電路學(EE2210)第十二次隨堂考

2015年12月30日
時間：15 分鐘
Close Book

學號：________________________________

姓名：________________________________

For the active filter circuit along with its impedance model (where s is a shorthand notation for $j\omega$) as shown in the following figures,

(a) Write the node equations for $\mathbf{V_1}$ and $\mathbf{V_2}$ for the impedance model.

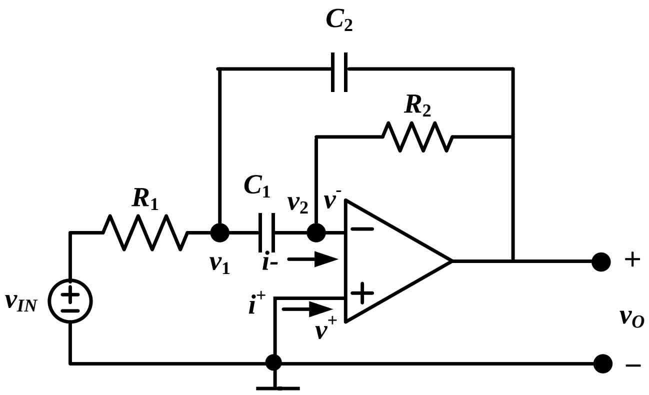
(b) Simplify these equations from (a) by using the ideal Op Amp assumptions in the impedance model, that is, $\mathbf{V}^{-} \approx \mathbf{V}^{+}$ and $\mathbf{I}^{-} = \mathbf{I}^{+} \approx 0$.

(c) Find the transfer function $\mathbf{H}(j\omega) = \mathbf{V_O}(j\omega)/\mathbf{V_{IN}}(j\omega)$ for this circuit from (b).

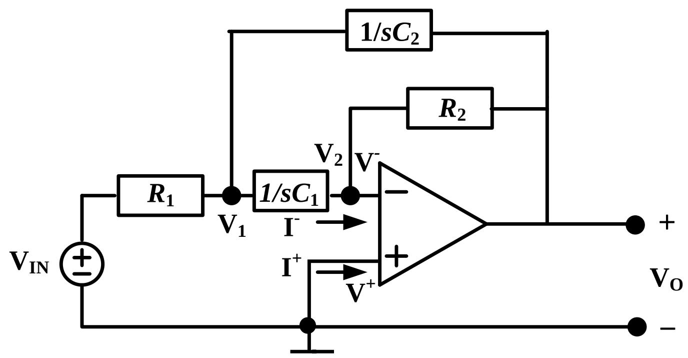
(d) Indicate the frequency at which the peak occurs, the magnitude of the transfer function at the peak, and the Q of the resonance. Use the following numerical values:
$C_1 = C_2 = 0.01\ \mu\text{F}$, $R1 = 10\ \Omega$, $R2 = 1\ \text{k}\Omega$

(e) Is this RC active filter a low-pass, high-pass, or bandpass filter?

(1) Circuit



(2) Impedance model



Solution:

(a)

Node $\mathbf{V_1}$:

$$\frac{\mathbf{V}_1 - \mathbf{V}_{IN}}{R_1} + \frac{\mathbf{V}_1 - \mathbf{V}_O}{1/sC_2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1/sC_1} = 0$$

$$\boxed{\rightarrow \mathbf{V}_1\left(\frac{1}{R_1} + sC_2 + sC_1\right) = \frac{\mathbf{V}_{IN}}{R_1} + sC_2\mathbf{V}_O + sC_1\mathbf{V}_2}$$

Node $\mathbf{V_2}$:

$$\boxed{\frac{\mathbf{V}_2 - \mathbf{V}_1}{1/sC_1} + \mathbf{i}^- + \frac{\mathbf{V}_2 - \mathbf{V}_O}{R_2} = 0}$$

(b)

The ideal Op Amp assumptions, $\mathbf{V_2} = \mathbf{V^-} = \mathbf{V^+} \approx 0$ and $\mathbf{I^-} = \mathbf{I^+} \approx 0$

Node $\mathbf{V_1}$:

$$\boxed{\mathbf{V}_1\left(\frac{1}{R_1} + sC_2 + sC_1\right) = \frac{\mathbf{V}_{IN}}{R_1} + sC_2\mathbf{V}_O}$$

Node $\mathbf{V_2}$:

$$\boxed{\mathbf{V}_1 = -\frac{\mathbf{V}_O}{sR_2C_1}}$$

(c)

$$-\frac{\mathbf{V}_O}{sR_2C_1} \times \left(\frac{1}{R_1} + sC_2 + sC_1\right) = \frac{\mathbf{V}_{IN}}{R_1} + sC_2\mathbf{V}_O$$

$$\rightarrow \frac{\mathbf{V}_O}{\mathbf{V}_{IN}} = -\frac{sR_2C_1}{1 + s(R_1C_1 + R_1C_2) + s^2R_1R_2C_1C_2}$$

$$\boxed{\rightarrow \frac{\mathbf{V}_O}{\mathbf{V}_{IN}} = -\frac{s/R_1C_2}{s^2 + s\left(\frac{C_1 + C_2}{R_2C_1C_2}\right) + \frac{1}{R_1R_2C_1C_2}}}$$

(d)

Find quality factor $Q = \frac{\omega_0}{2\alpha}$

$$\Delta\omega = 2\alpha = \frac{C_1 + C_2}{R_1C_1C_2}$$

$$\alpha = \frac{C_1 + C_2}{2R_2C_1C_2} = \frac{2 \times 0.01 \times 10^{-6}}{2 \times 1000 \times (0.01 \times 10^{-6})^2} = 10^5 \text{ rad/sec}$$

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}} = \sqrt{\frac{1}{10 \times 1000 \times (0.01 \times 10^{-6})^2}} = 10^6 \text{ rad/sec}$$

$$Q = \frac{\omega_0}{2\alpha} = \frac{10^6}{2 \times 10^5} = 5$$

(e)

bandpass filter

(a) Node $\mathbf{V_1}$ = ____,

Node $\mathbf{V_2}$ = ____,

(b) Node $\mathbf{V_1}$ = ____,

Node $\mathbf{V_2}$ = ____,

(c) $\mathbf{H}(j\omega)$ = ____,

(d) ω_0 = ____________________, Q = ____________________

(e) ____________________.