電路學(EE2210)第六次隨堂考

2016年11月09日
時間：10 分鐘
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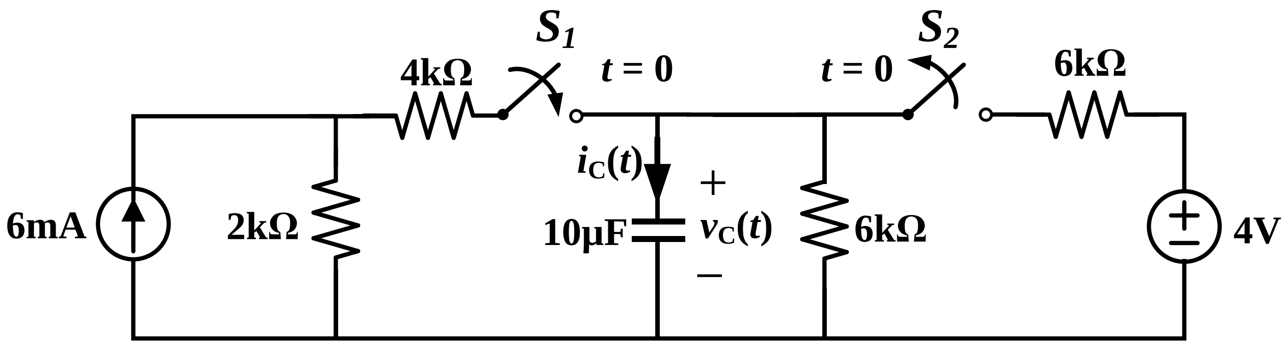
學號：________________________________

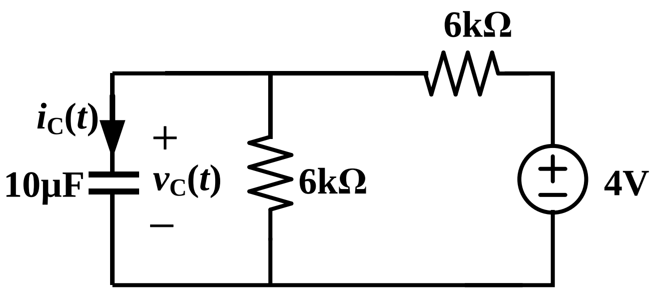
姓名：________________________________

For the circuit as shown in the following figure, the switch S_1 has been opened for a long time before it is closed at t = 0 and the switch S_2 has been closed for a long time before it is opened at t = 0. Find the $v_{C,\ initial}$, the $v_{C,\ final}$, the time constant (τ). Write down and sketch the $v_C(t)$ for $t > 0^+$.



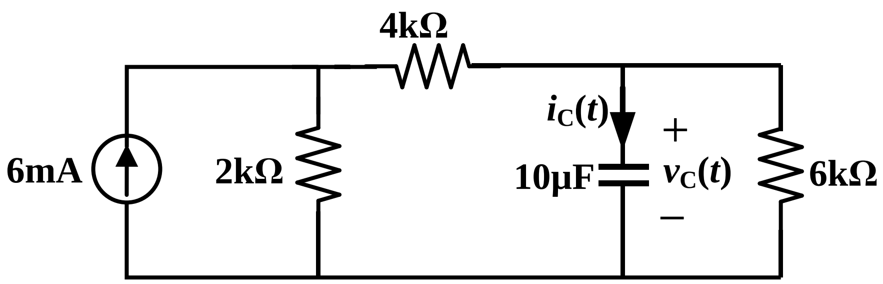
Solutions:

Because the switch S_2 has been closed for a long time before t = 0, the capacitor can be regarded as opened terminal, the equivalent circuit is

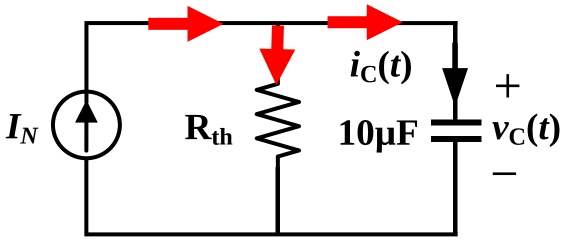


$$v_{C,initial} = v_C(0^+) == 4 \times \frac{6}{6+6} = 2\text{V}$$

After the switch S_1 is closed at t = 0,and switch S_2 is opened, the equivalent circuit is



The Norton equivalent circuit of this circuit network can be drawn as follows:



Find the Norton equivalent resistance I_N and R_{th}

$$i_N = 2mA$$
$$R_{th} = 3K\Omega$$

Apply node method:

$$i_N = i_{th} + i_C$$

$$\Rightarrow i_N = \frac{v_C}{R_{th}} + C\frac{dv_C}{dt}$$

Differential equation for v_C

$$\frac{dv_C}{dt} + \frac{v_C}{R_{th}C} = \frac{i_N}{C}$$

Find the particular solution:

$$\frac{dv_{CP}}{dt} + \frac{v_{CP}}{R_{th}C} = \frac{i_N}{C}$$

Assume $V_{CP} = K$ solution that satisfies the above equation

$$\Rightarrow \frac{dK}{dt} + \frac{K}{R_{th}C} = \frac{i_N}{C}$$

$$\Rightarrow 0 + \frac{K}{R_{th}C} = \frac{I_0}{C}$$

$$\Rightarrow K = i_N R_{th}$$

$$\Rightarrow v_{CP} = i_N R_{th} = 2mA \times 3K\Omega = 6V$$

$$v_{C,final} = v_{CP} = 2mA \times 3K\Omega = 6V$$

Find the homogeneous solution:

Assume solution is of this form $V_{CH} = Ae^{st}$

$$\frac{dv_{CH}}{dt} + \frac{v_{CH}}{R_{th}C} = \frac{i_N}{C}$$

$$\Rightarrow sAe^{st} + \frac{Ae^{st}}{R_{th}C} = 0$$

$$\Rightarrow s + \frac{1}{R_{th}C} = 0$$

$$\Rightarrow s = -\frac{1}{R_{th}C} = -\frac{1}{\tau}$$

$$\Rightarrow v_{CH} = Ae^{-\frac{t}{R_{th}C}}$$

Time constant $\tau = R_{th}C = 3K\Omega \times 10\mu F = 30m(\sec) = 0.03(\sec)$

The total solution is the sum of the particular and homogeneous solutions:

$$v_C = v_{CH} + v_{CP}$$

$$v_C = I_N R_{th} + Ae^{-\frac{t}{R_{th}C}}$$

Use the initial conditions:

$$v_C(0^+) = 2V$$

$$\Rightarrow 2 = I_N R_{th} + Ae^{-\frac{0}{(R_1+R_2)C}}$$

$$\Rightarrow A = 2 - I_N R_{th}$$

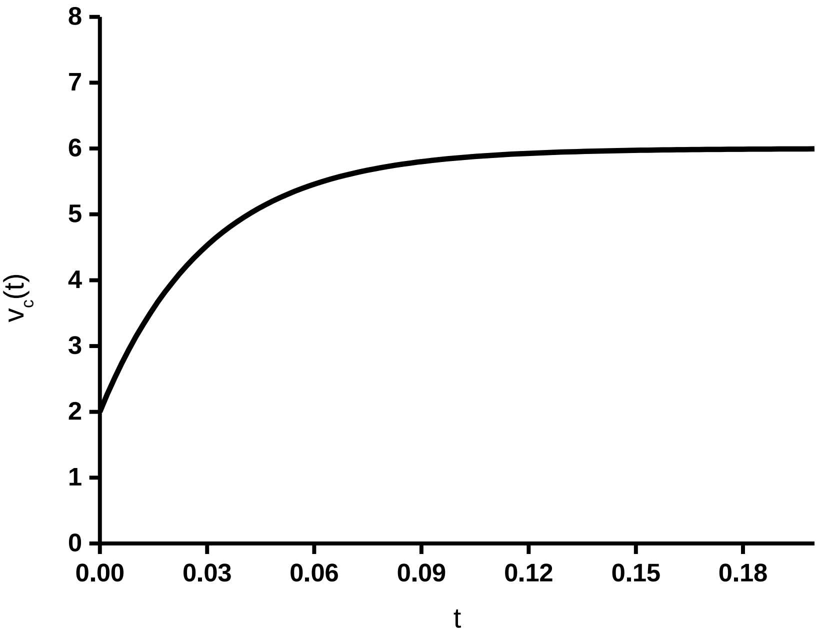
$$v_C = I_N R_{th} + (2 - I_N R_{th})e^{-\frac{t}{R_{th}C}}$$

$$v_C = v_{C,final} + (v_{C,initial} - v_{C,final})e^{-\frac{t}{\tau}}$$

$$\Rightarrow v_C(t) = 6 + (2-6)e^{-\frac{t}{30ms}} = 6 - 4e^{-\frac{t}{30ms}}$$

$v_{C,\,initial}$ = ____________ 2V ______________, (20%) $v_{C,\,final}$ = ____________ 6V ______________, (20%)

Time constant (τ) = ______ $30m(\sec)$ ______, (20%) $v_C(t)$, $t > 0^+$ = _________ $6 - 4e^{-\frac{t}{30ms}}$ ______, (20%)



(20%)