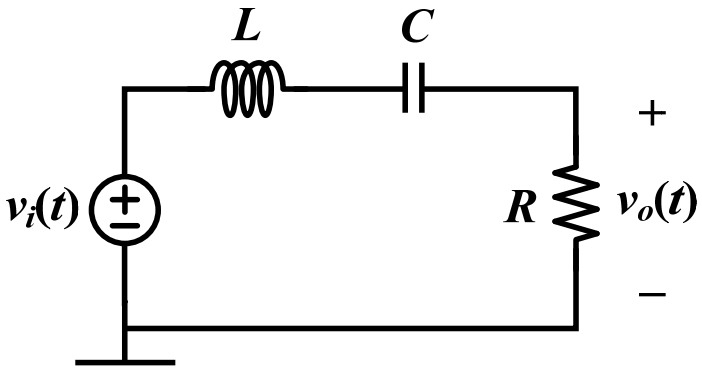
電路學(EE2210)第十次隨堂考

2015年6月3日
時間：20 分鐘
Close Book

學號：_____

姓名：_____

Find the sinusoidal steady state $v_o(t)$ for the following circuit by
(a) covert the $v_i(t)$ in time domain into the frequency domain $\mathbf{V_i}$,
(b) find the transfer function $\mathbf{H}(j\omega)$ in the frequency domain,
(c) find the numerical value of the $\mathbf{H}(j\omega)$ by plugging the frequency ω given in the figure,
(d) find the $\mathbf{V_o}$ in the frequency domain from (a) and (c),
(e) convert the $\mathbf{V_o}$ in the frequency domain into $v_o(t)$ in time domain.
(Assuming L = 2 mH, R = 2 kΩ, C = 0.2 μF, and $v_i(t)$ = 5cos(100t) + 1cos(50000t) V.)



Solution:
(a)

$$v_i(t) = \mathrm{Re}\left\{\mathbf{V_i}e^{j\omega t}\right\} = \mathrm{Re}\left\{5e^{j0}e^{j100t} + 1e^{j0}e^{j50000t}\right\}$$

$$\therefore \mathbf{V_i}\big|_{\omega=100} = 5\mathrm{V} \text{ and } \mathbf{V_i}\big|_{\omega=50000} = 1\mathrm{V}$$

(b)

$$\mathbf{H}(j\omega) = \frac{\mathbf{V_o}}{\mathbf{V_i}}(j\omega) = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega RC}\right)} = \frac{1}{1 + j\left(\frac{\omega}{1\times10^6} - \frac{2500}{\omega}\right)}$$

$$\therefore \left|\mathbf{H}(j\omega)\right| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{1\times10^6} - \frac{2500}{\omega}\right)^2}}, \text{ and } \angle\mathbf{H}(j\omega) = -\tan^{-1}\left(\frac{\omega}{1\times10^6} - \frac{2500}{\omega}\right)$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = 50000\text{rads/sec}$$

(c)
For $\omega = 100$,

$$\mathbf{H}(j100) = 0.039e^{j(87.7^\circ)} \cong 0.04e^{j(90^\circ)}$$

For $\omega = 50000$,

$$\mathbf{H}(j50000) = 1e^{j(0^\circ)}$$

(d)

$$\mathbf{V_o}(j\omega)\big|_{\omega=100} = \mathbf{V_i}(j100) \times \mathbf{H}(j100) = 5e^{j(0^\circ)} \times 0.04e^{j(90^\circ)} = 0.2e^{j(90^\circ)}$$

$$\mathbf{V_o}(j\omega)\big|_{\omega=50000} = \mathbf{V_i}(j50000) \times \mathbf{H}(j50000) = 1e^{j(0^\circ)} \times 1e^{j(0^\circ)} = 1e^{j(0^\circ)}$$

(e)

$$v_o(t) = v_o(t)\big|_{\omega=100} + v_o(t)\big|_{\omega=50000} = 0.2\cos(100t + 90^\circ) + 1\cos(50000t + 0^\circ)$$

(a) $\mathbf{V_i}$ (at ω = 100rads/sec) = _____,

$\mathbf{V_i}$ (at ω = 50000rads/sec) = _____,

(b) $\mathbf{H}(j\omega)$ = _____,

(c) $\mathbf{H}(j\omega) = |\mathbf{H}| \angle \phi$

where $|\mathbf{H}|$ (at ω = 100rads/sec) = _____,

$|\mathbf{H}|$ (at ω = 50000rads/sec) = _____,

and ϕ (at ω = 100rads/sec) = _____,

ϕ (at ω = 50000rads/sec) = _____,

(d) $\mathbf{V_o}$ (at ω = 100rads/sec) = _____,

$\mathbf{V_o}$ (at ω = 50000rads/sec) = _____,

(e) $v_o(t)$ = _____.