電路學(EE2210)第十次隨堂考

2015年12月16日
時間：20 分鐘
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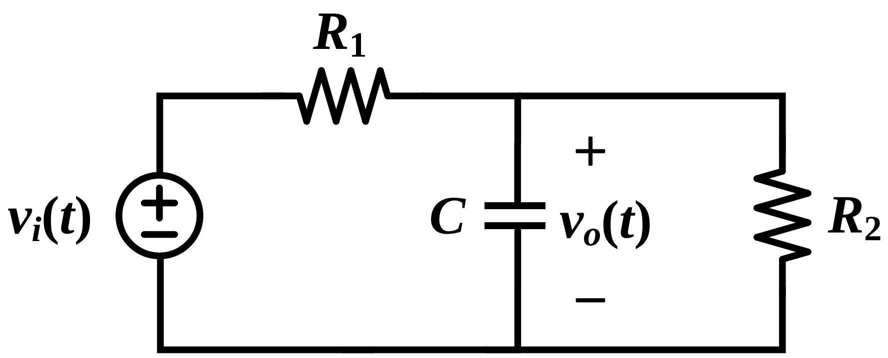
Find the sinusoidal steady state $v_o(t)$ for the following circuit by
(a) covert the $v_i(t)$ in time domain into the phasor expression $\mathbf{V_i}(j\omega)$ in the frequency domain,
(b) find the transfer function $\mathbf{H}(j\omega)$ in the frequency domain,
(c) find the numerical value of the $\mathbf{H}(j\omega)$ for $\omega = 5$ rad/s and $\omega = 50000$ rad/s,
(d) find the $\mathbf{V_o}(j\omega)$ in the frequency domain from (a) and (c),
(e) convert the $\mathbf{V_o}(j\omega)$ in the frequency domain into $v_o(t)$ in time domain.
(Assuming $R_1 = 6$ kΩ, $R_2 = 3$ kΩ, $C = 1$ uF, and $v_i(t) = 6\cos(5t) + 12\cos(50000t)$ V.)



Solution:

(a)

$$v_i(t) = \mathrm{Re}\left\{\mathbf{V_i} e^{j\omega t}\right\} = \mathrm{Re}\left\{6e^{j0}e^{j5t} + 12e^{j0}e^{j50000t}\right\}$$

$$\therefore \boxed{\mathbf{V_i}\big|_{\omega=5} = 6\mathrm{V}} \text{ and } \boxed{\mathbf{V_i}\big|_{\omega=50000} = 12\mathrm{V}}$$

(b)

$$\mathbf{H}(j\omega) = \frac{\mathbf{V_o}}{\mathbf{V_i}}(j\omega) = \frac{R_2 // \frac{1}{j\omega C}}{R_1 + R_2 // \frac{1}{j\omega C}} = \frac{\frac{\frac{1}{j\omega C}R_2}{\frac{1}{j\omega C}+R_2}}{R_1 + \frac{\frac{1}{j\omega C}R_2}{\frac{1}{j\omega C}+R_2}} = \frac{\frac{\frac{1}{j\omega C}R_2}{\frac{1}{j\omega C}+R_2}}{\frac{\left(\frac{1}{j\omega C}+R_2\right)R_1 + \frac{1}{j\omega C}R_2}{\frac{1}{j\omega C}+R_2}}$$

$$= \frac{\frac{R_2}{j\omega C}}{\left(\frac{1}{j\omega C}+R_2\right)R_1 + \frac{1}{j\omega C}R_2} = \frac{\frac{R_2}{j\omega C}}{R_1R_2 + \frac{R_1+R_2}{j\omega C}} = \frac{R_2}{(R_1+R_2) + j\omega C R_1 R_2} = \frac{1}{\left(\frac{R_1+R_2}{R_2}\right) + j\omega C R_1}$$

$$= \frac{\frac{R_2}{R_1+R_2}}{1 + \frac{j\omega C R_1}{\left(\frac{R_1+R_2}{R_2}\right)}} = \frac{\frac{R_2}{R_1+R_2}}{1 + \frac{j\omega C R_1}{\left(\frac{R_1+R_2}{R_2}\right)}} = \frac{\frac{R_2}{R_1+R_2}}{1 + j\omega C \frac{R_1R_2}{R_1+R_2}}$$

$$= \frac{\frac{3000}{6000+3000}}{1 + j\left(\omega \frac{1}{1000000}\frac{3000\times 6000}{3000+60000}\right)} = \frac{1/3}{1 + j\left(\frac{\omega}{500}\right)}$$

(c)

$$\mathbf{H}(j\omega) = |\mathbf{H}|\angle\phi$$

$$\therefore |\mathbf{H}(j\omega)| = \frac{1/3}{\sqrt{1^2 + \left(\frac{\omega}{500}\right)^2}} \quad \text{and} \quad \phi = \angle\mathbf{H}(j\omega) = -\tan^{-1}\left(\frac{\omega}{500}\right)$$

$$\boxed{|\mathbf{H}(j\omega)|_{\omega=5} = \frac{1/3}{\sqrt{1^2 + \left(\frac{\omega}{500}\right)^2}} = \frac{1/3}{\sqrt{1^2 + \left(\frac{5}{500}\right)^2}} = 0.33 \cong 1/3}$$

$$\left|\mathbf{H}(j\omega)\right|_{\omega=50000} = \frac{1/3}{\sqrt{1^2+\left(\frac{\omega}{500}\right)^2}} = \frac{1/3}{\sqrt{1^2+\left(\frac{50000}{500}\right)^2}} = 3.33\times10^{-3}$$

$$\angle\mathbf{H}(j\omega)_{\omega=5} = -\tan^{-1}\left(\frac{5}{500}\right) = -0.57^\circ \cong 0^\circ$$

$$\angle\mathbf{H}(j\omega)_{\omega=50000} = -\tan^{-1}\left(\frac{50000}{500}\right) = -89.43^\circ \cong -90^\circ$$

(d)

$$\mathbf{V_o}(j\omega)\Big| = \mathbf{V_i}(j\omega)\times\mathbf{H}(j\omega)$$

$$\mathbf{V_o}(j\omega)\Big|_{\omega=5} = \mathbf{V_i}(j5)\times\mathbf{H}(j5) = 6e^{j(0^\circ)}\times \tfrac{1}{3}e^{j(-0.57^\circ)} = 2e^{j(-0.57^\circ)} \cong 2e^{j(0^\circ)}\text{V}$$

$$\mathbf{V_o}(j\omega)\Big|_{\omega=50000} = \mathbf{V_i}(j50000)\times\mathbf{H}(j50000) = 12e^{j(0^\circ)}\times 3.33\times10^{-3}e^{j(-89.43^\circ)} = 0.03996\,e^{j(-89.43^\circ)} \cong 0.04e^{j(-90^\circ)}\text{V}$$

(e)

$$\begin{aligned} v_o(t) &= v_o(t)\Big|_{\omega=100} + v_o(t)\Big|_{\omega=50000} \\ &= 2\cos(5t-0.57^\circ) + 0.04\cos(50000t-89.43^\circ) \\ &\cong 2\cos(5t) + 0.04\cos(50000t-90^\circ)\text{V} \end{aligned}$$

(a) $\mathbf{V_i}(j\omega)$ (at ω = 5 rad/s) = ________________________________,

$\mathbf{V_i}(j\omega)$ (at ω = 50000 rad/s) = ________________________________,

(b) $\mathbf{H}(j\omega)$ = ________________________________,

(c) $\mathbf{H}(j\omega) = |\mathbf{H}|\angle\phi$

where $|\mathbf{H}|$ (at ω = 5 rad/s) = ________________________________,

$|\mathbf{H}|$ (at ω = 50000 rad/s) = ________________________________,

and ϕ (at ω = 5 rad/s) = ________________________________,

ϕ (at ω = 50000 rad/s) = ________________________________,

(d) $\mathbf{V_o}(j\omega)$ (at ω = 5 rad/s) = ________________________________,

$\mathbf{V_o}(j\omega)$ (at ω = 50000 rad/s) = ________________________________,

(e) $v_o(t)$ = ________________________________.