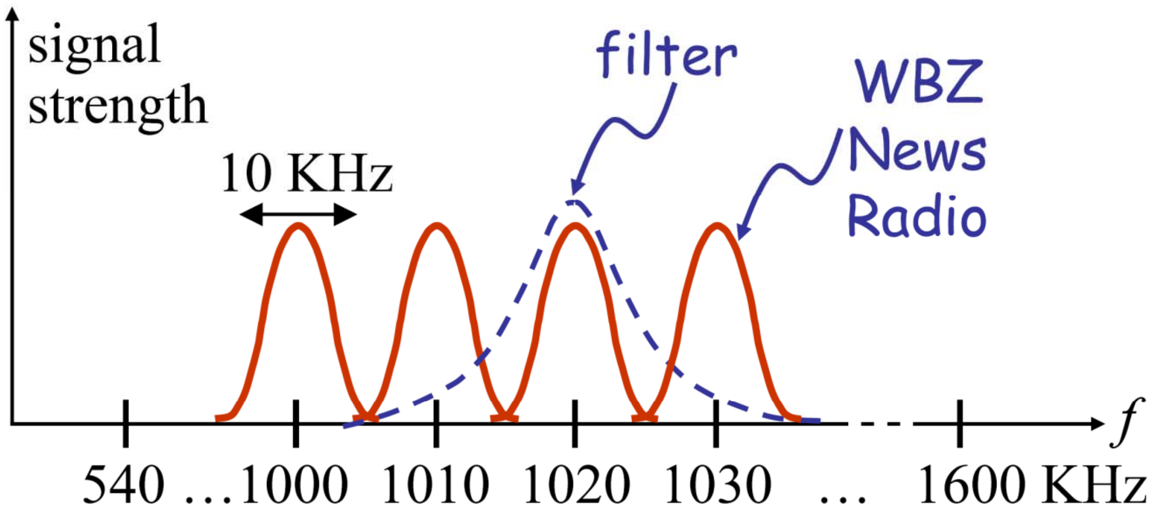
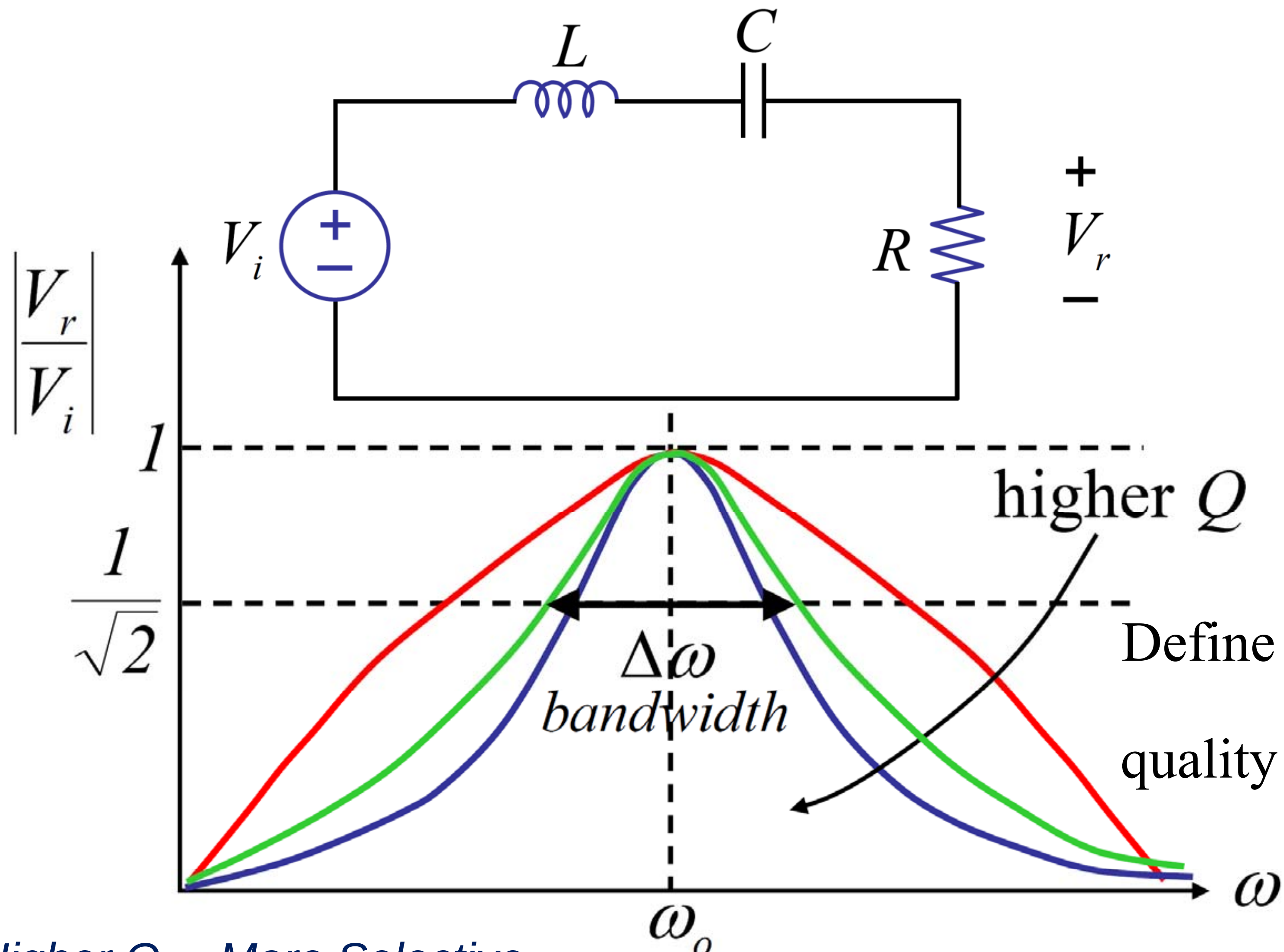
- Receiver "Selectivity" is important — relates to a parameter "Q" of the filter.





Selectivity: Look at series RLC in more detail



Define: $Q = \frac{\omega_0}{\Delta\omega}$

quality factor

Higher Q ⇒More Selective

Quality Factor Q

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- To find quality factor $Q = \dfrac{\omega_0}{\Delta\omega}$
- We need to find ω_0 and $\Delta\omega = ?$
- Recall

$$\frac{\mathbf{V_r}}{\mathbf{V_i}} = \frac{R}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{1}{1 + j\left(\omega\dfrac{L}{R} - \dfrac{1}{\omega CR}\right)}$$

- At ω_0, we have $\omega_o \dfrac{L}{R} - \dfrac{1}{\omega_0 CR} = 0$
- ω_0 is simply : $\boxed{\omega_o = \dfrac{1}{\sqrt{LC}}}$

Quality Factor Q

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- Δω is the bandwidth, i.e. ω between magnitude fall to -3dB
- We need to find

$$\left|\frac{\mathbf{V_r}}{\mathbf{V_i}}\right| = \frac{V_r}{V_i} = \frac{1}{\sqrt{2}} = \left|\frac{1}{1 + j\left(\omega\frac{L}{R} - \frac{1}{\omega CR}\right)}\right| = \left|\frac{1}{1 \pm j}\right|$$

- That is $\omega\frac{L}{R} - \frac{1}{\omega CR} = \pm 1$
- Or $\omega^2 \mp \frac{R}{L}\omega - \frac{1}{LC} = 0$
- The roots of both equations are

$$\omega_1 = \frac{R}{2L} + \frac{1}{2}\sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}$$

$$\omega_2 = -\frac{R}{2L} + \frac{1}{2}\sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}$$

- Δω is simply : $\Delta\omega = \omega_1 - \omega_2 = \frac{R}{L}$

Quality Factor Q

- The quality factor $Q = \frac{\omega_0}{\Delta\omega}$

- Q is simply : $Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{\frac{R}{L}} = \frac{\omega_0 L}{R}$ $\qquad \omega_o = \frac{1}{\sqrt{LC}}$

- Q is simply : $Q = \frac{\omega_0 L}{R}$

- The lower the R (for series RLC), the sharper the peak.

- Another way of looking at Q:

$$Q = 2\pi \frac{\text{Energy Stored}}{\text{Energy lost per cycle}} = 2\pi \frac{\frac{1}{2} L |I_r|^2}{\frac{1}{2} R |I_r|^2 \frac{2\pi}{\omega_0}} = \frac{\omega_0 L}{R}$$

Parallel RLC

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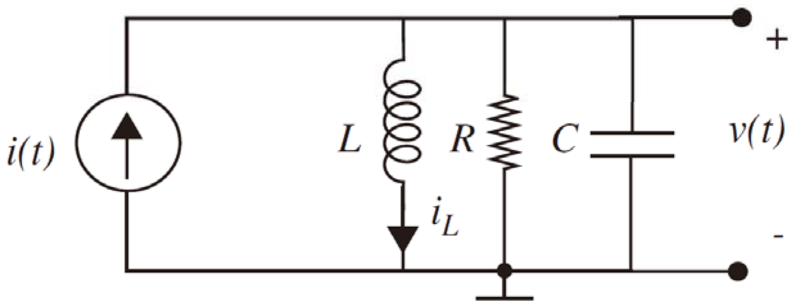

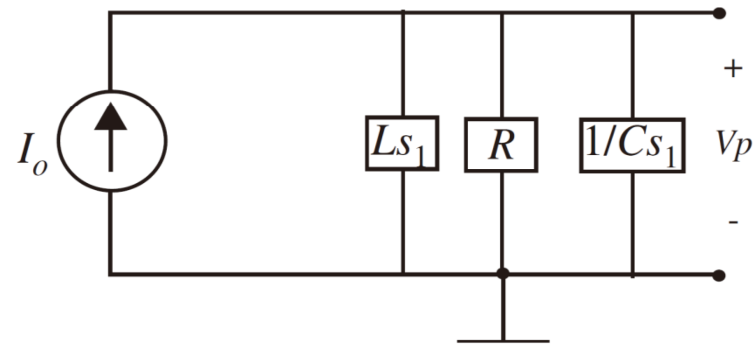

- To find $\mathbf{V_p}$ $$\mathbf{V_p} = \mathbf{I_0}\frac{1}{\frac{1}{j\omega L}+\frac{1}{R}+j\omega C} = \mathbf{I_0}\frac{R}{1+j\left(\omega CR - \frac{R}{\omega L}\right)}$$

$$\frac{\mathbf{V_p}}{\mathbf{I_0}} = \frac{R}{1+j\left(\omega CR - \frac{R}{\omega L}\right)} = \frac{1}{G+j\left(\omega C - \frac{1}{\omega L}\right)}$$

Quality Factor Q

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- To find quality factor $Q = \dfrac{\omega_0}{\Delta\omega}$

- We need to find ω_0 and $\Delta\omega = ?$

- Recall

$$\frac{\mathbf{V_p}}{\mathbf{I_0}} = \frac{R}{1 + j\left(\omega CR - \dfrac{R}{\omega L}\right)}$$

- At ω_0, we have $\omega_0 CR - \dfrac{R}{\omega_o L} = 0$

- ω_0 is simply : $\omega_o = \dfrac{1}{\sqrt{LC}}$

Quality Factor Q

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- Δω is the bandwidth, i.e. ω between magnitude fall to -3dB
- We need to find $\left|\frac{\mathbf{V_i}}{\mathbf{I_0}}\right| = \frac{V_i}{I_0} = \frac{R}{\sqrt{2}} = \left|\frac{R}{1+j\left(\omega CR - \frac{\mathrm{R}}{\omega L}\right)}\right| = \left|\frac{1}{1 \pm j}\right|$
- That is $\omega CR - \frac{\mathrm{R}}{\omega L} = \pm 1$
- Or $\omega^2 \mp \frac{1}{RC}\omega - \frac{1}{LC} = 0$
- The roots of both equations are

$$\omega_1 = \frac{1}{2RC} + \frac{1}{2}\sqrt{\frac{1}{R^2C^2} + \frac{4}{LC}}$$

$$\omega_2 = -\frac{1}{2RC} + \frac{1}{2}\sqrt{\frac{1}{R^2C^2} + \frac{4}{LC}}$$

- Δω is simply : $\Delta\omega = \omega_1 - \omega_2 = \frac{1}{RC}$

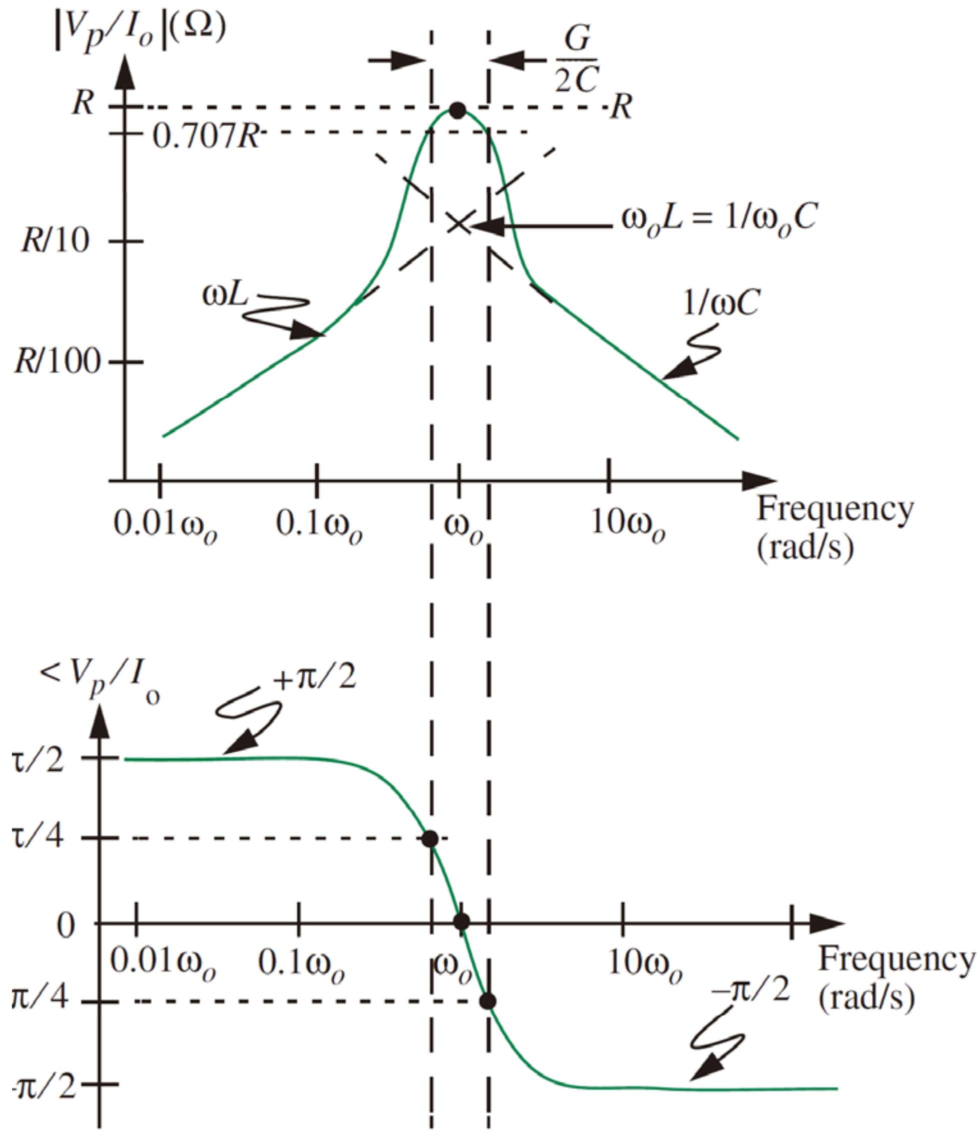
Transfer Function

- **Transfer function**

$$\frac{\mathbf{V_p}}{\mathbf{I_0}} = \frac{R}{1 + j\left(\omega CR - \dfrac{R}{\omega L}\right)}$$

- Magnitude and

  Phase plots.



Example

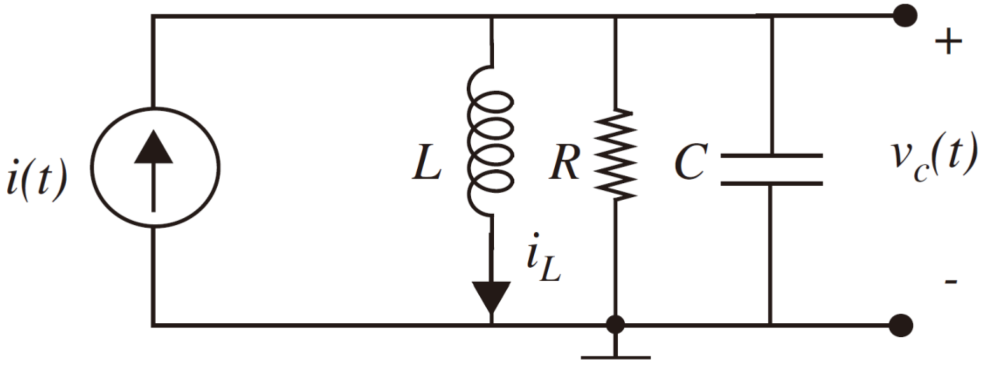
- $L = 0.1$ mH, $C = 1$ μF and $R = 10\ \Omega$
- **Transfer function**

$$\frac{\mathbf{V_c}}{\mathbf{I}_0} = \frac{R}{1 + j\left(\omega CR - \dfrac{R}{\omega L}\right)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^5 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^5}{2} \text{ rad/s}$$

$$Q = \frac{\omega_0}{2\alpha} = 1$$



Magnitude and Phase plots

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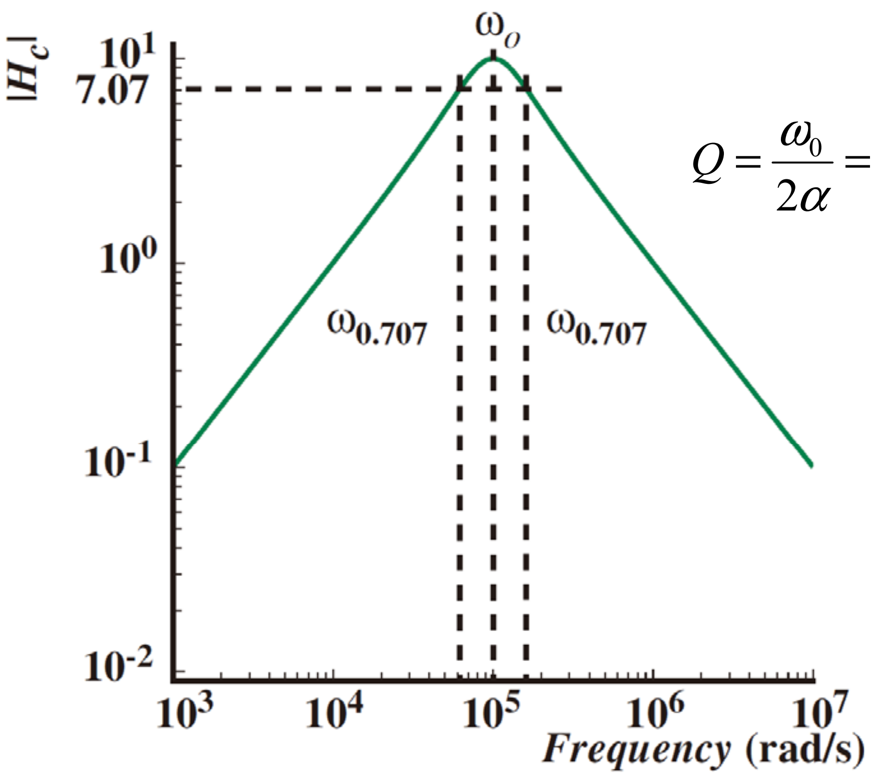
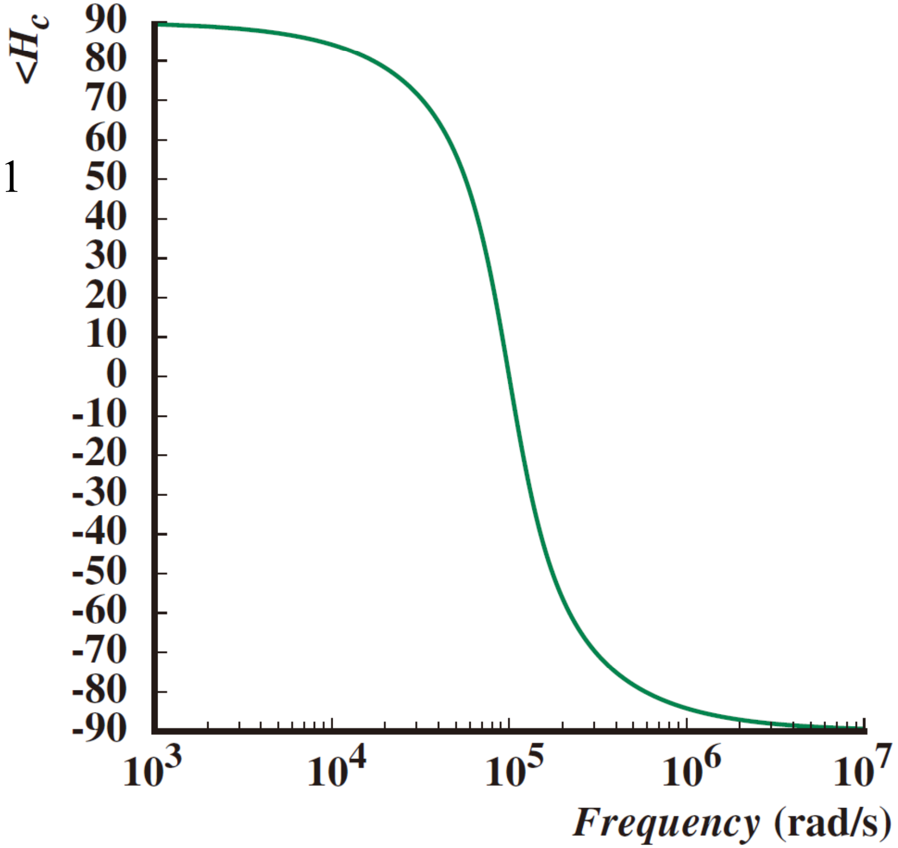
- **Log scale**



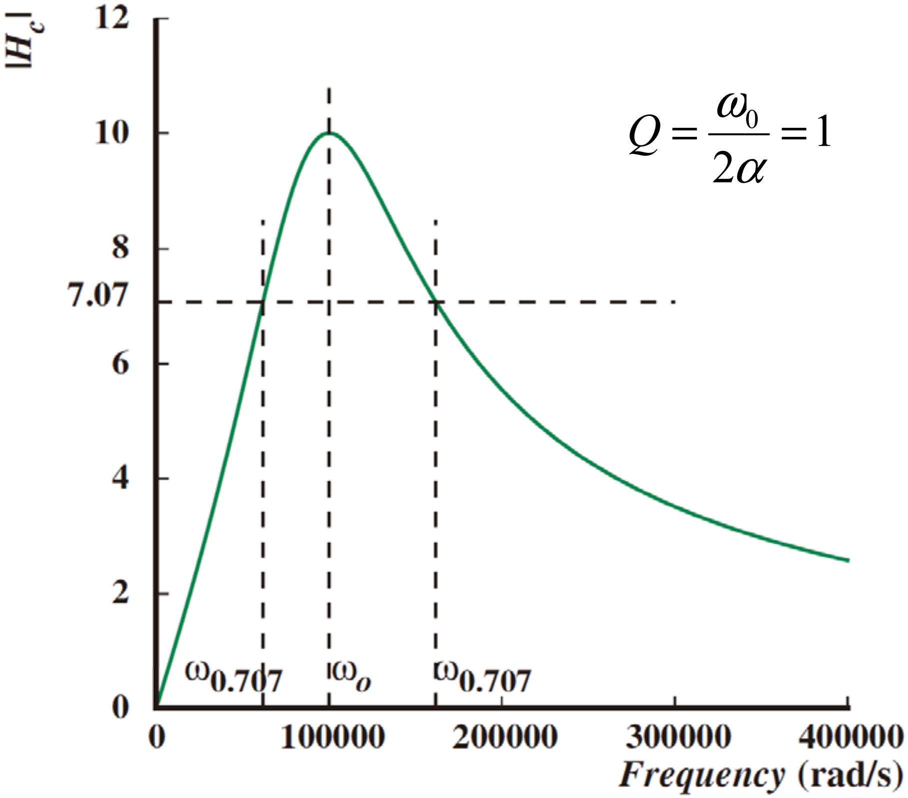
$$Q = \frac{\omega_0}{2\alpha} = 1$$



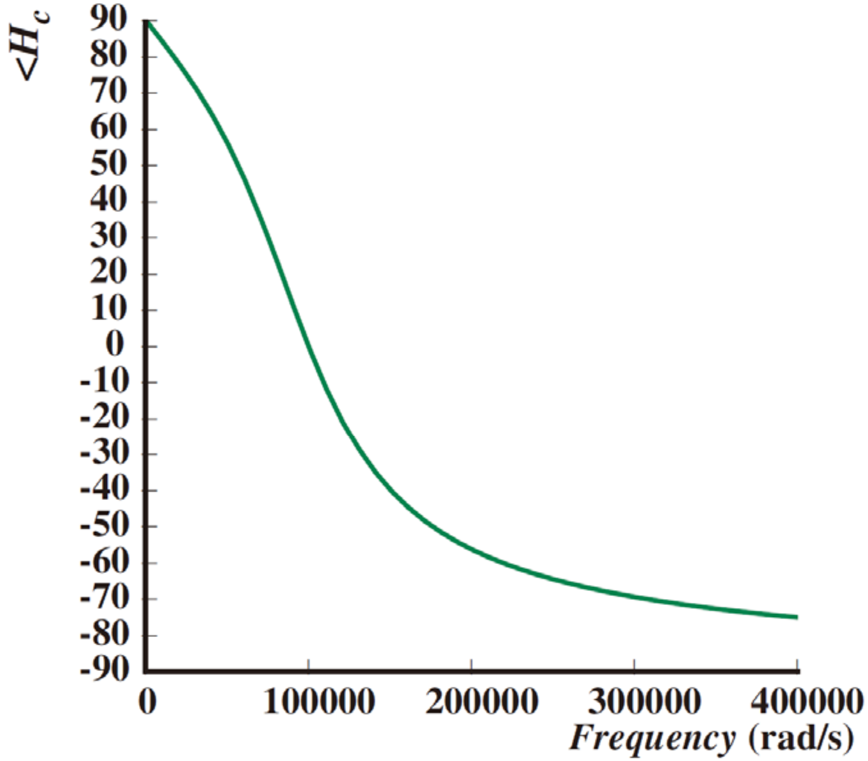
$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^5 \text{ rad/s}$$

Magnitude and Phase plots

- **Linear scale**



$$Q = \frac{\omega_0}{2\alpha} = 1$$



$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^5 \ \text{rad/s}$$

Transfer Function and Q

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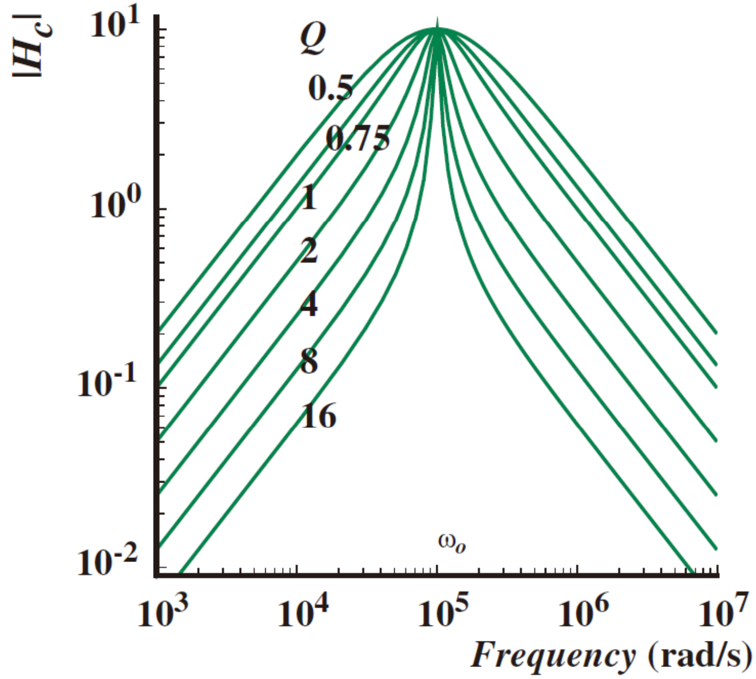
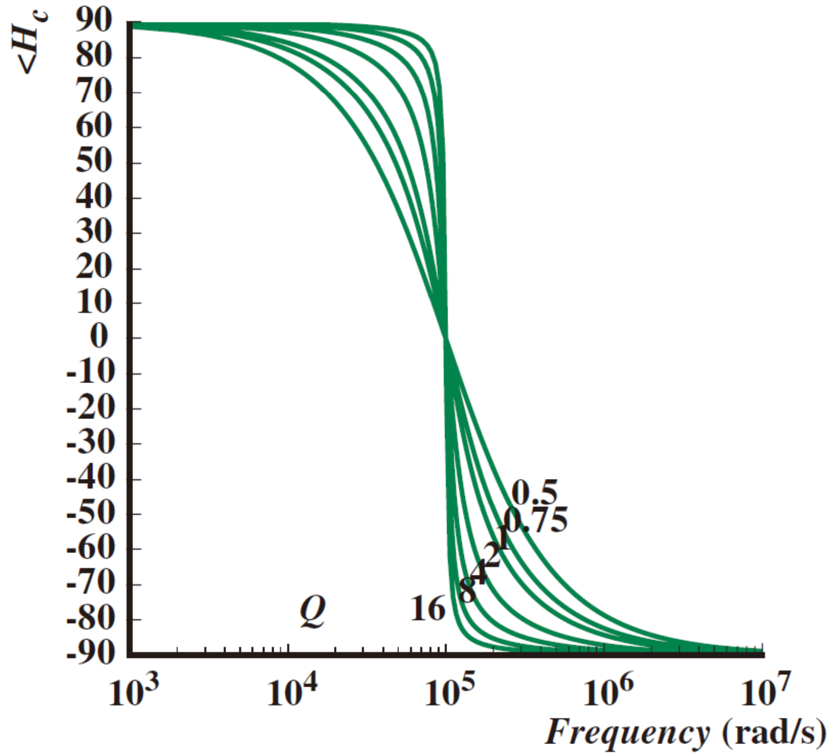
- **Transfer function** $$\frac{V_p}{I_0} = \frac{R}{1 + j\left(\omega CR - \frac{R}{\omega L}\right)}$$

- Magnitude and Phase plots.





Revisit of the LPF with L

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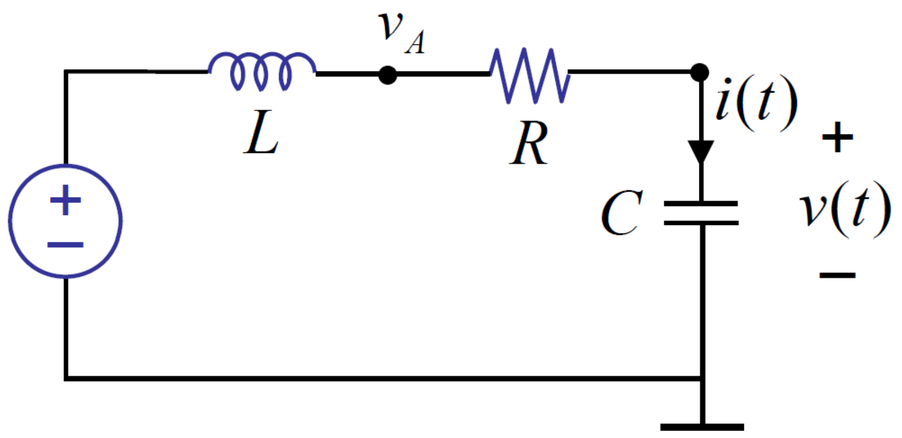
- $L = 20$ mH, $C = 13$ nF and $R = 50\ \Omega$
- **Transfer function**



$$\frac{\mathbf{V_c}}{\mathbf{V_i}} = \frac{\dfrac{1}{j\omega RC}}{1 + j\left(\omega \dfrac{L}{R} - \dfrac{1}{\omega RC}\right)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 6.2\times10^4 \ \text{rad/s}$$

$$\alpha = \frac{1}{2RC} = 76.9\times10^4 \ \text{rad/s} = 12.4\omega_0$$

$$Q = \frac{\omega_0}{2\alpha} = 24.8$$

Time-domain Vs Frequency-domain

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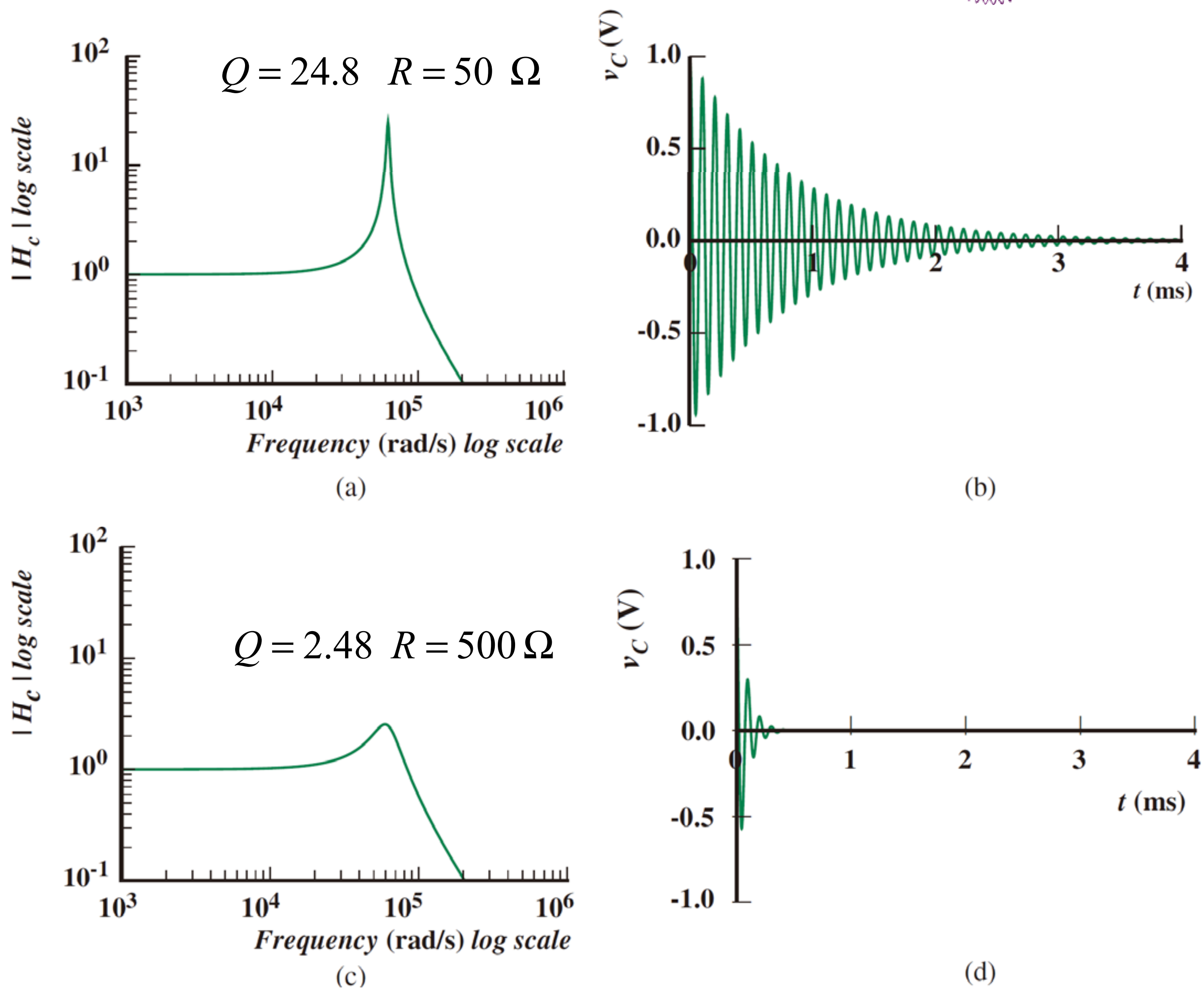
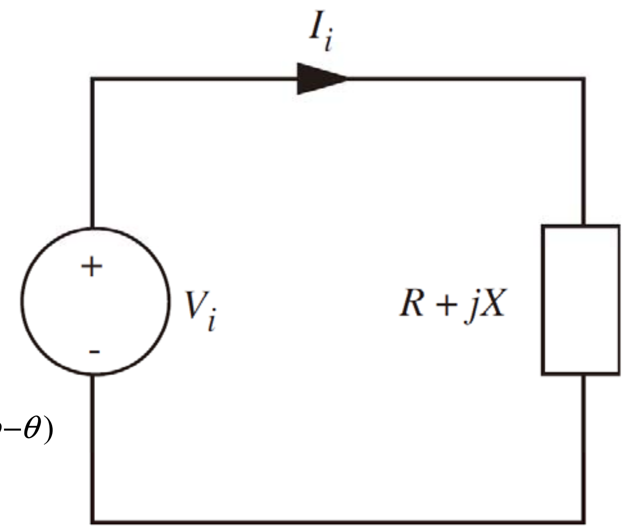


Power and Energy in an Impedance

- Power and energy are critical issues in the design of circuits.
- The power delivered to some arbitrary impedance Z = R + jX by a sinusoidal source. $v_i(t) = V_i \cos(\omega t + \phi)$



- In Phasor notation $\mathbf{V_i} = V_i e^{j\phi}$
- And

$$\mathbf{I_i} = \frac{\mathbf{V_i}}{Z} = \frac{V_i e^{j\phi}}{R + jX} = \frac{V_i}{\sqrt{R^2 + X^2}} e^{j(\phi - \theta)} = I_i e^{j(\phi-\theta)}$$

Where $\theta = \tan^{-1}\dfrac{X}{R}$

- Because power is not a linear function of *v* and *i*, we must be cautious about using impedance concepts in power calculations.

Time expressions

- Let's start with time expressions rather than complex amplitudes.
- The current and voltage as a function of time are.

$$i_i(t) = \mathrm{Re}(\mathbf{I_i} e^{j\omega t}) = \frac{V_i}{\sqrt{R^2 + X^2}} \cos(\omega t + \phi - \theta)$$

$$v_i(t) = V_i \cos(\omega t + \phi)$$

- Then, the instantaneous power is given by:

$$p(t) = v_i i_i = \frac{V_i^2}{\sqrt{R^2 + X^2}} \cos(\omega t + \phi) \cos(\omega t + \phi - \theta)$$

$$= \frac{1}{2} \frac{V_i^2}{\sqrt{R^2 + X^2}} [\cos(2\omega t + 2\phi - \theta) + \cos\theta]$$

- The instantaneous power for sinusoidal drive has a sinusoidal component at twice the frequency of the input signal, and the DC component.

Average power

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- Because the average value of cos(2ωt) is zero, the average power flowing into an arbitrary impedance is just the DC term of the expression below.

$$p(t) = \frac{1}{2}\frac{V_i^2}{\sqrt{R^2 + X^2}}[\cos(2\omega t + 2\phi - \theta) + \cos\theta]$$

- The average power :

$$\bar{p} = \frac{1}{2}\frac{V_i^2}{\sqrt{R^2 + X^2}}\cos\theta$$

$$\bar{p} = \frac{1}{2}V_i I_i \cos\theta$$

- The average power in terms of complex amplitudes of voltages and currents is one-half the product of the two magnitudes multiplied by the cosine of the angle between them.
- The term cosθ is often called **the power factor**.

Average power

- The average power can also be written directly in terms of complex voltage and complex current.

$$\bar{p} = \frac{1}{2}\mathrm{Re}[\mathbf{V_i}\mathbf{I_i^*}] = \frac{1}{2}\mathrm{Re}[\mathbf{V_i^*}\mathbf{I_i}]$$

Where $\mathbf{V_i^*}$ is the complex conjugate of $\mathbf{V_i}$.

$\mathbf{I_i^*}$ is the complex conjugate of $\mathbf{I_i}$.

- Using this notation, 1/2 **VI*** is often called complex power, whence the **real part** of the complex power is the average power, the **"real" power**, and the **imaginary part** is called **reactive power**.

Pure Resistance

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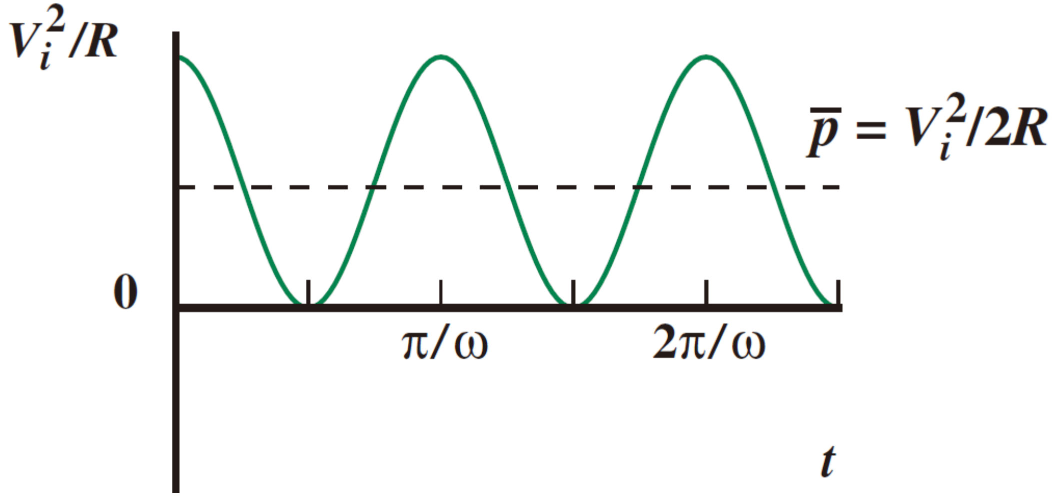
- Assume that the impedance is a pure resistance R, that is, $X = 0$ and $\phi = 0$.

$$p(t) = \frac{1}{2}\frac{V_i^2}{\sqrt{R^2}}[\cos(2\omega t) + \cos 0] = \frac{V_i^2}{2R}[1 + \cos(2\omega t)]$$

- The average power dissipated in the resistor is:

$$\overline{p} = \frac{V_i^2}{2R}$$

- This is exactly one half of the power delivered by the DC voltage of the same amplitude.



Root-Mean-Square (rms) Voltage

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- The root-mean-square voltage, abbreviated rms, which is related to the peak amplitude of the sinewave by the square root of two.

$$V_{rms} = \frac{V_i}{\sqrt{2}}$$

- In terms of the rms unit, average power is :

$$\overline{p} = \frac{V_i^2}{2R} = \frac{(V_{rms})^2}{R}$$

- For non-sinusoidal voltages, the general definition of rms voltage is, as the name implies, $V_{rms} = \sqrt{\overline{v}^2(t)}$

- Thus, the 110-V AC power from a wall socket is 110 volts rms,
- Or $110 \times \sqrt{2} = 155.6$ volts peak.

Pure Reactance

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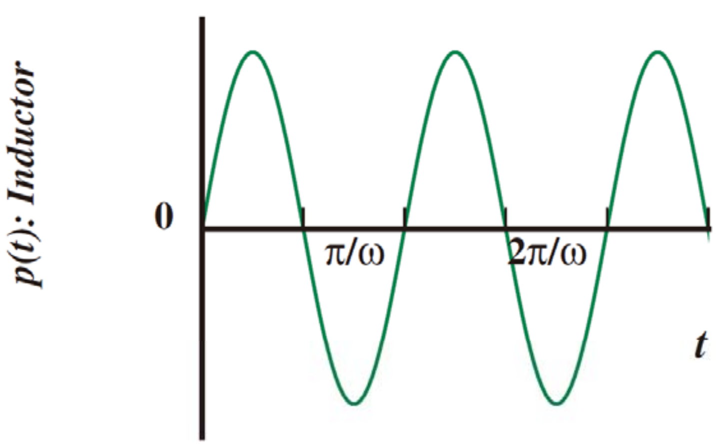
- If the impedance consists only of inductor ($\theta = \pi/2$), that is, R = 0. Also assume that $\phi = 0$.

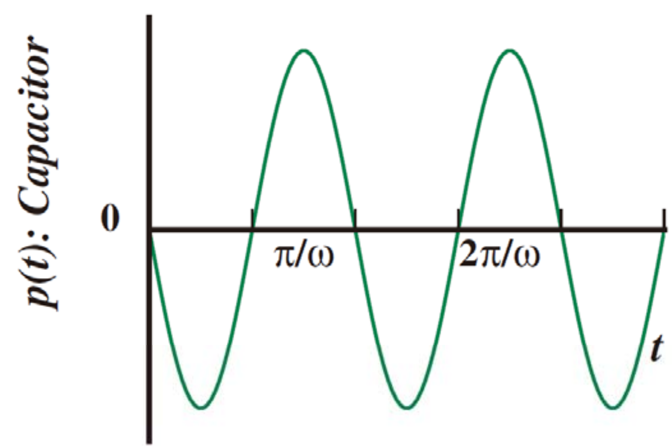
$$p(t) = \frac{V_i^2}{2X}[\cos(2\omega t - \frac{\pi}{2}) + \cos\frac{\pi}{2}] = \frac{V_i^2}{2X}\cos(2\omega t - \frac{\pi}{2}) = \frac{V_i^2}{2X}\sin(2\omega t)$$

- If the impedance consists only of capacitor ($\theta = -\pi/2$):

$$p(t) = \frac{V_i^2}{2X}[\cos(2\omega t + \frac{\pi}{2}) + \cos(-\frac{\pi}{2})] = \frac{V_i^2}{2X}\cos(2\omega t + \frac{\pi}{2}) = -\frac{V_i^2}{2X}\sin(2\omega t)$$

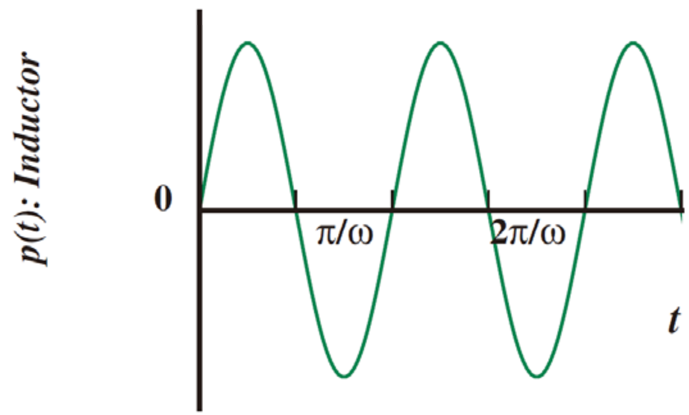
- In both cases, the average power is zero.

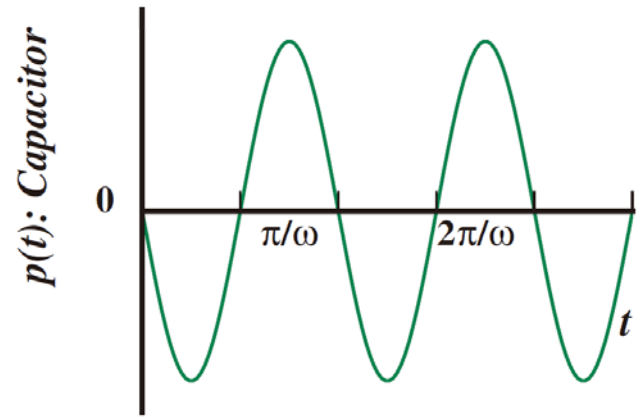




Pure Reactance

- The L's and C's absorb power for two quarters of each cycle, and deliver the power back to the source during the other two quarter cycles.





- Power companies are not happy about this state of affairs, because they still must supply the power and pay for the power losses in the transmission line.
- Although there is no average power supplied to this lossless circuit in the sinusoidal steady state, there is **energy stored** on the average.

Average Stored Energies

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- For a capacitor, the stored energy is.

$$W_C = \frac{1}{2}Cv^2(t) = \frac{1}{2}C[V_i\cos(\omega t)]^2 = \frac{1}{2}CV_i^2\left(\frac{1}{2}+\frac{1}{2}\cos(2\omega t)\right)$$

- Again a DC term and a double-frequency term.
- The average stored energy is

$$W_C = \frac{1}{2}CV_i^2\left(\frac{1}{2}\right) = \frac{1}{4}CV_i^2$$

- A similar derivation for an inductor yields:

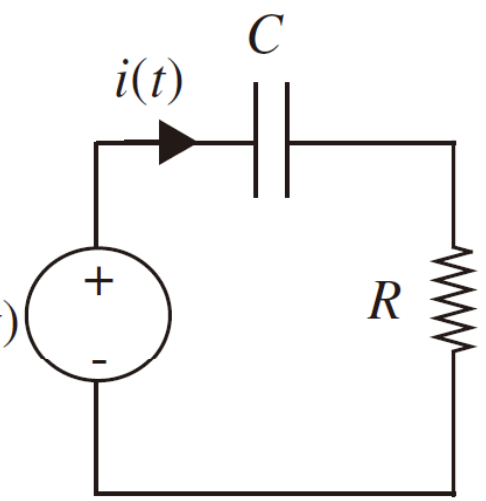
$$\overline{W}_L = \frac{1}{4}LI_i^2 \qquad \text{where } i_L(t) = I_i\cos(\omega t)$$

Example

- **The complex amplitude of the current**

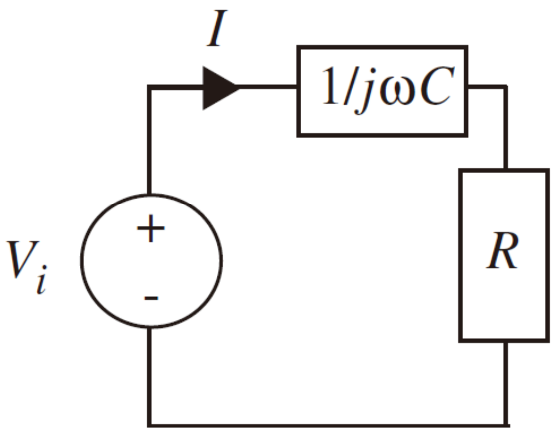
$$\mathbf{I_i} = \frac{\mathbf{V}_i}{Z} = \frac{\mathbf{V_i}}{R + \frac{1}{j\omega C}} = \frac{\mathbf{V_i}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} e^{-j\theta}$$

Where $\theta = \tan^{-1} \dfrac{1}{\omega RC}$



- **The average power dissipated in the circuit is**

$$\overline{p} = \frac{1}{2} \frac{V_i^2}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \cos\theta = \frac{1}{2} \frac{V_i^2}{|Z|} \cos\theta$$



- **At the break frequency or corner frequency**

$$\overline{p} = \frac{1}{2} \frac{V_i^2}{R\sqrt{1^2 + 1^2}} \cos\frac{\pi}{4} = \frac{1}{2} \frac{V_i^2}{2R}$$

- Hence the frequency $\omega = 1/RC$ is also called the half-power frequency

Peak Voltage in Resonant Circuit

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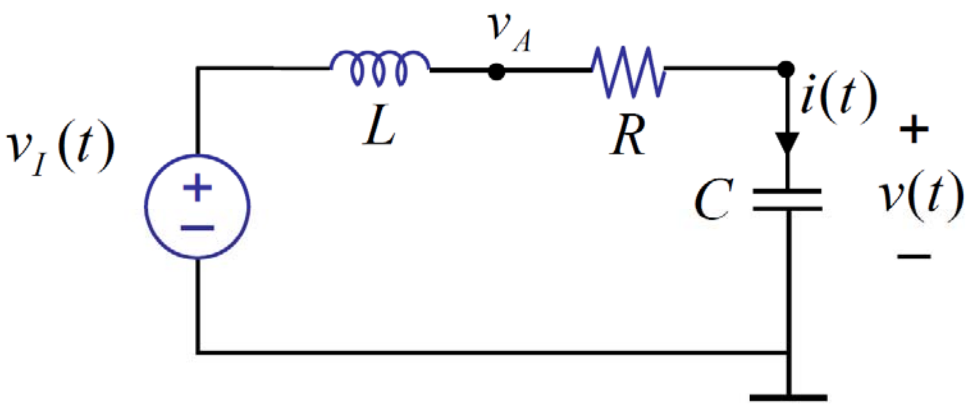
- Assuming a cosine wave for the voltage source: $v_i(t) = V_i \cos \omega t$
- Assume that the circuit is being driven at its resonant frequency.

$$\mathbf{I} = \frac{\mathbf{V_i}}{1 + j\left(\omega_0 \frac{L}{R} - \frac{1}{\omega_0 RC}\right)} = \frac{\mathbf{V_i}}{R}$$

$$\mathbf{V_c} = \frac{\mathbf{I}}{j\omega_0 C} = \frac{\mathbf{V_i}}{j\omega_0 RC} = -j\frac{\omega_0 L}{R}\mathbf{V_i}$$

$$\Rightarrow V_c = \frac{\omega_0 L}{R} V_i = QV_i$$

$$\mathbf{V_l} = j\omega_0 L\mathbf{I} = j\frac{\omega_0 L}{R}\mathbf{V_i} \Rightarrow V_l = \frac{\omega_0 L}{R} V_i = QV_i$$



- The voltage across either the capacitor or the inductor in a series resonant circuit is Q times the input voltage when the circuit is driven at its resonant frequency.

Stored Energy in Resonant Circuit

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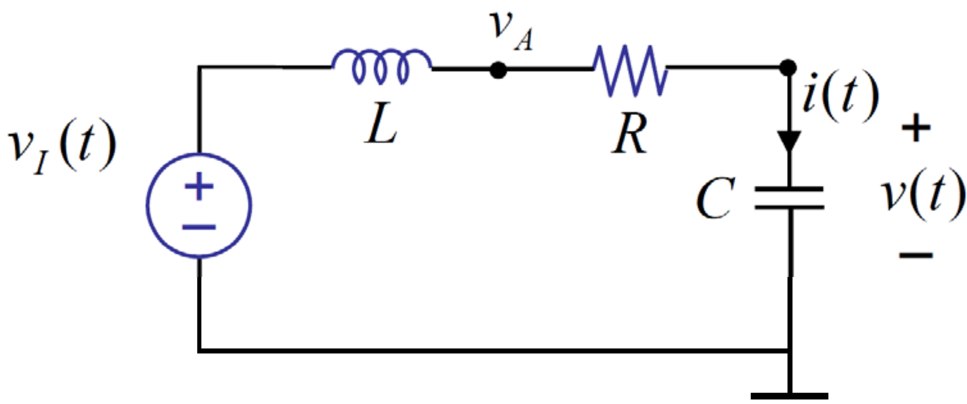
- Assuming a cosine wave for the voltage source: $v_i(t) = V_i \cos \omega t$
- The voltage across the capacitor is:

$$v_C(t) = \mathrm{Re}[\mathbf{V_c} e^{j\omega_0 t}] = \mathrm{Re}[-jV_i Q e^{j\omega_0 t}] = V_i Q \sin \omega t$$

- The energy stored in the capacitor

$$w_C = \frac{1}{2} C V_i^2 Q^2 \sin^2(\omega_0 t)$$
$$= \frac{1}{2} C V_i^2 Q^2 [1 - \cos(2\omega_0 t)]$$



- The energy stored in the inductor

$$i_L(t) = \mathrm{Re}[\mathbf{I} e^{j\omega_0 t}] = \frac{V_i}{R} \cos \omega_0 t$$

$$w_L = \frac{1}{2} L \frac{V_i^2}{R^2} \cos^2(\omega_0 t)$$
$$= \frac{1}{2} L \frac{V_i^2}{R^2} [1 + \cos(2\omega_0 t)]$$

Summary

- The performance of a resonant circuit is summarized by its frequency response. The frequency response comprises plots of magnitude and phase angle versus frequency.
- The magnitude plot is sketched by drawing the low-frequency and the high-frequency asymptotes. The two asymptotes intersect at the break frequency.
- The quality factor Q, the resonant frequency ω_0, and the damping factor α are three key parameters that characterize the behavior of resonant systems.
- The average power in terms of complex amplitudes of voltages and currents is one-half the product of the two magnitudes multiplied by the cosine of the angle between them.
- The bandwidth is related to the resonant frequency by the quality factor:

$$Q = \frac{\text{Resonant frequency}}{\text{Bandwidth}}$$