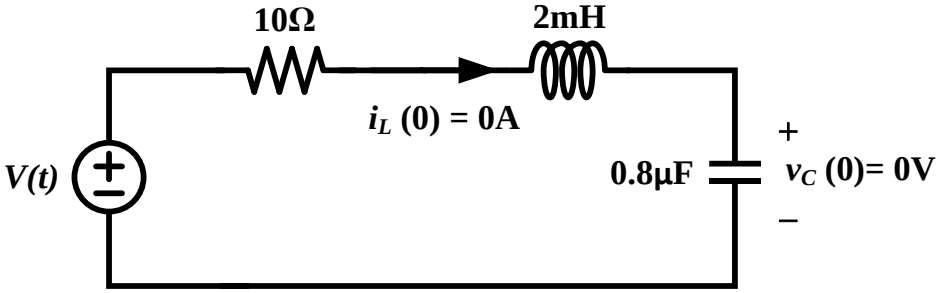
電路學(10510EE221002)第七次隨堂考

2016年11月23日
時間：15 分鐘
Close Book

學號：_____

姓名：_____

For the following circuit as shown, assume $V(t) = V_I u(t)$, V_I =5V, the initial state of the capacitor $v_C(0)$ is 0V and that of inductor $i_L(0)$ = 0 A, answer that following questions.



(1) Find the undamped natural frequence, ω_0. (15%)
(2) Find the damping factor, α. (15%)
(3) Find the approximate damped-natural frequence, ω_d. (15%)
(4) Find the approximate period of the ringing, T. (15%)
(5) Find the quality factor, Q. (15%)
(6) Sketch $v_C(t)$ for $t \geq 0$. (25%)

To analysis the response for the undriven RLC circuit, we write KVL equation for its loop.

KVL: $v_R + v_L + v_C = 0$

$\Rightarrow i_C R + L\frac{di_L}{dt} + v_C = 0$, where $i_L = i_C = C\frac{dv_C}{dt}$

$\Rightarrow RC\frac{dv_C}{dt} + LC\frac{d^2v_C}{dt^2} + v_C = 0$

$\Rightarrow \frac{d^2v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C = 0$

Thus, the characteristic equation can be written as

$s^2 + 2\alpha s + \omega_0^{\,2} = s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

Where

(1) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\text{mH} \times 0.8\mu\text{F}}} = 25000\text{rads/sec}$

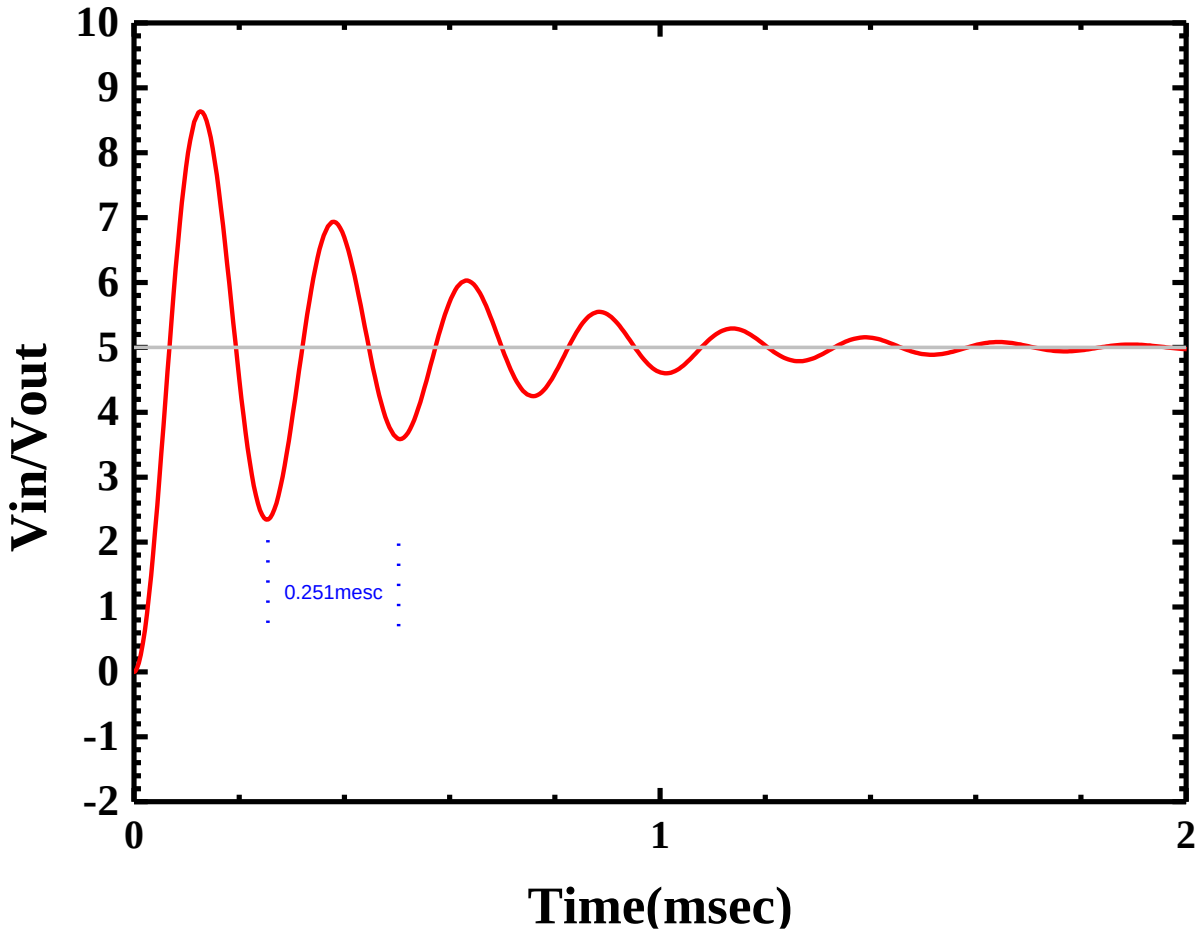
(2) $\alpha = \frac{R}{2L} = \frac{10}{2 \times 2\text{m}} = 2500\text{rads/sec}$

$\because \alpha < \omega_0 \Rightarrow$ under-damped dynamics

(3) ω_d is approximate equal to ω_0, or the accurate value is $\omega_d = \sqrt{\omega_0^{\,2} - \alpha^2} \approx 24874.69\text{rads/sec}$.

(4)Thus, the period is $T = \frac{2\pi}{\omega_0} = 0.251\text{msec}$, or the accurate value is $T = \frac{2\pi}{\omega_d} = 0.252\text{msec}$

(5)The quality factor is $Q = \frac{\omega_0}{2\alpha} = 5$.



ω_0 = 25000rad/sec, α = 2500 rad/sec, $\omega_d \approx$ 24874.69rad/sec,

$T \approx$ 0.251msec, Q = 5,