Sinusoidal Steady State
Impedance and Frequency Response

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Response to Sinusoidal Drive

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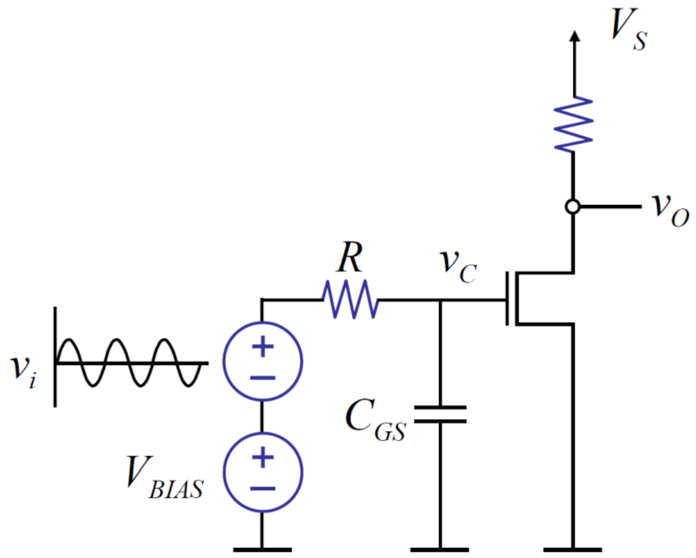
- Response of networks to sinusoidal drive.



- Sinusoids is important because signals can be represented as a sum of sinusoids.
- Response to sinusoids of various frequencies –also called as frequency response --tells us a lot about the system.

Sinusoids

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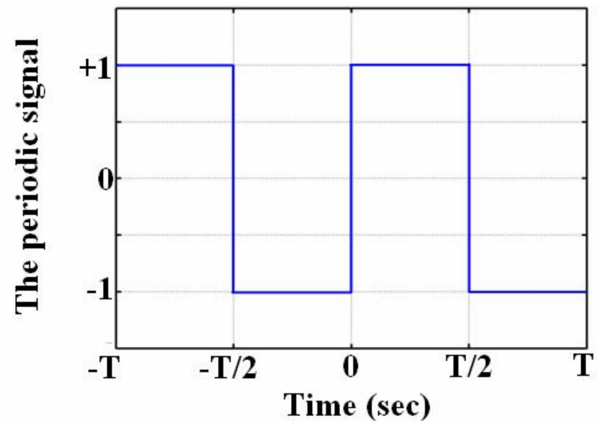
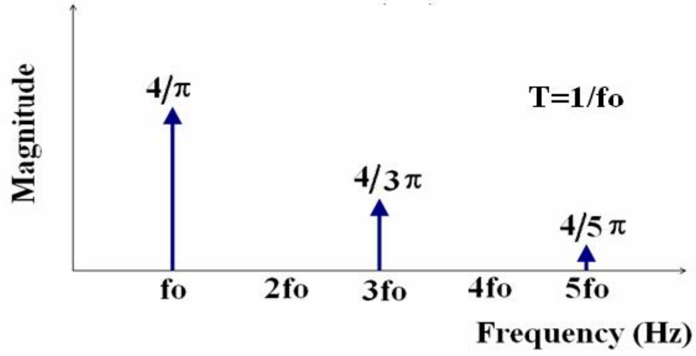
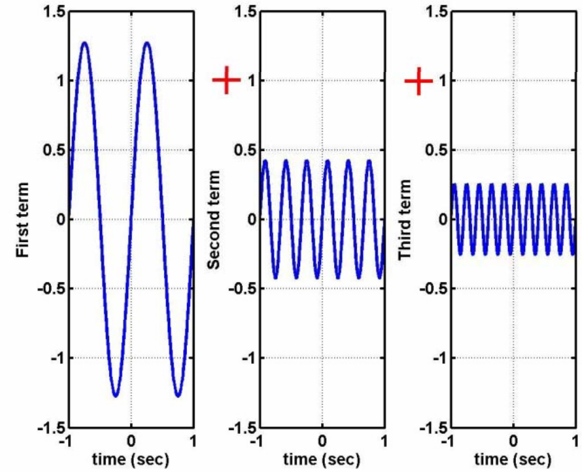
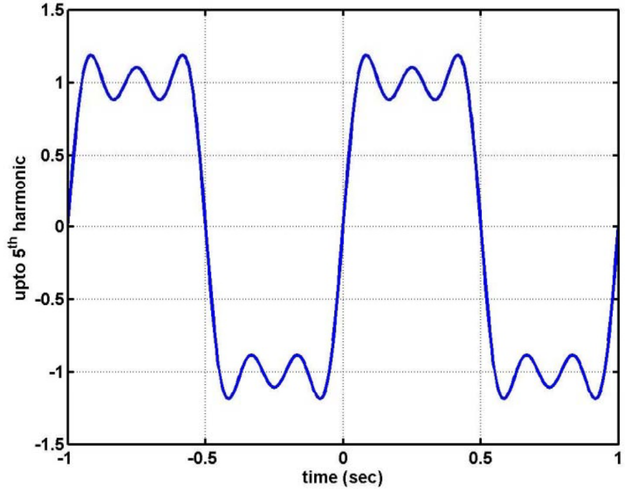
- Sinusoids is important because signals can be represented as a sum of sinusoids. $$x(t)=\sum_{n=1,3,5,\ldots}^{\infty}\frac{4}{n\pi}\sin\left(n\omega_0 t\right)$$









Response to Sinusoidal Drive

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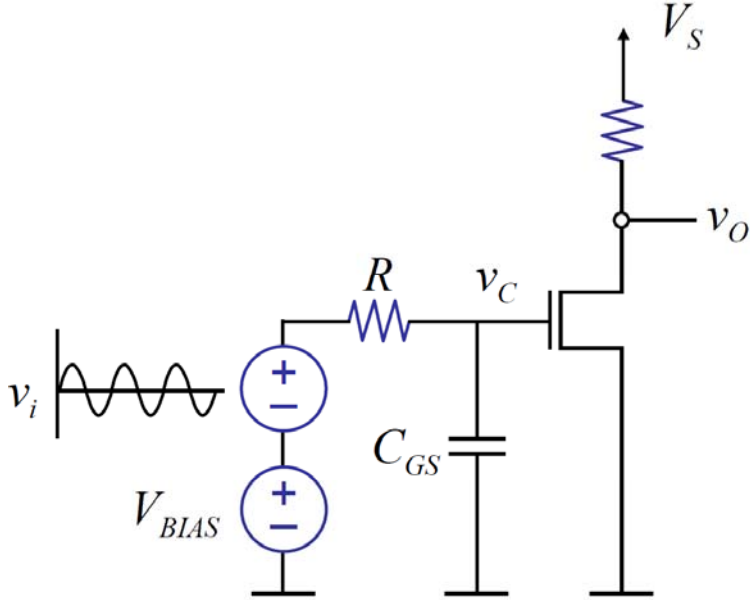
- What is the response of the following amplifier to a sinusoidal drive?



- Observing the amplitude of v_o as the frequency of the input v_o changed.
- We found the amplitude of v_o decreased with frequency.
- We also found that v_o shifted as frequency changes (phase).
- Need to study behavior of networks for sinusoidal drive.

Sinusoidal Response of RC Network

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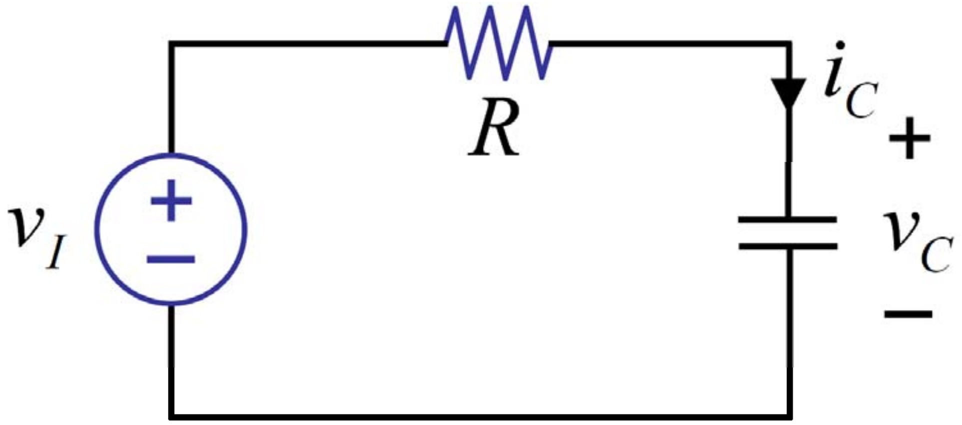
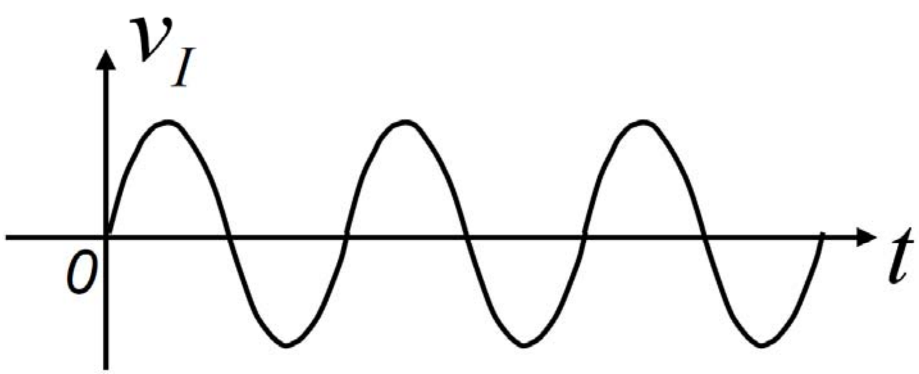
- The Circuits:



- The input:

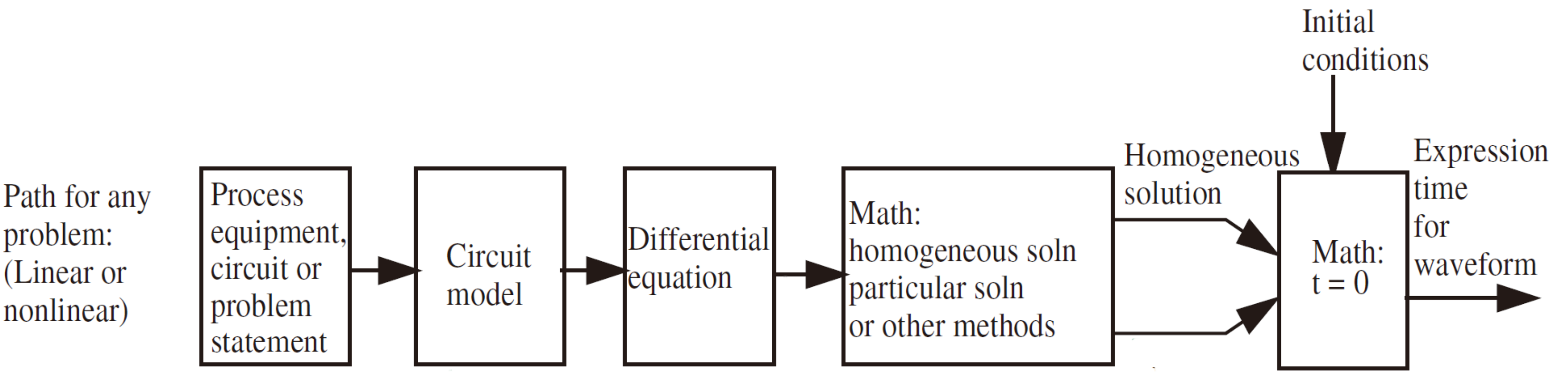
$$v_I(t) = V_i \cos(\omega t) \quad \text{for } t \geq 0 \quad V_i \text{ is real.}$$
$$= 0 \quad \text{for } t < 0$$



- Assume zero initial state:

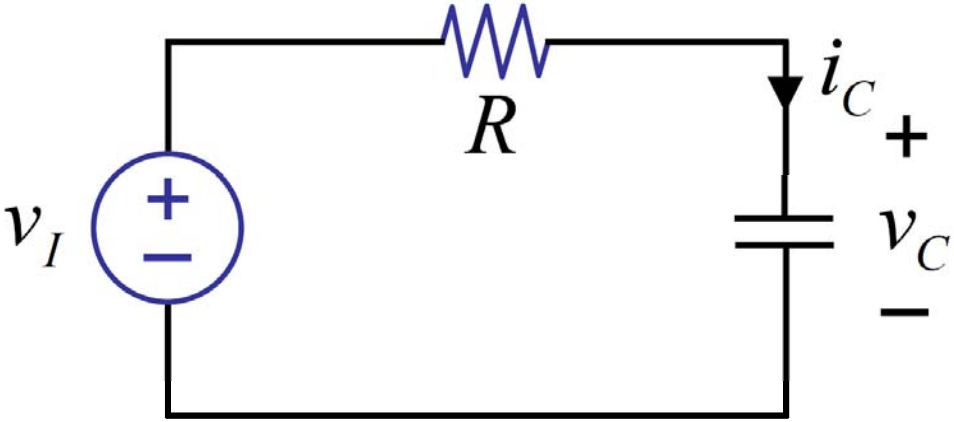
$$v_C(0) = 0 \ \text{V}$$

Usual Approach



- Set up the differential equation.
- Find the particular solution, v_P.
- Find the homogeneous solution , v_H.
- The total solution is the sum of the particular and homogeneous solutions , $v = v_P + v_H$.
- Use the initial conditions to solve for the remaining constants.

Usual Approach



- Set up the differential equation.

$$RC\frac{dv_C}{dt}+v_C=v_I=V_i\cos(\omega t)$$

- That was easy.

Usual Approach

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- Find the particular solution, v_P.

$$RC\frac{dv_P}{dt}+v_P=V_i\cos(\omega t)$$

- Try $v_P=K$ $\quad RC\frac{dK}{dt}+K=K\neq V_i\cos(\omega t)\Rightarrow \text{Noop}$
- Try $v_P=A\cos(\omega t)$ $\quad -A\omega RC\sin(\omega t)+A\cos(\omega t)\neq V_i\cos(\omega t)\Rightarrow \text{Noop}$
- Try $v_P=A\cos(\omega t+\phi)$

$$-A\omega RC\sin(\omega t+\phi)+A\cos(\omega t+\phi)=V_i\cos(\omega t)$$

$$-A\omega RC\sin(\omega t)\cos(\phi)-A\omega RC\cos(\omega t)\sin(\phi)+$$
$$A\cos(\omega t)\cos(\phi)-A\sin(\omega t)\sin(\phi)=V_i\cos(\omega t)$$

$$\Rightarrow\sqrt{1+\omega^2R^2C^2}\,A\cos(\omega t+\phi+\delta)=V_i\cos(\omega t)\quad \text{where }\ \delta=\tan^{-1}(\omega RC)$$

- Worked but what a trigonometry nightmare.

Sneaky Approach

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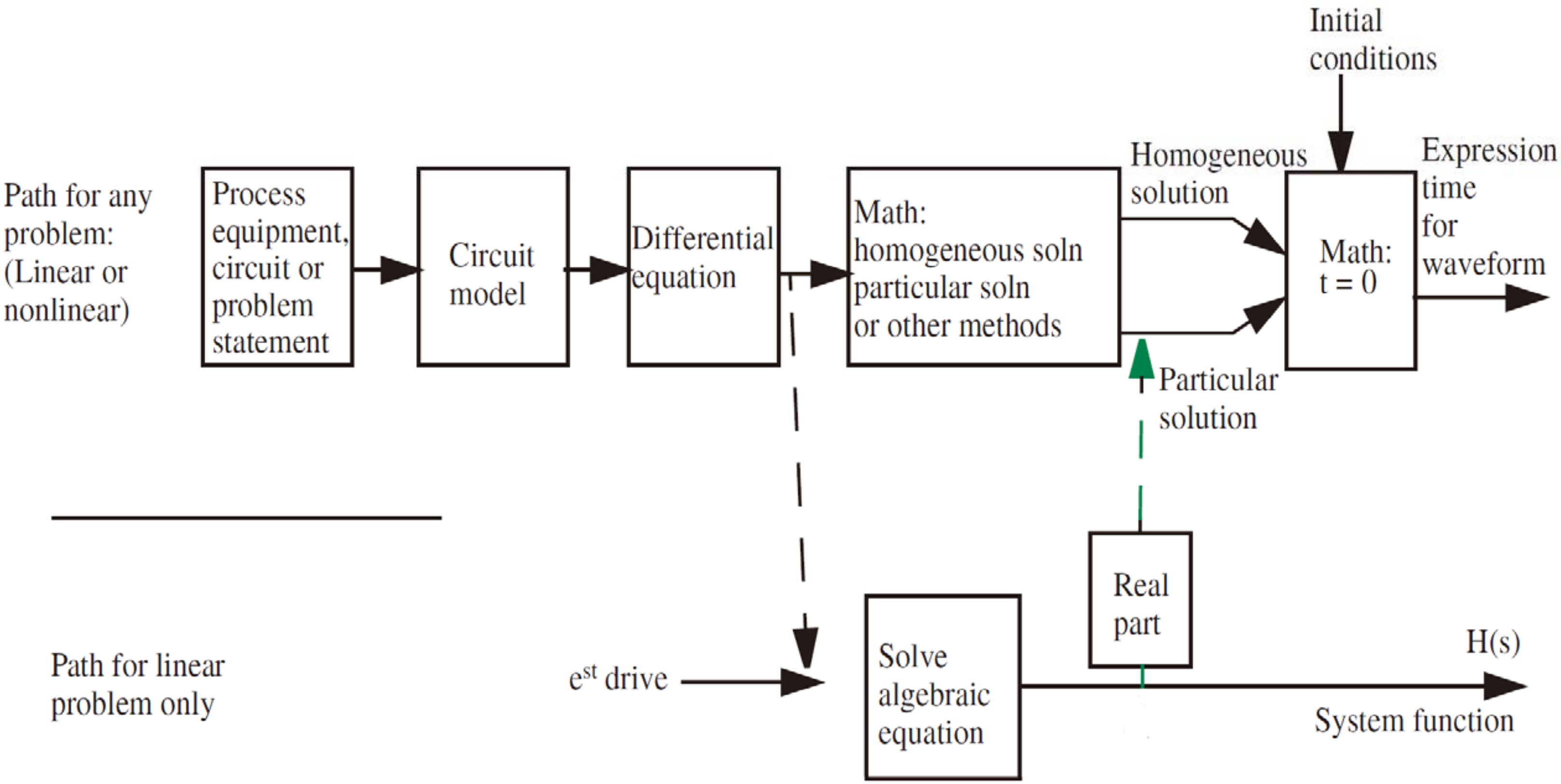
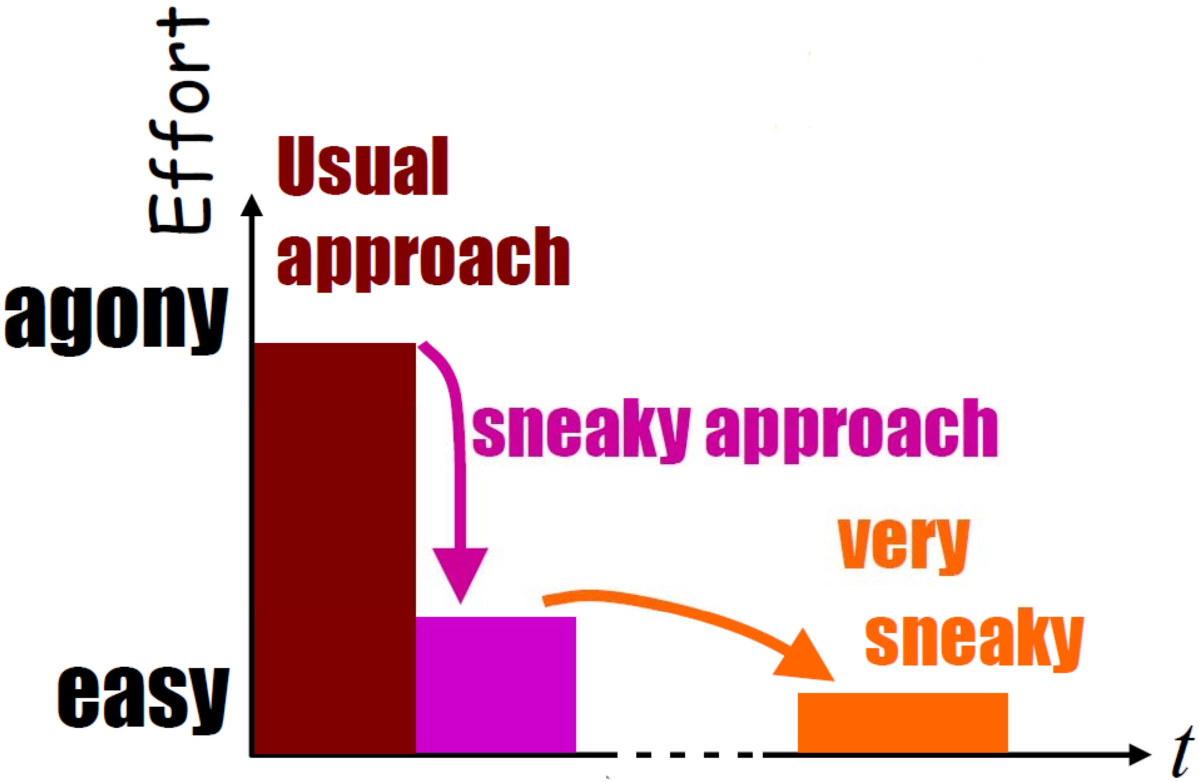


- Instead of input: $v_I(t) = V_i \cos(\omega t)$
- Find particular solution to another input: $v_I(t) = V_i e^{st}$

The Effort of Various Approaches



Charles Proteus Steinmetz

German-American mathematician and electrical engineer (1893)(1865-1923)

Sneaky Approach

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- The new drive: $v_I(t) = V_i e^{st}$
- Find the particular solution, v_{PS}:

$$RC\frac{dv_{PS}}{dt} + v_{PS} = V_i e^{st}$$

- Try solution: $v_{PS} = V_P e^{st}$

$$RC\frac{dV_P e^{st}}{dt} + V_P e^{st} = V_i e^{st} \Rightarrow \mathrm{s}RCV_P e^{st} + V_P e^{st} = V_i e^{st}$$

$$\Rightarrow (\mathrm{s}RC + 1)V_P = V_i$$

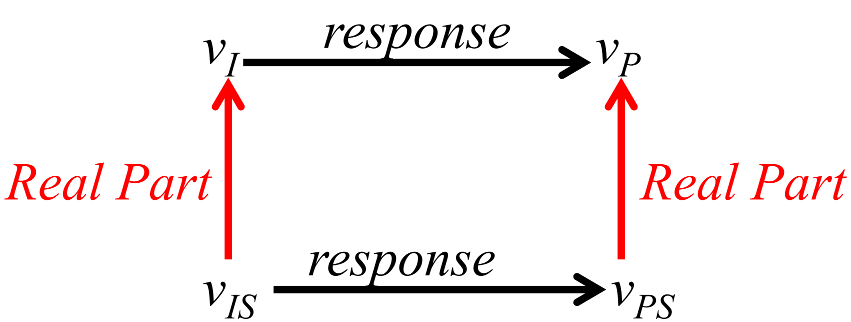
$$\Rightarrow V_P = \frac{V_i}{1 + \mathrm{s}RC} \qquad \Rightarrow v_{PS} = \frac{V_i}{1 + \mathrm{s}RC} e^{st}$$

- $v_{PS} = \frac{V_i}{1 + \mathrm{s}RC} e^{st}$ is particular solution for input $v_I(t) = V_i e^{st}$
- Easy!!

Sneaky Approach

- Similarly: $v_{PS} = \frac{V_i}{1 + j\omega RC} e^{j\omega t}$ is particular solution for input $v_I(t) = V_i e^{j\omega t}$
- We have complex amplitude V_P.
- Fact 1: Finding the response to $v_{PS} = V_P e^{j\omega t}$ is easy.
- Fact 2: From Euler relation, $e^{j\omega t} = \cos \omega t + j \sin \omega t$

$$v_I(t) = V_i \cos \omega t = \mathrm{Re}[V_i e^{j\omega t}]$$



- An inverse superposition argument, assuming system is real, linear.

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- Let's try to find v_P from v_{PS}: $v_I(t) = V_i e^{st}$

$$v_P = \mathrm{Re}[v_{PS}] = \mathrm{Re}[V_p e^{j\omega t}]$$

$$= \mathrm{Re}\left[\frac{V_i}{1 + j\omega RC} e^{j\omega t}\right]$$

$$= \mathrm{Re}\left[\frac{V_i(1 - j\omega RC)}{1 + \omega^2 R^2 C^2} e^{j\omega t}\right]$$

$$= \mathrm{Re}\left[\frac{V_i}{\sqrt{1 + \omega^2 R^2 C^2}} e^{j\phi} e^{j\omega t}\right], \quad \tan\phi = -\omega RC$$

$$= \mathrm{Re}\left[\frac{V_i}{\sqrt{1 + \omega^2 R^2 C^2}} e^{j(\omega t + \phi)}\right]$$

- The particular solution, v_P for $v_I(t) = V_i \cos(\omega t)$

$$v_P = \frac{V_i}{\sqrt{1 + \omega^2 R^2 C^2}} \cos(\omega t + \phi)$$

Sneaky Approach

- Recalled from Chapter 10, the homogeneous solution , v_H :

$$v_H = Ae^{-\frac{t}{RC}}$$

- The total solution is the sum of the particular and homogeneous solutions , $v = v_P + v_H$:

$$v_C = v_P + v_H = \frac{V_i}{\sqrt{1+\omega^2 R^2 C^2}}\cos(\omega t+\phi) + Ae^{-\frac{t}{RC}} \qquad \text{where } \tan\phi = -\omega RC$$

- Given: $v_C(0) = 0\ \text{V} \quad \Rightarrow v_C = \frac{V_i}{\sqrt{1+\omega^2 R^2 C^2}}\cos(\omega 0+\phi) + Ae^{-\frac{0}{RC}} = 0$
- So

$$A = -\frac{V_i}{\sqrt{1+\omega^2 R^2 C^2}}\cos(\phi)$$

Done! Phew!

Sinusoidal Steady State (SSS)

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- The total solution :

$$v_C = \frac{V_i}{\sqrt{1+\omega^2 R^2 C^2}} \left(\cos(\omega t + \phi) - \cos\phi \cdot e^{-\frac{t}{RC}} \right)$$

where $\tan\phi = -\omega RC$

- We are usually interested only in the particular solution for sinusoids, i.e. after transients have died.
- Notice when $t \to \infty$, $v_C \to v_P$ as $e^{-\frac{t}{RC}} \to 0$

$$v_C = v_P = \frac{V_i}{\sqrt{1+\omega^2 R^2 C^2}} \cos(\omega t + \phi)$$

- Described as: **Sinusoidal Steady State** (SSS)

Sinusoidal Steady State

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- All information about **Sinusoidal Steady State** is contained in V_P

$$V_P = \frac{V_i}{1 + j\omega RC}$$

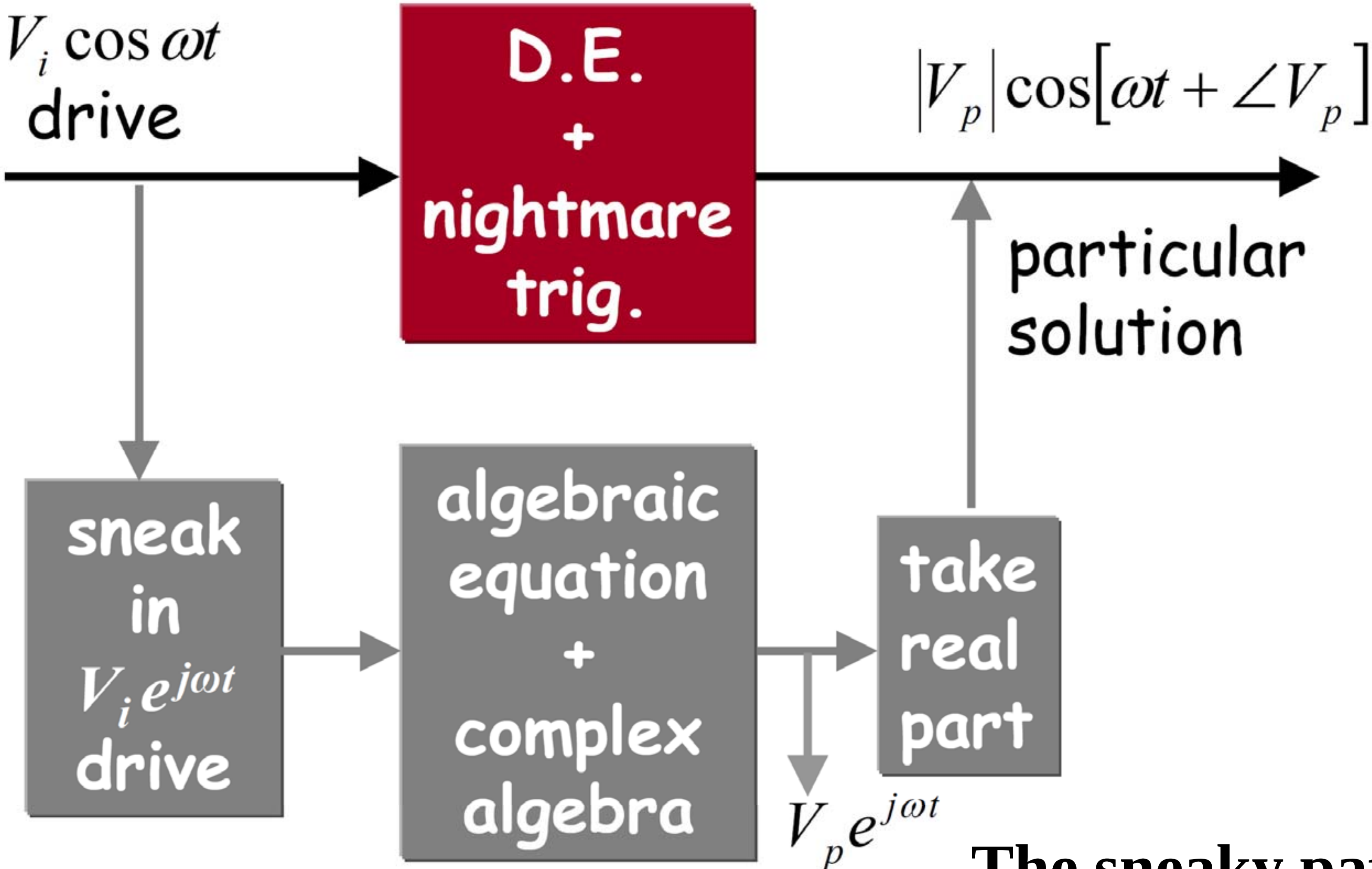
- A complex amplitude!
- Steps (1) find the homogeneous solution (2) find the total solution and determine remaining constants from the initial conditions **were a waste of time!**
- Let's rewrite $\frac{V_P}{V_i} = \frac{1}{1 + j\omega RC} = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} e^{j\phi}$ where $\tan\phi = -\omega RC$

$$\text{magnitude}: \left|\frac{V_P}{V_i}\right| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$\text{Phase } \phi: \quad \angle\frac{V_P}{V_i} = -\tan^{-1}\omega RC$$

Sinusoidal Steady State

- Visualizing the process of finding the particular solution v_P



The sneaky path!

Magnitude Plot

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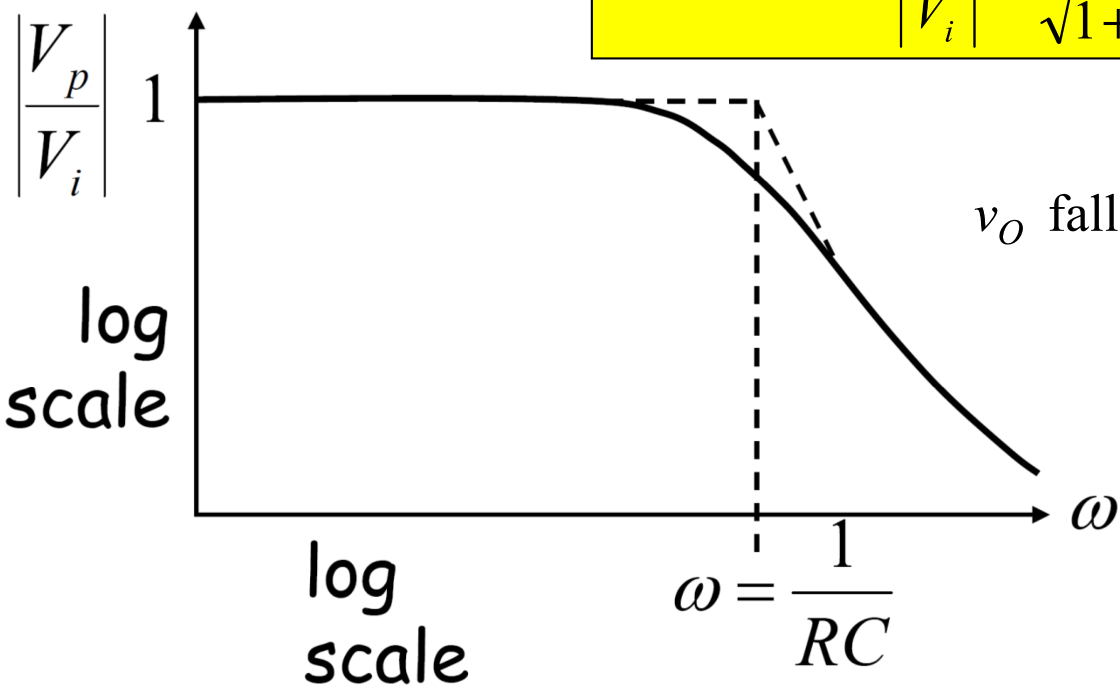
- **Transfer function** $H(j\omega)=\dfrac{V_P}{V_i}$
- **Transfer function**, also known as a system function, is the ratio of the complex amplitude of the network output to the complex amplitude of the input.

$$\text{Magnitude}: \left|\frac{V_P}{V_i}\right| = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}}$$



v_O fall off for high frequencies!

Bel and Decibel (dB)

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- Curious units called **"decibels"** are used by EEs to measure electric power, voltage, current, the gain or loss of amplifiers, and the insertion loss of filters.
- A **bel** (symbol **B**) is a unit of measure of ratios of power levels, i.e. relative power levels.
  - **1B =** $\mathbf{log_{10}(\mathit{P}_1/\mathit{P}_2)}$ where P_1 and P_2 are power levels.
  - The bel is a logarithmic measure
  - 1 bels corresponds to a ratio of 10:1
- The bel is too large for everyday use, so the **decibel (dB)**, equal to 0.1B, is more commonly used.
  - $10\text{dB} = \log_{10}(P_1/P_2)$
  - 10 dB corresponds to a ratio of 10:1
- The word decibel is a reference to powers of ten and to Alexander Graham Bell.

Logarithmic Measure for Power

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- To express a power in terms of decibels, one starts by choosing a reference power, $P_{reference}$, and writing.

  Power P in decibels = $10 \log_{10}(P/P_{reference})$

- Example:
  - Express a power of 50 mW in decibels relative to 1 watt.
  - P (dB) =$10 \log_{10} (50 \times 10^{-3}) = - 13$ dB
- Use logarithmic scale to express power ratios varying over a large range

dB: $$10\log\left(\frac{P_1}{P_2}\right)(\mathrm{dB})$$

Note: dB is not a unit for a physical quantity since power ratio is unitless. It is just a notation to remind us we are in the log scale.

Decibels for Measuring Transfer Function Magnitude?

- Decibles provide a measure of relative power levels.
- They can also be used in transfer functions.
- The key is in realizing that $P \propto V^2 \propto I^2$
- Thus

$$\mathrm{dB} = 10\log\left(\frac{V_{out}^2}{V_{in}^2}\right) = 20\log\left(\frac{V_{out}}{V_{in}}\right)$$

- **Transfer function**

$$\left|H(j\omega)\right|_{\text{in dB}} = 20\log\left(H(j\omega)\right)$$

Neper

- The "Neper" (after John Napier 1550-1617) is a unit based on Naperian logarithms to the base *e*.

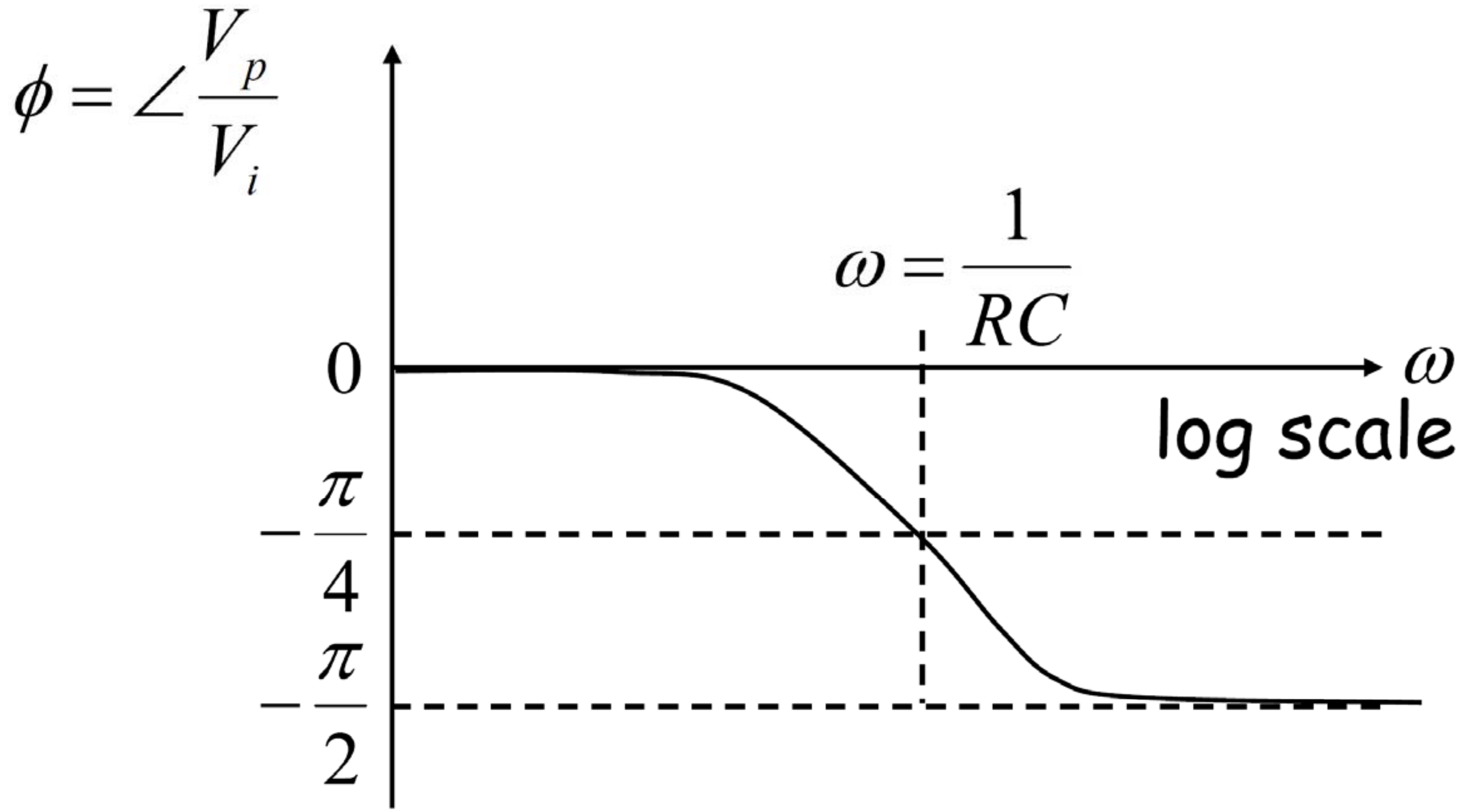
$$\text{Neper} = \log_e \sqrt{\frac{p_{out}}{p_{in}}} = \frac{1}{2}\log_e \frac{p_{out}}{p_{in}} = \log_e \frac{V_{out}}{V_{in}}$$

- The growth in popularity of the deciBel, since 1929, has been so great that it is now almost a household word throughout all branches of Electrical Engineering and Acoustics.
- The Neper is used in some European countries, but is less commonly encountered than the dB.

Phase Plot

- Transfer function $H(j\omega) = \dfrac{V_P}{V_i}$

Phase ϕ: $\quad \phi = \angle \dfrac{V_P}{V_i} = -\tan^{-1} \omega RC$



Is there an even simpler way to get V_P?

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- Let us look more closely at V_P

$$V_P = \frac{V_i}{1 + j\omega RC}$$

- Divide numerator and denominator by $j\omega C$.

$$V_P = V_i \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R}$$

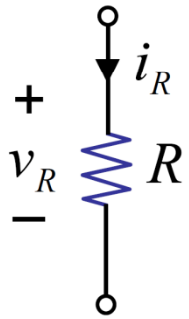
- Hmmm…looks like a voltage divider relationship.

$$V_P = V_i \frac{Z_C}{Z_C + R}$$

- Let's explore further…

The Impedance Model

- Consider resistor:



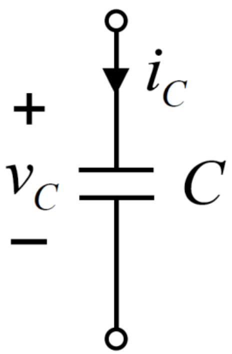
$$i_R = I_r e^{j\omega t} \quad \text{and} \quad v_R = V_r e^{j\omega t}$$

$$v_R = R i_R \;\Rightarrow\; V_r e^{j\omega t} = R I_r e^{j\omega t}$$

- For resistor $\boxed{V_r = R I_r}$

- For capacitor

$$i_C = I_c e^{j\omega t} \quad \text{and} \quad v_C = V_c e^{j\omega t}$$



$$i_C = C\frac{dv_C}{dt} \;\Rightarrow\; I_c e^{j\omega t} = C V_c j\omega e^{j\omega t}$$

$$\boxed{V_c = \frac{1}{j\omega C} I_c = Z_C I_c}$$

The Impedance Model

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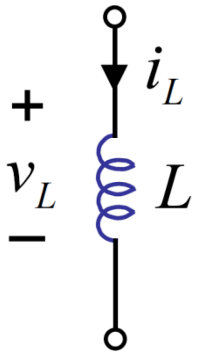
- For inductor:



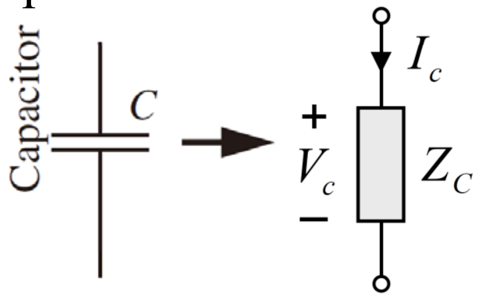
$$i_L = I_l e^{j\omega t} \quad \text{and} \quad v_L = V_l e^{j\omega t}$$

$$v_L = L\frac{di_L}{dt} \Rightarrow V_l e^{j\omega t} = L I_l j\omega e^{j\omega t}$$

$$V_l = j\omega L I_l = Z_L I_l$$

The Impedance Model

- In other words,
- Capacitor



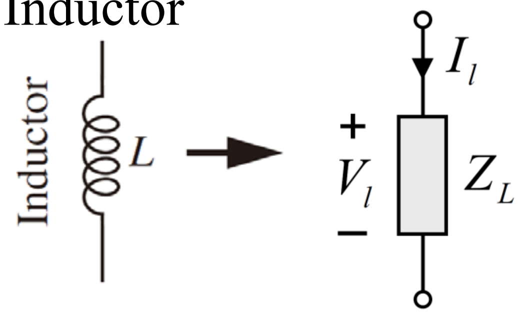
$$V_c = Z_C I_c$$

$$Z_C = \frac{1}{j\omega C}$$

Z_C is called **impedance**

For a drive of the form $V_C e^{j\omega t}$, complex amplitude V_C is related to the complex amplitude I_C algebraically, by a **generalization of Ohm's Law**.

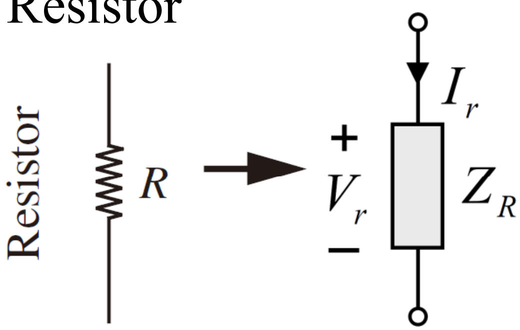
- Inductor



$$V_l = Z_L I_l$$

$$Z_L = j\omega L$$

- Resistor



$$V_r = Z_R I_r$$

$$Z_R = R$$

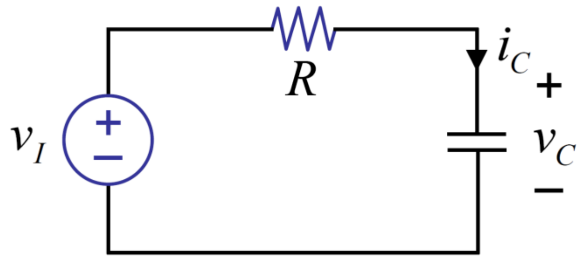
Z_C, Z_L, and Z_R, are called **impedance**

Symbol Conventions

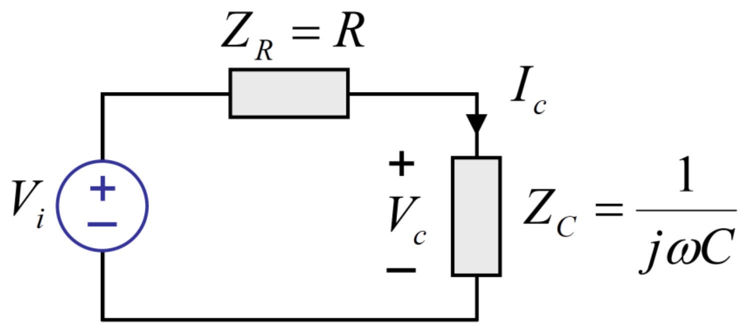
- **DC or operating-point variables**: uppercase symbols with uppercase subscripts (for example, V_A).
- **Total instantaneous variables**: lowercase symbols with uppercase subscripts (for example, v_A).
- **Incremental instantaneous variables**: lowercase symbols with lowercase subscripts (for example, v_a).
- **Complex amplitudes** or complex amplitudes of incremental components, and real amplitudes of sinusoidal input sources: uppercase symbols with lowercase subscripts (for example, V_a).

Back to RC Network

- Circuit:



- Impedance model:



- To find V_P

$$V_P = V_i \frac{Z_C}{Z_C + R} = V_i \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \quad \Rightarrow \quad V_P = \frac{1}{1 + j\omega RC} V_i$$

Done!

- **All our old friends apply! KVL, KCL, superposition…**

The Impedance Method

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- **1st Step**: Replace the (sinusoidal) sources by their complex (or real) amplitudes. $v_A = V_a \cos(\omega t)$ is replaced V_a.
- **2nd step**: Replace circuit elements by their impedances. The resulting diagram is called the impedance model of the network..
- **3rd Step**: Determine the complex amplitudes of the voltages and currents in the circuit by any standard linear circuit analysis.
- **4th Step**: Obtain the time variables from the complex amplitudes. For example, the time variable corresponding to node variable V_o is given by $v_o(t) = |V_o| \cos(\omega t + \angle V_o)$. This step is usually not necessary.

Phasor Overview

- Phasor analysis is first developed by Charles Proteus Steinmetz (1865-1923) in 1893 while working for General Electric.
- Phasor is a technique which uses complex number to analyze circuits at sinusoidal steady state;
  - Definition of phasors;
  - Comparison between time domain and phasor domain;
  - Circuit Theories (KCL, KVL, ..) in the phasor domain.

Sinusoidal Excitation

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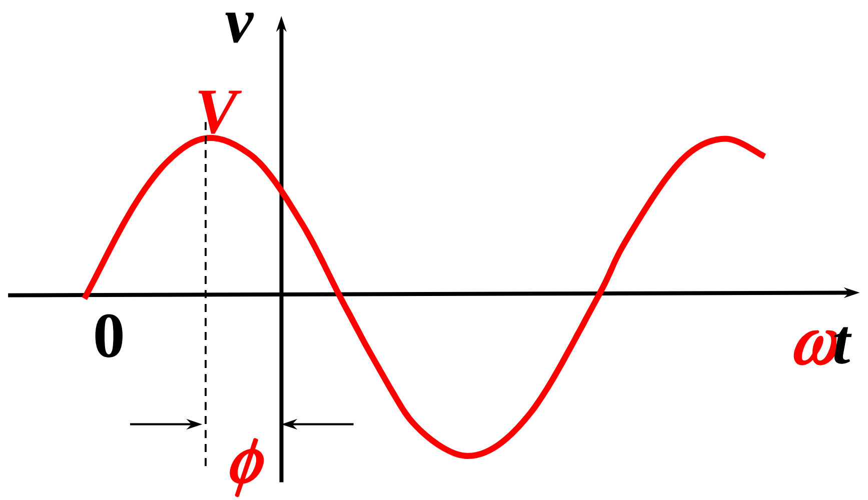
- Sinusoidal Excitation:

$$v(t)=V\cos(\omega t+\phi)$$

- There are 3 parameters: Amplitude V, angular frequency ω, and phase angle ϕ

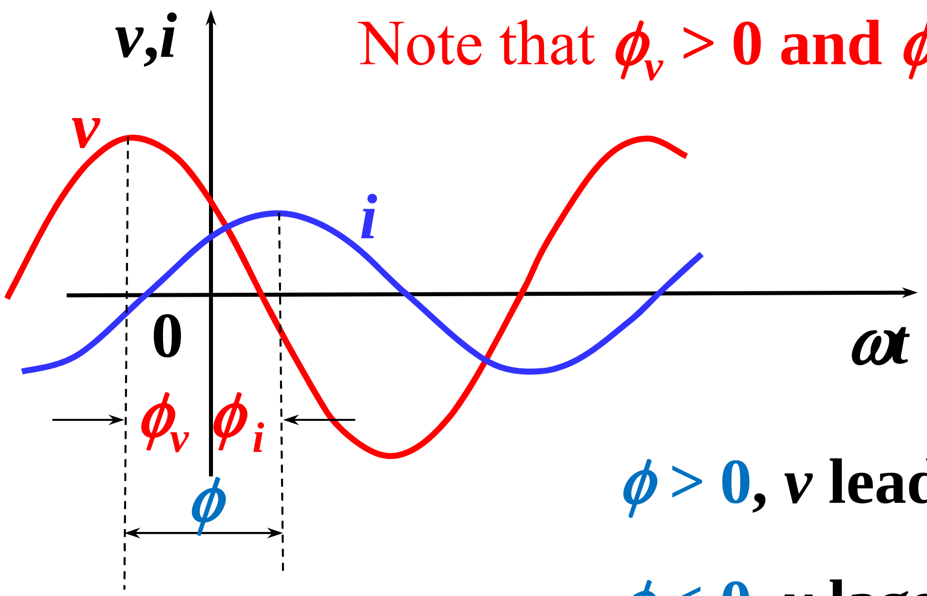


Lead and lag

- Sinusoidal Excitation:

For $v(t) = V\cos(\omega t + \phi_v)$ and $i(t) = I\cos(\omega t + \phi_i)$

- The phase difference $\phi = \omega t + \phi_v - \omega t - \phi_i = \phi_v - \phi_i$



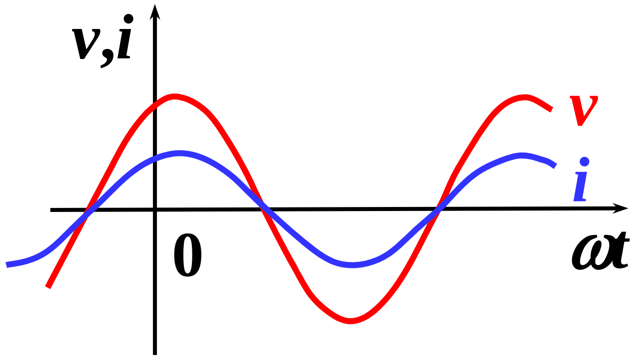
Note that $\phi_v > 0$ and $\phi_i < 0$

$\phi > 0$, v leading i, or i lagging v

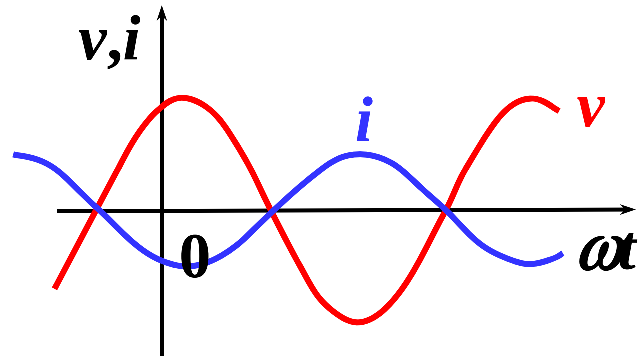
$\phi < 0$, v lagging i, or i leading v

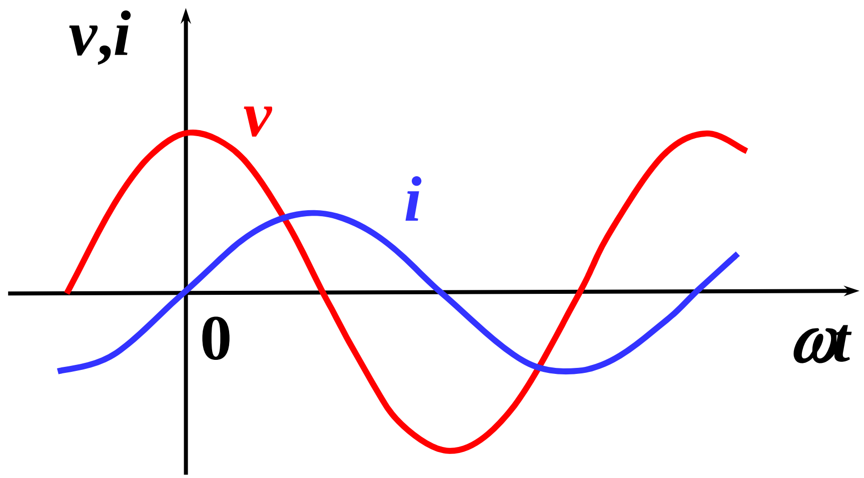
Special Phase Relation

Same Phase: $\phi = 0$



Inverting: $\phi = \pm\pi \ (\pm 180^0)$





$\phi = 90^\circ$

v **leading** ***i by*** **90°**
or ***i*** **lagging** ***v by*** **90°**
Not ***v*** **leading** ***i by*** **270°**
or ***I*** **lagging** ***v by*** **270°**

Convention： $|\phi| \le \pi \ (180^\circ)$

Phasor

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- Sinusoidal Excitation:

$$v(t)=V\cos(\omega t+\phi)$$

- For sinusoidal excitation with the same frequency ω, there are 2 parameters, amplitude V and phase angle ϕ, left.
- What can be used to represent amplitude V and phase angle ϕ at the same time?
- ***Complex number***

Phasor

- Assuming a source voltage is a sinusoid time-varying function
  Sinusoidal Excitation (Time Domain) is a time function :

$$v(t)=V\cos(\omega t+\phi)$$

- We can write

$$v(t)=V_m\cos(\omega t+\phi)=V_m\,\mathrm{Re}\left[e^{j(\omega t+\phi)}\right]=\mathrm{Re}\left[V_m e^{j\phi}e^{j\omega t}\right]=\mathrm{Re}\left[\mathbf{V}e^{j\omega t}\right]$$

- Define Phasor as

$$\mathbf{V}=V_m e^{j\phi}=V_m\angle\phi$$

- Phasor (Frequency domain) is a complex number:

$$\mathbf{V}=V_m e^{j\phi}\ \ or\ \ V_m\angle\phi=V_m\cos\phi+jV_m\sin\phi$$

$$v(t)=V_m\cos(\omega t+\phi)\ \leftrightarrow\ \mathbf{V}=V_m e^{j\phi}\ \text{or}\ V_m\angle\phi$$

Time Domain Frequency domain

Complex Numbers

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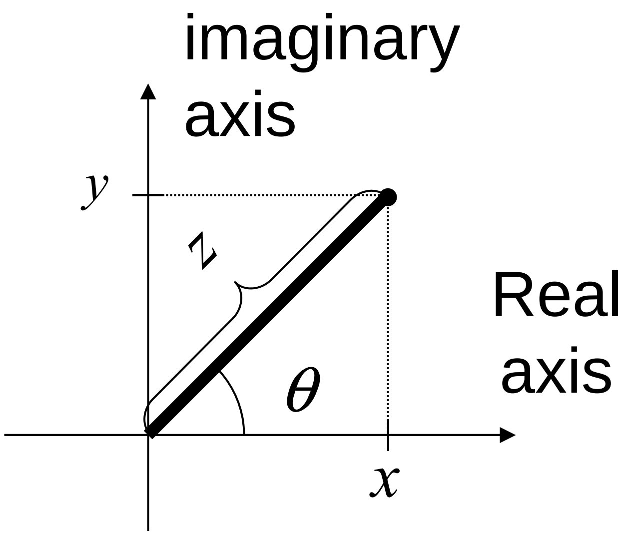

- x is the real part
- y is the imaginary part
- z is the magnitude
- θ is the phase
- Rectangular Coordinates: $\mathbf{Z} = x + jy$
- Polar Coordinates: $\mathbf{Z} = z \angle \theta$

$$x = z\cos\theta \qquad z = \sqrt{x^2 + y^2} \qquad \mathbf{Z} = z(\cos\theta + j\sin\theta)$$

$$y = z\sin\theta \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) \qquad \mathbf{Z} = ze^{j\theta} = z\angle\theta$$

- Exponential Form: $\mathbf{V} = z\, e^{j\theta}$

$$1 = 1e^{j0^\circ} = 1\angle 0^\circ$$

$$j = 1e^{j90^\circ} = 1\angle 90^\circ$$

Addition

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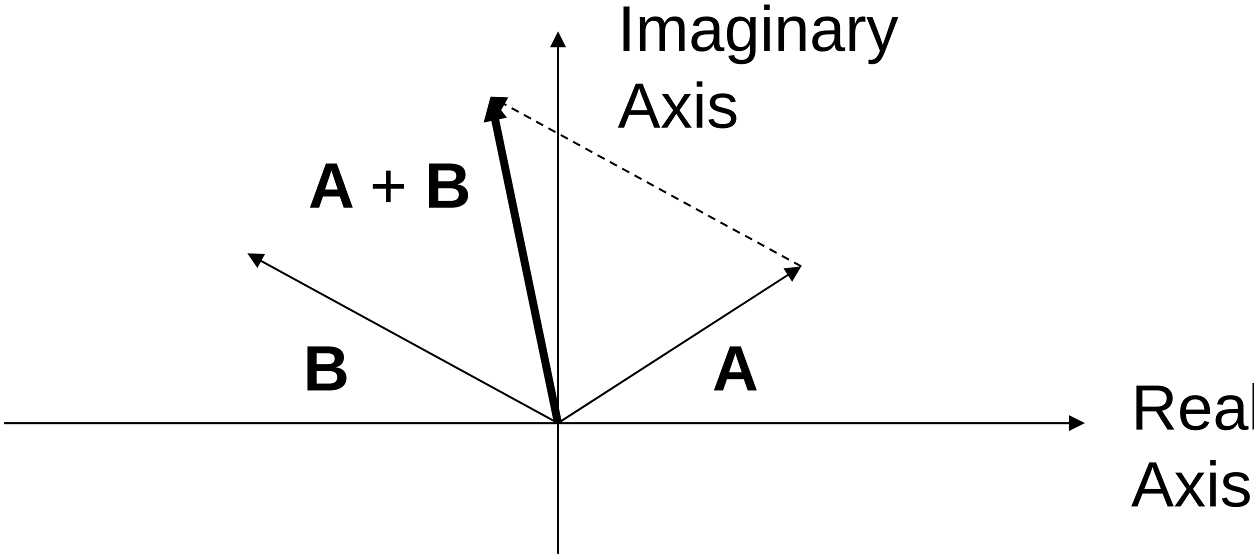
- Addition is most easily performed in rectangular coordinates:

$$\mathbf{A} = a + jb$$

$$\mathbf{B} = c + jd$$

$$\mathbf{A} + \mathbf{B} = (a + b) + j(c + d)$$



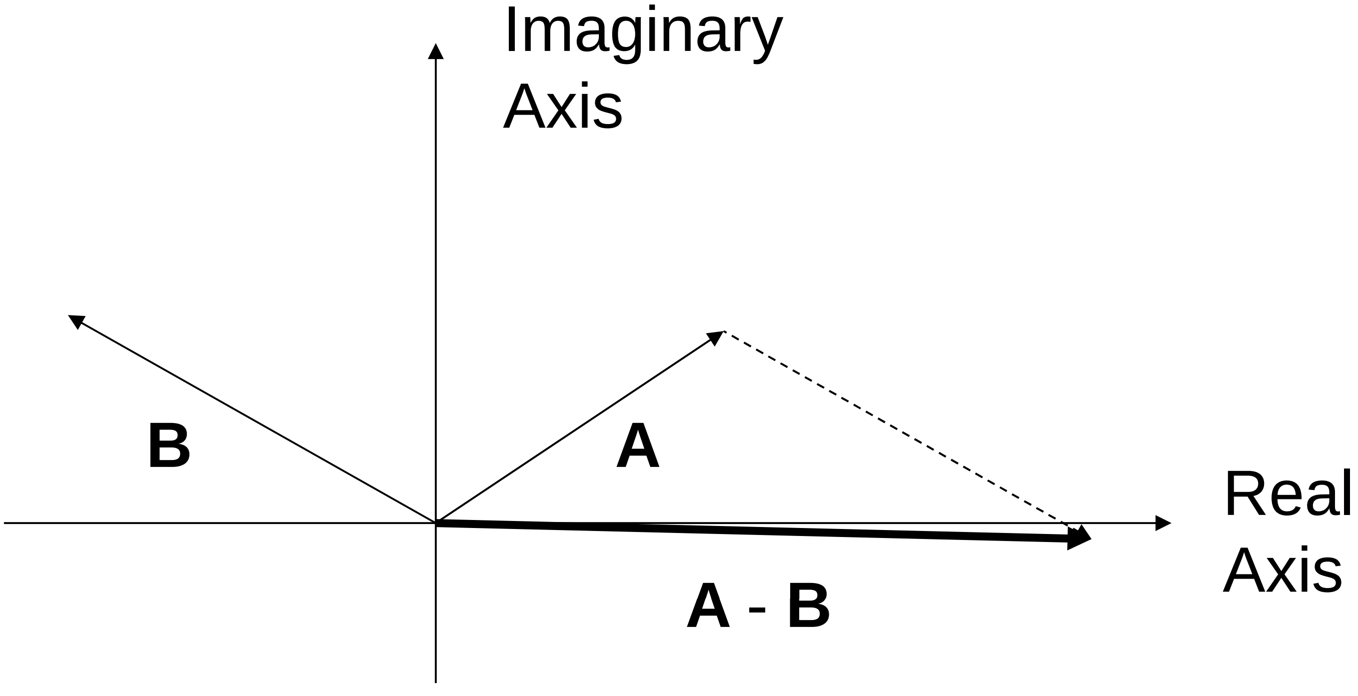
Subtraction

- Subtraction is most easily performed in rectangular coordinates:

$$\mathbf{A} = a + jb$$

$$\mathbf{B} = c + jd$$

$$\mathbf{A} - \mathbf{B} = (a - b) + j(c - d)$$



Multiplication

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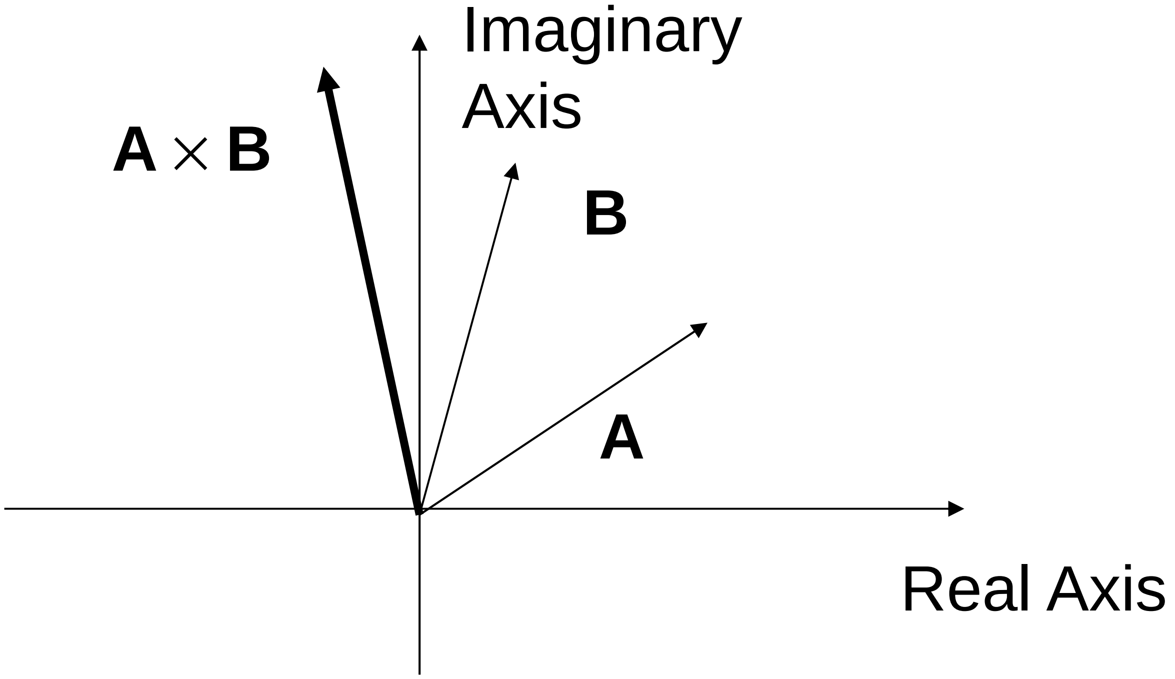
- Multiplication is most easily performed in polar coordinates :

$$\mathbf{A} = A_m e^{j\theta} = A_m \angle \theta$$

$$B = B_m e^{j\phi} = B_m \angle \phi$$

$$\mathbf{A} \times \mathbf{B} = (A_m \times B_m) e^{j(\theta+\phi)} = (A_m \times B_m) \angle (\theta + \phi)$$



Division

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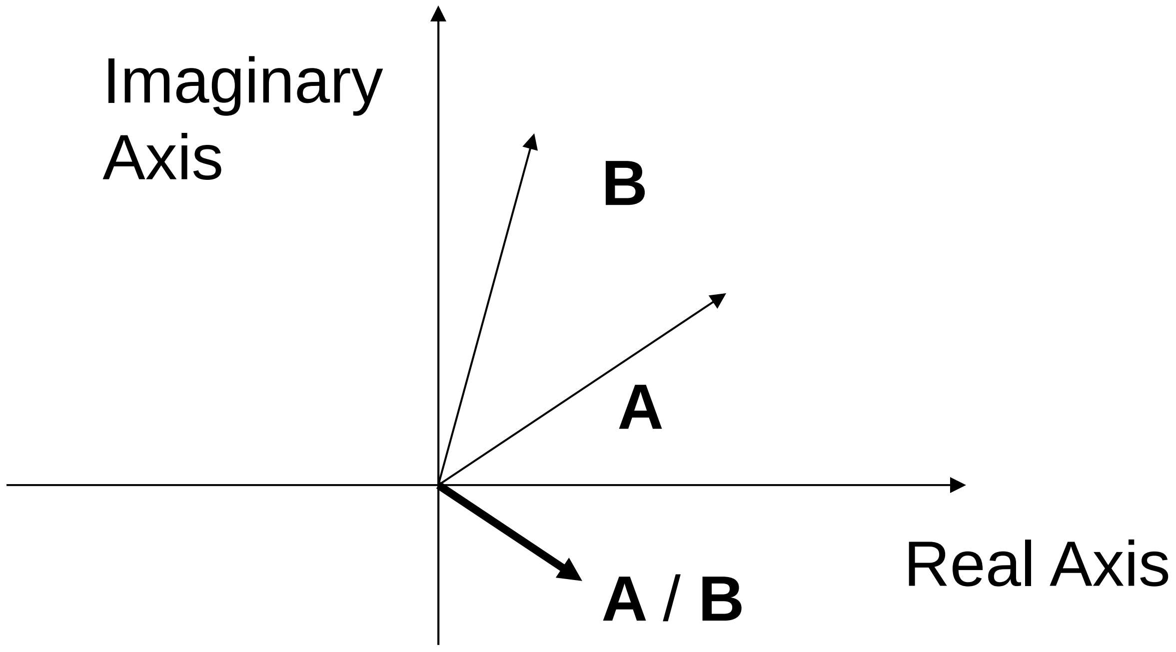
- Division is most easily performed in polar coordinates :

$$\mathbf{A} = A_m e^{j\theta} = A_m \angle \theta$$

$$B = B_m e^{j\phi} = B_m \angle \phi$$

$$\frac{\mathbf{A}}{\mathbf{B}} = \frac{A_m}{B_m} e^{j(\theta-\phi)} = \frac{A_m}{B_m} \angle(\theta - \phi)$$



$$v_1(t) = 6\cos(314t + 30^\circ)\ \text{V} \qquad \Rightarrow \qquad \mathbf{V_1} = 6\angle 30^\circ\ \text{V}$$

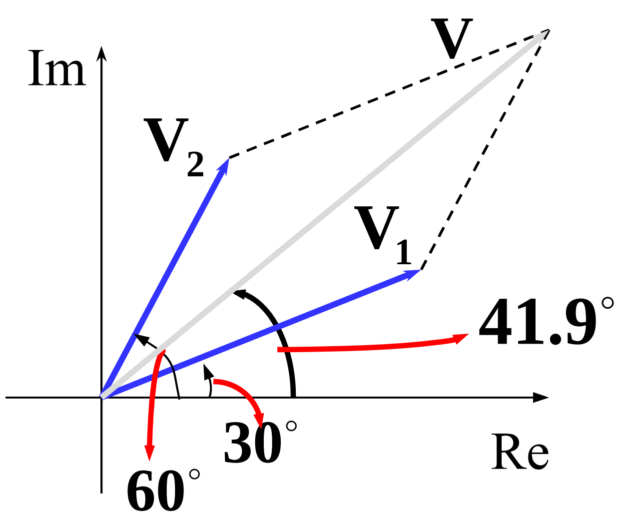
$$v_2(t) = 4\cos(314t + 60^\circ)\ \text{V} \qquad \Rightarrow \qquad \mathbf{V_2} = 4\angle 60^\circ\ \text{V}$$

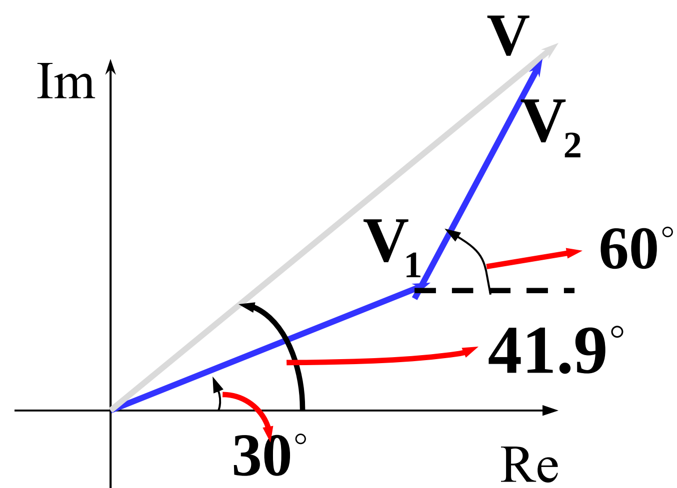
$$\mathbf{V} = \mathbf{V_1} + \mathbf{V_2} = 6\angle 30^\circ + 4\angle 60^\circ = 5.196 + j3 + 2 + j3.464$$

$$= 7.196 + j6.464 = 9.67\angle 41.9^\circ\ \text{V}$$

$$\therefore v(t) = v_1(t) + v_2(t) = 9.67\cos(314t + 41.9^\circ)\ \text{V}$$

The addition of two sinusoidal excitations can be found with the help of the phasor diagram。





Algebra With Complex Numbers

- To compute phasor voltages and currents, we need to be able to perform computation with complex numbers.

(1) $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$

(2) $\mathbf{A}+(\mathbf{B}+\mathbf{C})=(\mathbf{A}+\mathbf{B})+\mathbf{C}$

(3) $\mathbf{A}\times\mathbf{B}=\mathbf{B}\times\mathbf{A}$

(4) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=(\mathbf{A}\times\mathbf{B})\times\mathbf{C}$

(5) $\mathbf{A}\times(\mathbf{B}+\mathbf{C})=\mathbf{A}\times\mathbf{B}+\mathbf{A}\times\mathbf{C}$

(6) $(\mathbf{A}\times\mathbf{B})^{\mathbf{C}}=\mathbf{A}^{\mathbf{C}}\times\mathbf{B}^{\mathbf{C}}$

(7) $\mathbf{A}^{\mathbf{B}}\times\mathbf{A}^{\mathbf{C}}=\mathbf{A}^{(\mathbf{B}+\mathbf{C})}$

(8) $(\mathbf{A}^{\mathbf{B}})^{\mathbf{C}}=\mathbf{A}^{\mathbf{B}\times\mathbf{C}}$

(9) $\mathbf{A}+\mathbf{B}=\mathbf{C}$

(10) $\mathbf{B}=\mathbf{C}-\mathbf{A}$

(11) $\mathbf{A}\times\mathbf{B}=\mathbf{C}$

(12) $\mathbf{B}=\dfrac{\mathbf{C}}{\mathbf{A}}$

(13) $\mathbf{B}^{\mathbf{A}}=\mathbf{C}$

(14) $\mathbf{B}=\sqrt[\mathbf{A}]{\mathbf{C}}$

(15) $\mathbf{A}^{\mathbf{B}}=\mathbf{C}$

(16) $\mathbf{B}=\log_{\mathbf{A}}\mathbf{C}$

Kirchhoff's Laws for Phasors

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- Suitable for AC steady state.

KVL: $v_1 + v_2 + \cdot\cdot\cdot + v_n = 0$

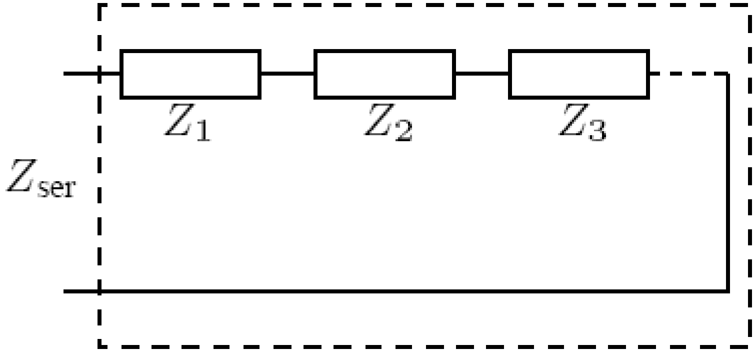
$$\rightarrow V_{m1}\cos(\omega t + \phi_1) + V_{m2}\cos(\omega t + \phi_2) + \cdots + V_{mn}\cos(\omega t + \phi_n) = 0$$

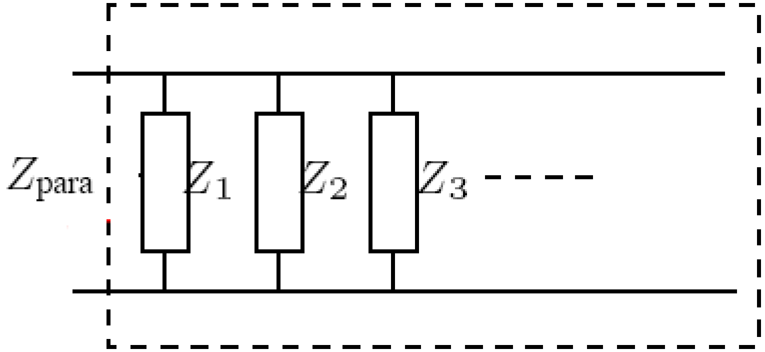
$$\rightarrow \mathrm{Re}\left[V_{m1}e^{j\phi_1}e^{j\omega t}\right] + \mathrm{Re}\left[V_{m2}e^{j\phi_2}e^{j\omega t}\right] + \cdots + \mathrm{Re}\left[V_{mn}e^{j\phi_n}e^{j\omega t}\right] = 0$$

$$\rightarrow \mathrm{Re}\left[(\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n)e^{j\omega t}\right] = 0$$

$$\rightarrow \mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n = 0$$

KCL: $i_1 + i_2 + \cdot\cdot\cdot + i_n = 0$ . . . $\rightarrow$ $\mathbf{I}_1 + \mathbf{I}_2 + \cdot\cdot\cdot + \mathbf{I}_n = 0$





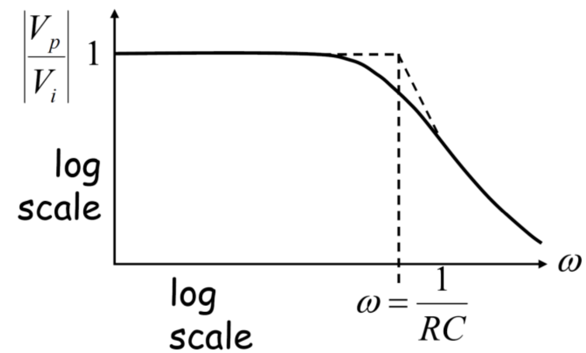
$$Z_{ser} = Z_1 + Z_2 + Z_3 + \cdots \qquad {}^1/Z_{para} = {}^1/Z_1 + {}^1/Z_2 + \cdots$$

Transfer Function

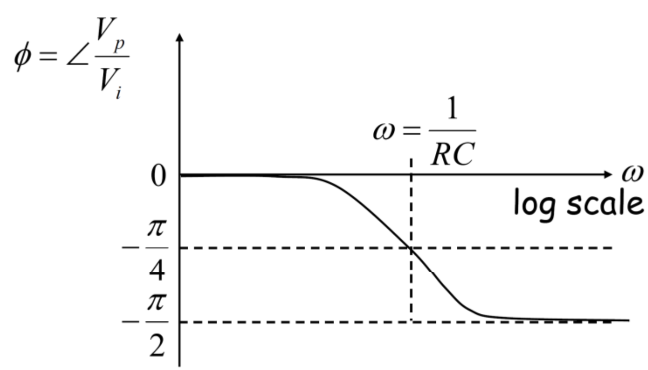
- Map system input to output $\mathbf{V_o} = H(j\omega)\mathbf{V_i}$
- Covert an input $v_i(t)$=$V_i\cos(\omega t+\phi)$ into phasor $\mathbf{V_i}$. Plug into above Eq.
- Get an output $\mathbf{V_o}$=H($j\omega$)$\mathbf{V_i}$. Covert back to time domain form.

$$v_o(t) = |H(j\omega)| V_i \cos[\omega t + \phi + \angle H(j\omega)]$$

- Output is scaled by $|H(j\omega)|$



- Shift in time by $\dfrac{\angle H(j\omega)}{\omega}$



The Effort of Various Approaches

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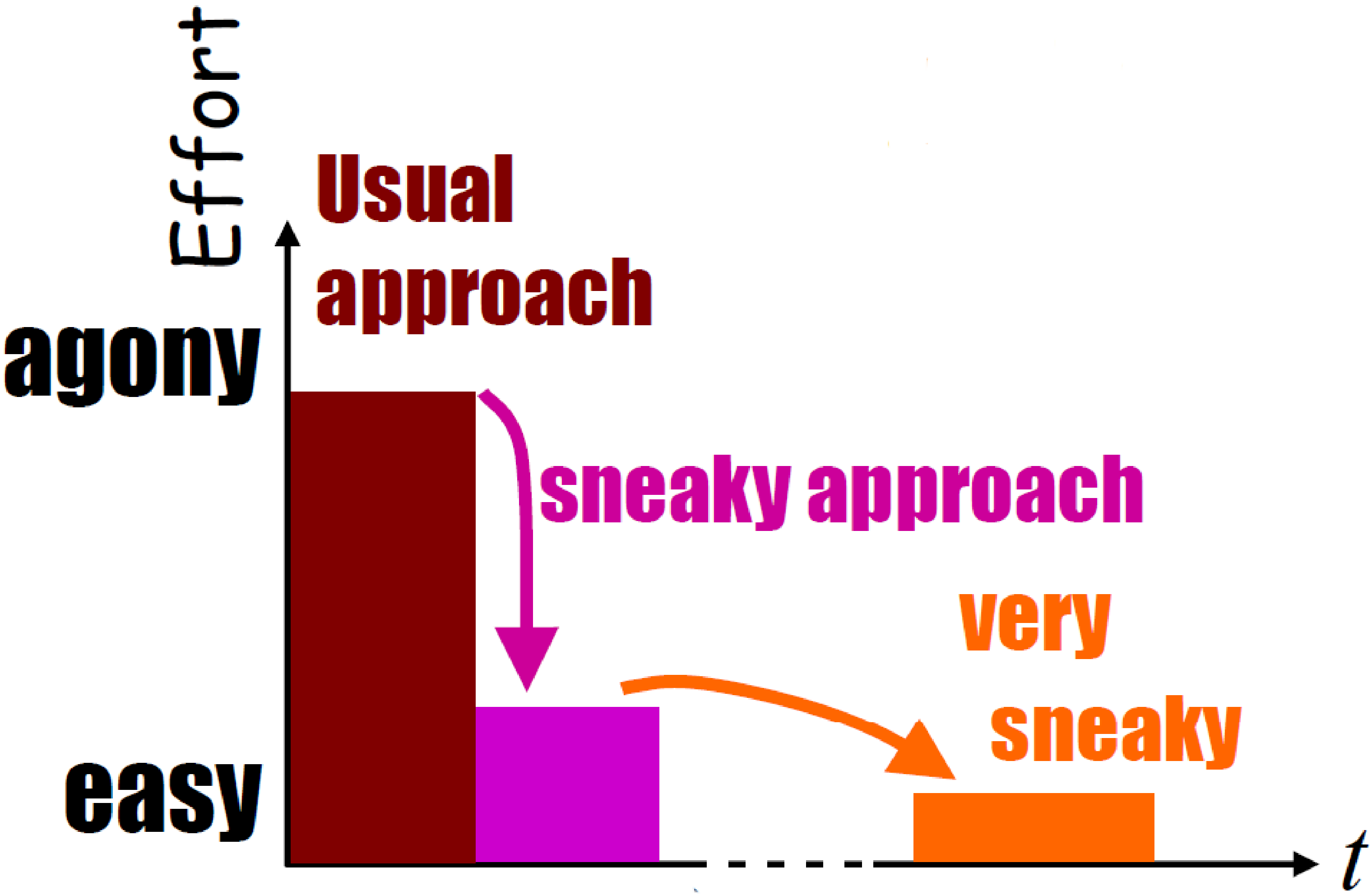
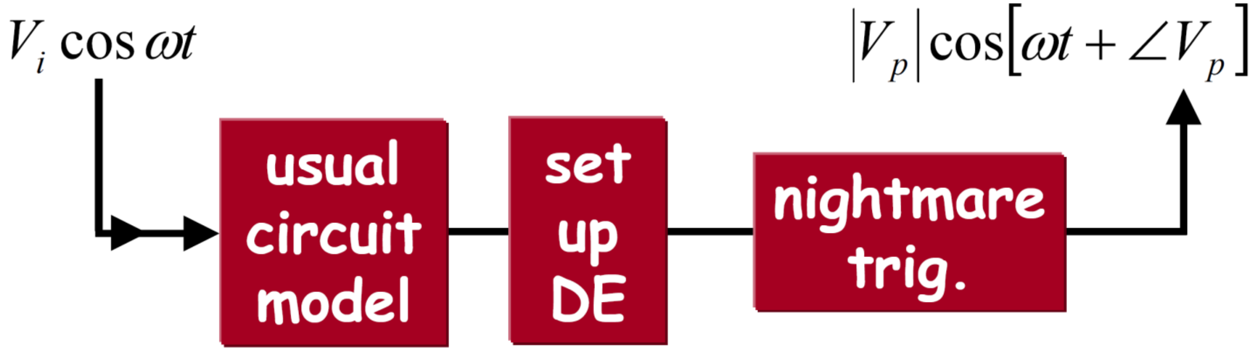


The Big Picture



The Big Picture

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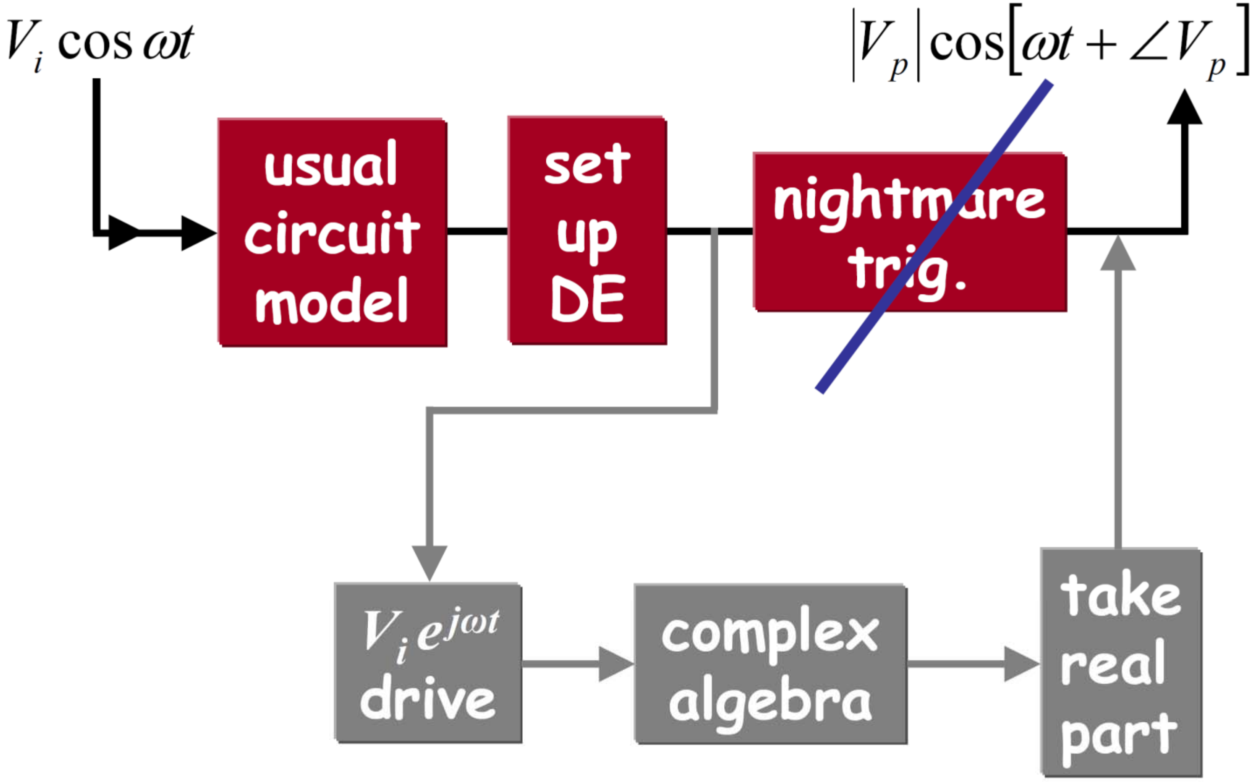

The Big Picture

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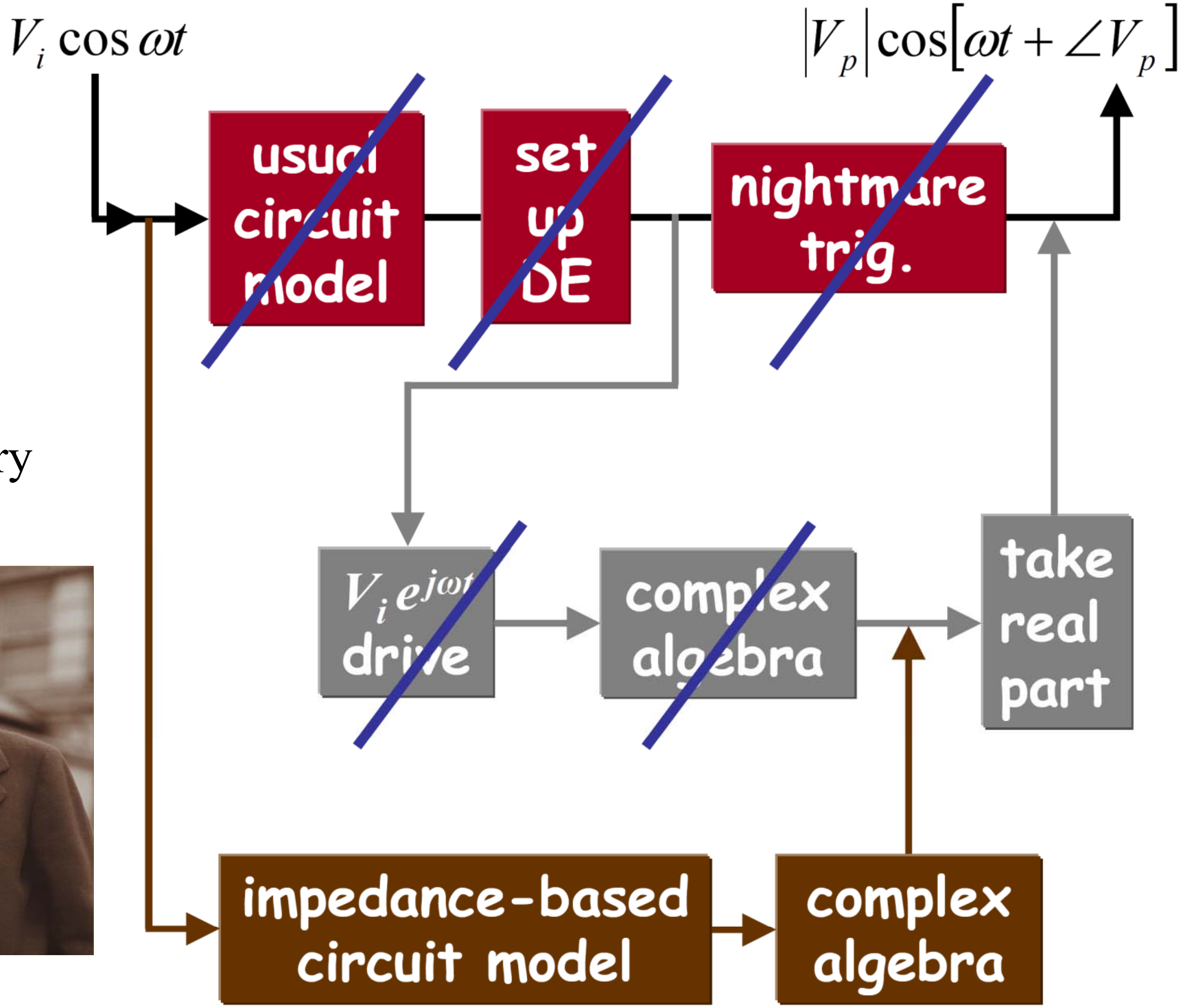
No Differential Equations

No Trigonometry



The Big Picture

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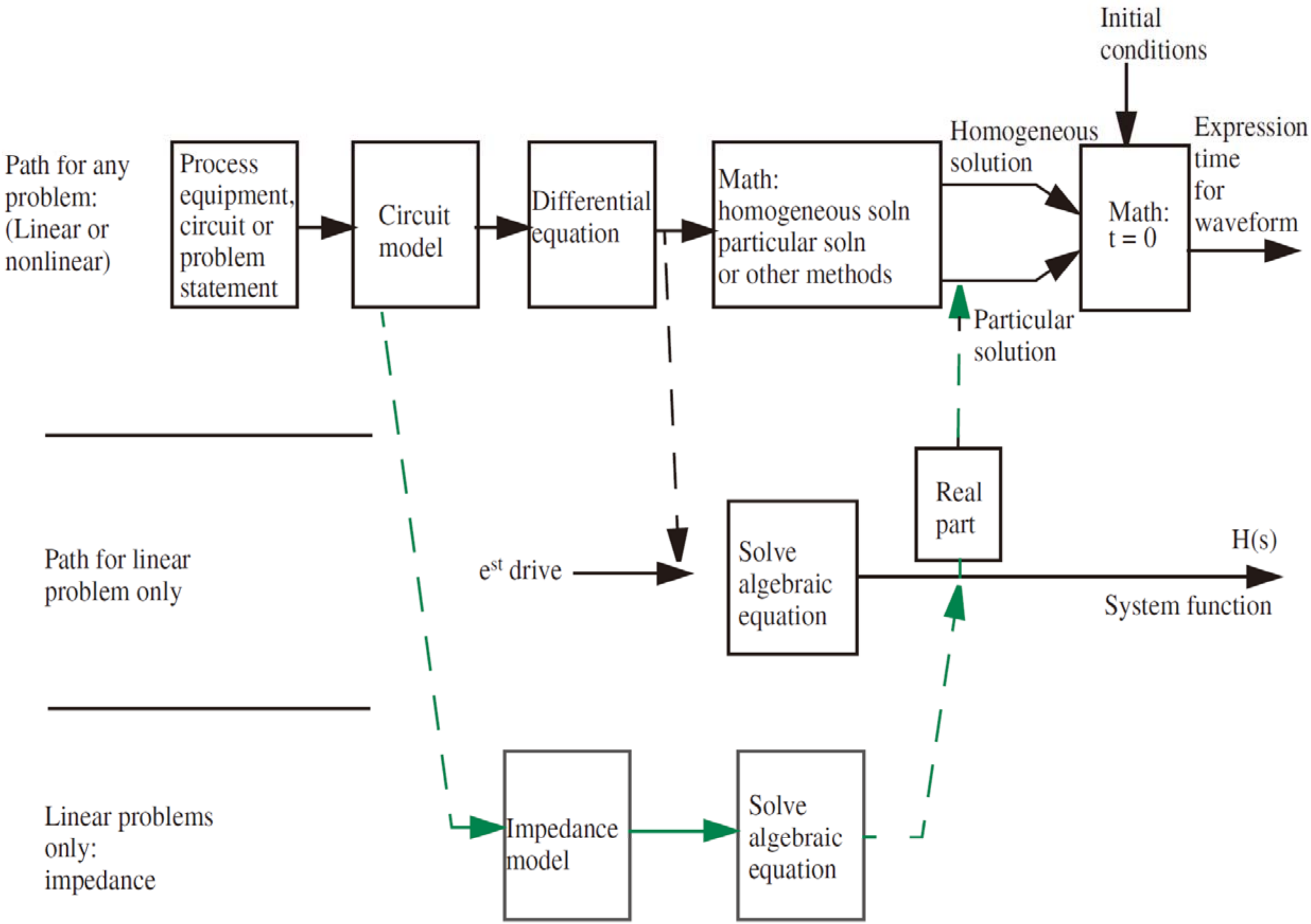

Summary

- Sinusoidal steady state is an important characterization of a linear system. It comprises a frequency response, which includes a gain plot and a phase plot as a function of frequency.
- By assuming complex exponential drives instead of sinusoidal drives for linear time-invariant circuits, the differential equations describing circuit behavior reduce to algebraic equations.
- The impedance method allows us to determine with ease the steady-state response of any linear RLC network for a sinusoidal input.
- The impedance method allows us to determine with ease the steady-state response of any linear RLC network for a sinusoidal input.
- The frequency response characterizes the behavior of a network as a function of frequency. A frequency response plot is a convenient way of summarizing how a network behaves as function of frequency. A frequency response plot has two graphics: the gain plot and the phase plot