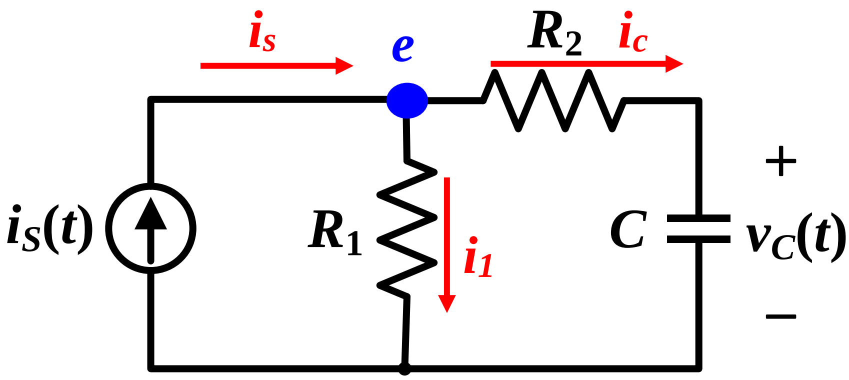
電路學(EE2210)第七次隨堂考

2015年11月11日
時間：10 分鐘
Close Book

學號：________________________________

姓名：________________________________

For the circuit as shown, assume $i_S(t) = I_0u(t)$. Find (1) the linear first-order ordinary differential equation with constant coefficients for v_C, (2) $v_C(0^+)$, (3) the particular solution v_{CP}, (4) the homogeneous solution v_{CH}, (5) the time constant (τ) and (6) the total solution $v_C(t)$.



(1)

$v_C(0^-) = 0V$

Apply node method at node e:

$$i_s = i_1 + i_C$$

$$\Rightarrow i_s = \frac{V_{R_2} + V_C}{R_1} + C\frac{dV_C}{dt}$$

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$$\Rightarrow i_s = \frac{R_2 \times C\frac{dV_C}{dt} + V_c}{R_1} + C\frac{dV_C}{dt}$$

$$\Rightarrow i_s = (\frac{R_2}{R_1} + 1)C\frac{dV_C}{dt} + \frac{V_C}{R_1}$$

Let us rearrange terms to find the differential equation for v_C

$$\frac{dv_C}{dt}+\frac{v_C}{(\frac{R_2}{R_1}+1)R_1C}=\frac{i_s}{(\frac{R_2}{R_1}+1)C}$$

Replacing i_s with $i_s = I_0 u(t)$, we have the differential equation for v_C as:

$$\boxed{\frac{dv_C}{dt}+\frac{v_C}{(\frac{R_2}{R_1}+1)R_1C}=\frac{I_0}{(\frac{R_2}{R_1}+1)C}} \quad \text{for} \quad t>0$$

(2) $\boxed{v_C(0^+) = v_C(0^-) = 0V}$

(3) The particular solution:

To find the particular solution, let us assume $v_{CP} = K$ that satisfies the following equation:

$$\frac{dv_{CP}}{dt}+\frac{v_{CP}}{(\frac{R_2}{R_1}+1)R_1C}=\frac{I_0}{(\frac{R_2}{R_1}+1)C}$$

$$\Rightarrow \frac{dK}{dt}+\frac{K}{(\frac{R_2}{R_1}+1)R_1C}=\frac{I_0}{(\frac{R_2}{R_1}+1)C}$$

$$\Rightarrow 0+\frac{K}{(\frac{R_2}{R_1}+1)R_1C}=\frac{I_0}{(\frac{R_2}{R_1}+1)C}$$

$$\Rightarrow K = I_0 R_1$$

$$\boxed{v_{CP} = I_0 R_1}$$

(4) Find the homogeneous solution:

$$\frac{dv_{CH}}{dt}+\frac{v_{CH}}{(\frac{R_2}{R_1}+1)R_1C}=0$$

To find the homogeneous solution, by plugging $v_{CH} = Ae^{st}$ into the above equation, we have.

$$sAe^{st}+\frac{Ae^{st}}{(R_1+R_2)C}=0$$

$$\Rightarrow s = -\frac{1}{(R_1+R_2)C} = -\frac{1}{\tau}$$

$$v_{CH} = Ae^{-\frac{t}{(R_1+R_2)C}}$$

(5) Time constant $\tau = (R_1 + R_2)C$

(6) The total solution is simply the sum of the particular solution and the homogeneous solution:

$$v_C = V_{CP} + v_{CH}$$

$$v_C = I_0 R_1 + Ae^{-\frac{t}{(R_1+R_2)C}}$$

The constant A can be found from the initial conditions.

$$v_C(0^+) = 0V$$

$$\Rightarrow 0 = I_0 R_1 + Ae^{-\frac{0}{(R_1+R_2)C}}$$

$$\Rightarrow A = -I_0 R_1$$

$$v_C = I_0 R_1 - I_0 R_1 e^{-\frac{t}{(R_1+R_2)C}}$$

$$v_C(t) = I_0 R_1\left(1 - e^{-\frac{t}{(R_1+R_2)C}}\right)$$

Differential equation for v_C :____ (20%)

$v_C(0^+)$ = ________________, (16%) v_{CP} = ____________________, (16%) v_{CH} =___________________(16%)

Time constant (τ) = _________________, (16%) $v_C(t)$ = _______________________________________. (16%)