電路學(EE2210)第十一次隨堂考

2015年12月23日
時間：15 分鐘
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學號：________________________________

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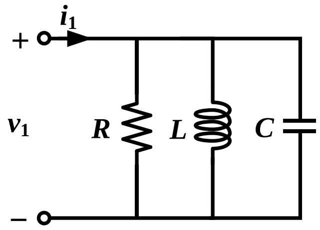
For each of the circuits shown in the figure, select the magnitude of the frequency response for the system function (that is, impedance, admittance, or transfer function) from those given. It is not necessary to relate the critical frequencies to the circuit parameters, and you may choose a magnitude response more than once. Please note that the magnitude responses are sketched on a log-log scale, with slopes labeled.

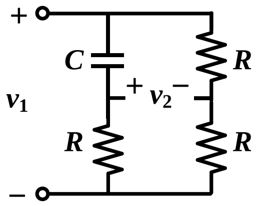
(a)

R = 10**Ω**, L = 0.1 mH, C = 1μF



$$\mathbf{Z}(j\omega)=\frac{\mathbf{V}_1(j\omega)}{\mathbf{I}_1(j\omega)}$$

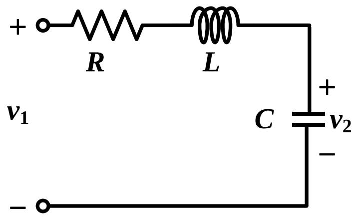
(b)



$$\mathbf{H}(j\omega)=\frac{\mathbf{V}_2(j\omega)}{\mathbf{V}_1(j\omega)}$$

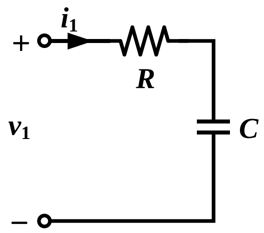
(c)

R = 5**Ω**, L = 10 mH, C = 1μF

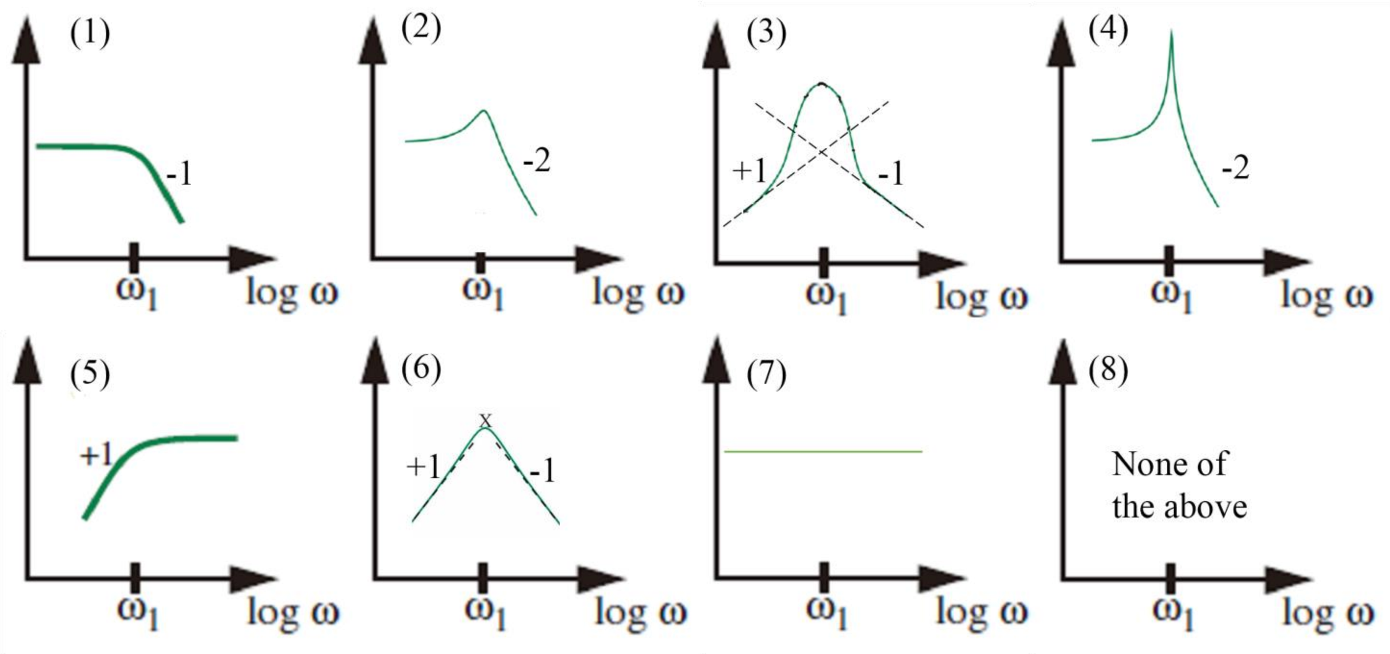


$$\mathbf{H}(j\omega)=\frac{\mathbf{V}_2(j\omega)}{\mathbf{V}_1(j\omega)}$$

(d)



$$\mathbf{Z}(j\omega)=\frac{\mathbf{V}_1(j\omega)}{\mathbf{I}_1(j\omega)}$$



Solution:

(a)

$$\mathbf{Z}(j\omega)=\frac{\mathbf{V}_1(j\omega)}{\mathbf{I}_1(j\omega)}=\frac{1}{\frac{1}{R}+\frac{1}{j\omega L}+j\omega C}=\frac{R}{1+j\left(\omega RC-\frac{R}{\omega L}\right)}$$

If $\omega\to 0\Rightarrow |\mathrm{Z}|\cong \omega L\propto\omega^1$. The slope of the plot is +1.

If $\omega\to\infty\Rightarrow |\mathrm{Z}|\cong \frac{1}{\omega C}\propto\omega^{-1}$. The slope of the plot is −1.

We need to find quality factor $Q=\frac{\omega_0}{\Delta\omega_0}$ to determine the peaking in the plot at resonance frequency.

Let's find ω_0 first, $\omega_0 CR-\frac{R}{\omega_0 L}=0$

$$\omega=\omega_0=\sqrt{\frac{1}{LC}}$$

Then, let's find find the bandwidth $\Delta\omega_0$.

$$\left|\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|=\frac{\mathbf{V}_1}{\mathbf{I}_1}=\frac{R}{\sqrt{2}}=\left|\frac{R}{1+j\left(\omega RC-\frac{R}{\omega L}\right)}\right|=\left|\frac{R}{1\pm j}\right|$$

so, $\omega RC-\frac{R}{\omega L}=\pm 1\to\omega^2\mp\frac{1}{RC}\omega-\frac{1}{LC}=0$

The roots of both equations are $\omega_1=\frac{1}{2RC}+\frac{1}{2}\sqrt{\frac{1}{R^2C^2}+\frac{4}{LC}}$, $\omega_2=-\frac{1}{2RC}+\frac{1}{2}\sqrt{\frac{1}{R^2C^2}+\frac{4}{LC}}$

$$\Delta\omega_0=\omega_1-\omega_2=\frac{1}{RC}=2\alpha$$

$$\therefore Q=\frac{\omega_0}{\Delta\omega_0}=\frac{\frac{1}{\sqrt{LC}}}{\frac{1}{RC}}=\frac{\frac{1}{\sqrt{0.1\times10^{-3}\times1\times10^{-6}}}}{\frac{1}{10\times1\times10^{-6}}}=1$$

Figure 6 meet the characteristics of the plot described above.

(b)

$$\mathbf{H}(j\omega)=\frac{\mathbf{V}_2(j\omega)}{\mathbf{V}_1(j\omega)}=\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}-\frac{R}{R+R}=\frac{1}{1+j\omega RC}-\frac{1}{2}=\frac{\frac{1}{2}(1-j\omega RC)}{1+j\omega RC}$$

$$|\mathrm{H}|=\frac{1}{2}\frac{\sqrt{1^2+(\omega RC)^2}}{\sqrt{1^2+(\omega RC)^2}}=\frac{1}{2}$$

This is an all pass filter and Figure 7 meet the requirements.

(c)

$$\mathbf{H}(j\omega)=\frac{\mathbf{V}_2(j\omega)}{\mathbf{V}_1(j\omega)}=\frac{\frac{1}{j\omega C}}{R+j\omega L+\frac{1}{j\omega C}}=\frac{\frac{1}{j\omega C}}{R+j\left(\omega L-\frac{1}{\omega C}\right)}=\frac{\frac{1}{j\omega RC}}{1+j\left(\omega\frac{L}{R}-\frac{1}{\omega RC}\right)}$$

If $\omega\to 0\Rightarrow|\mathbf{H}|\cong 1$

If $\omega\to\infty\Rightarrow|\mathrm{H}|\cong\frac{1}{\omega^2 LC}\propto\omega^{-2}$. The slope of the plot is –2.

We need to find quality factor $Q=\frac{\omega_0}{2\alpha}$ to determine the peaking in the plot at resonance frequency.

Let's find ω_0 first, $\omega_0\frac{L}{R}-\frac{1}{\omega_0 RC}=0$

$$\omega=\omega_0=\sqrt{\frac{1}{LC}}$$

Then, let's find find the damping factor α.

From above characteristic equation, $2\alpha=\frac{R}{L}$

$$\alpha=\frac{R}{2L}$$

$$\therefore Q=\frac{\omega_0}{2\alpha}=\frac{\frac{1}{\sqrt{LC}}}{\frac{R}{L}}=\frac{\frac{1}{\sqrt{10\times10^{-3}\times1\times10^{-6}}}}{\frac{5}{10\times10^{-3}}}=20\cdot$$

Figure 4 meet the characteristics of the plot described above.

(d)

$$\mathrm{Z}(j\omega)=\frac{\mathrm{V}_1(j\omega)}{\mathrm{I}_1(j\omega)}=R+\frac{1}{j\omega C}$$

If $\omega\to 0\Rightarrow|\mathrm{Z}|\cong\frac{1}{\omega C}\to\infty$

If $\omega\to\infty\Rightarrow|\mathrm{Z}|\cong R$

All the plots Figure 1-8 do not fit the above conditions and the answer is Figure 8

(a) → ________________, (b) → ________________, (c) → ________________, (d) → ________________.