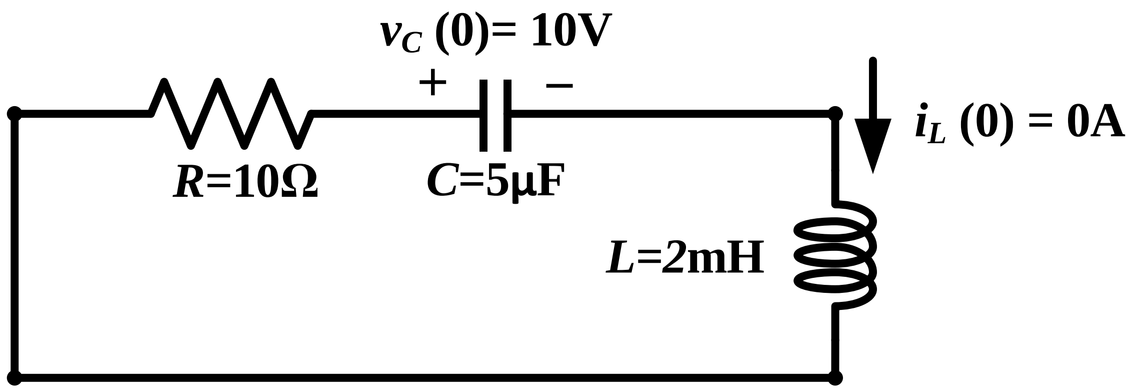
電路學(EE2210)第八次隨堂考

2015年11月25日
時間：20分鐘
Close Book

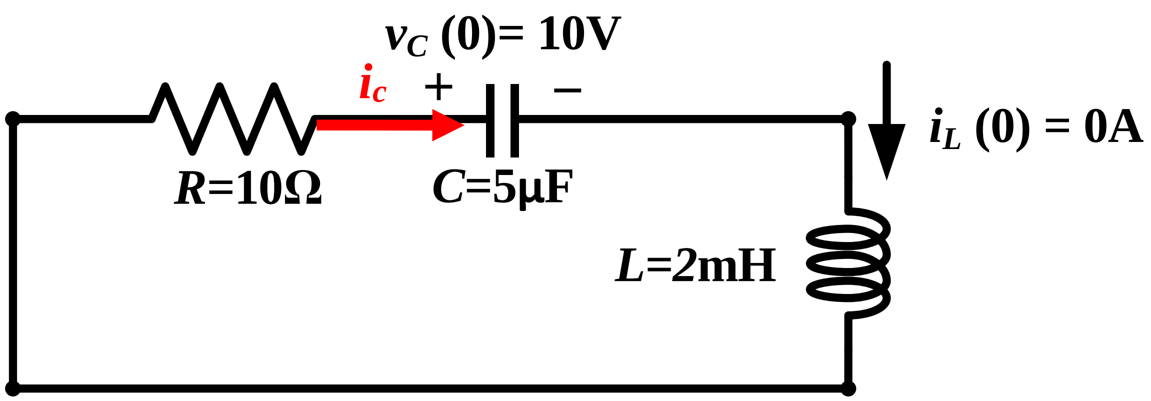
學號：______________________________

姓名：______________________________

For the circuit as shown in the following figure, assume the initial state of the capacitor $v_C(0)$ is 10V and that of inductor $i_L(0)$ = 0 A, answer that following questions.



(1) Find the differential equation with constant coefficients for i_L (10%)
(2) Find $i_L(0^+)$. (10%)
(3) Find $\frac{di_L(0^+)}{dt}$. (10%)
(4) Find the undamped natural frequence, ω_0. (10%)
(5) Find the damping factor, α. (10%)
(6) Find the approximate damped-natural frequence, ω_d. (10%)
(7) Find the approximate period of the ringing, T. (10%)
(8) Find the quality factor, Q. (10%)
(9) Sketch the zero-input response $i_L(t)$ for $t \geq 0$. (Label the key values in your sketch.) (20%)



To analysis the response for the undriven RLC circuit, we apply KVL to the loop first.

KVL: $v_R + v_L + v_C = 0$

$$v_R = i_L R \ , \ v_L = L\frac{di_L}{dt} \ , \ v_C = \frac{1}{C}\int i_C dt \Rightarrow i_C = C\frac{dv_C}{dt} \Rightarrow \frac{i_C}{C} = \frac{dv_C}{dt}$$

$$\Rightarrow i_L R + L\frac{di_L}{dt} + v_C = 0$$

By differentiating above equation with *t*, we have

$$\Rightarrow R\frac{di_L}{dt} + L\frac{d^2 i_L}{dt^2} + \frac{dv_C}{dt} = 0$$

$$\Rightarrow R\frac{di_L}{dt} + L\frac{d^2 i_L}{dt^2} + \frac{i_C}{C} = 0$$

where $i_L = i_C$ from KCL.

Let us rearrange terms to find the differential equation for i_L

$$\Rightarrow \frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{i_L}{LC} = 0$$

(1) Differential equation for i_L,

$$\boxed{\frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{i_L}{LC} = 0}$$

(2) $i_L(0^+)$.

$$\boxed{i_L(0^+) = i_L(0^-) = 0\text{A}}.$$

(3) $\frac{di_L(0^+)}{dt}$,

$$\boxed{\frac{di_L(0^+)}{dt} = \frac{-v_C(0^+)}{L} = \frac{v_L(0^+)}{L} = \frac{-10V}{2mH} = \frac{-10}{2\times 10^{-3}} = -5000\, rad/\text{sec}}$$

The characteristic equation can be found from the following steps.

$$\frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{i_L}{LC} = s^2 Ae^{st} + \frac{R}{L}sAe^{st} + \frac{1}{LC}Ae^{st} = 0$$

$$\Rightarrow s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 2\alpha s + \omega_0^{\ 2} = 0$$

(4) $\omega_0^{\ 2} = \frac{1}{LC} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

$$\boxed{\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\text{mH}\times 5\mu\text{F}}} = \frac{1}{\sqrt{2\times 10^{-3}\times 5\times 10^{-6}}} = 10^4\, rad/\text{sec}}$$

(5) $2\alpha = \frac{R}{L} \Rightarrow \alpha = \frac{R}{2L}$

$$\alpha = \frac{R}{2L} = \frac{10\Omega}{2\times 2\text{mH}} = \frac{10\Omega}{4\times 10^{-3}} = 2500\,{}^{rad}\!/_{\sec}$$

$\alpha < \omega_0 \Rightarrow$ under-damped

(6)

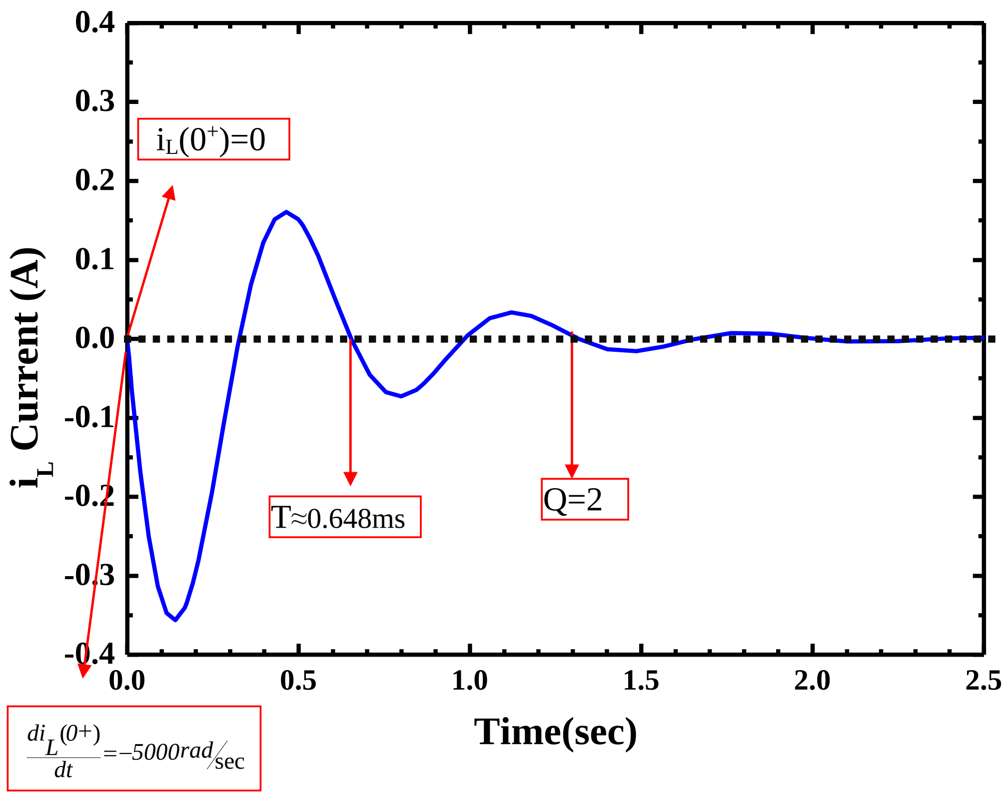
$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 9682.45\ \text{rad/sec}$. Note that ω_d is approximate equal to ω_0.

(7) The period $T \approx \frac{2\pi}{\omega_0} = 0.628\text{ms}$ approximately, or $T = \frac{2\pi}{\omega_d} = 0.648\text{ms}$ exactly.

(8)Finally, the quality factor Q is

$Q = \frac{\omega_0}{2\alpha} = \frac{10000}{2\times 2500} = 2$.

(9)



Differential equation for i_L : $\frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{i_L}{LC} = 0$,

$i_L(0^+) =$ 0 A, $\frac{di_L(0^+)}{dt} =$ -5000 rad/sec, $\omega_0 =$ 10^4 rad/sec,

$\alpha =$ 2500 rad/sec, $\omega_d \approx$ 9682.45 rad/sec, $T \approx$ 0.648ms, $Q =$ 2