電路學(EE2210)第九次隨堂考

2013年12月4日
時間：15 分鐘
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學號：________________________

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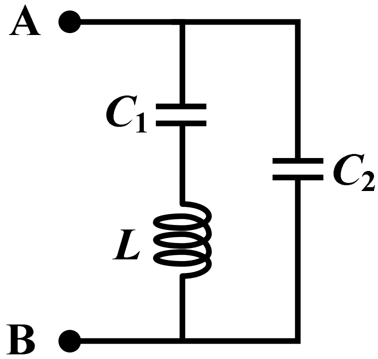
For the following circuit as shown, answer that following questions.



(1) Find the impedance between terminal A and B at frequency ω. (30%)
(2) Find the frequency ω_0 at which the impedance between terminal A and B is zero. (10%)
(3) Find the frequency range at which the impedance between terminal A and B behave like an inductor. (30%)
(4) Find the frequency range at which the impedance between terminal A and B behave like an capacitor. (30%)

Solution:

(a)

$$Z(j\omega)=\frac{\left(\frac{1}{j\omega C_1}+j\omega L\right)\frac{1}{j\omega C_2}}{\left(\frac{1}{j\omega C_1}+j\omega L\right)+\frac{1}{j\omega C_2}}=\frac{1-\omega^2 C_1 L}{j\left[\omega(C_1+C_2)-\omega^3 C_1 C_2 L\right]}=-j\frac{1-\omega^2 C_1 L}{\left[\omega(C_1+C_2)-\omega^3 C_1 C_2 L\right]}$$

$$=j\frac{\omega^2 LC_1-1}{\omega\left[(C_1+C_2)-\omega^2 C_1 C_2 L\right]}=j\frac{\omega LC_1-\frac{1}{\omega}}{\left[(C_1+C_2)-\omega^2 C_1 C_2 L\right]}$$

(b)

If $Z(j\omega_0)=j\frac{\omega_0 LC_1-\frac{1}{\omega_0}}{\left[(C_1+C_2)-\omega_0^{\,2} C_1 C_2 L\right]}=0$

$$\Rightarrow \omega_0 LC_1-\frac{1}{\omega_0}=0$$

$$\Rightarrow \omega_0=\sqrt{\frac{1}{LC_1}}$$

(c)

If the impedance $Z(j\omega)$ is like a inductor $j\omega L'$,

Case (1):

The conditions of $\omega LC_1 - \frac{1}{\omega} > 0$ and $\left[(C_1 + C_2) - \omega^2 C_1 C_2 L\right] > 0$ are must satisfied simultaneously,

$\Rightarrow \omega > \sqrt{\frac{1}{LC_1}}$ and $0 < \omega < \sqrt{\frac{1}{L\frac{C_1C_2}{C_1 + C_2}}}$

Because $\frac{C_1C_2}{C_1 + C_2} < C_1$, thus $\sqrt{\frac{1}{L\frac{C_1C_2}{C_1 + C_2}}} > \sqrt{\frac{1}{LC_1}}$,

Thus in this case, the completed region is $\sqrt{\frac{1}{LC_1}} < \omega < \sqrt{\frac{1}{L\frac{C_1C_2}{C_1 + C_2}}}$

Case (2):

The conditions of $\omega LC_1 - \frac{1}{\omega} < 0$ and $\left[(C_1 + C_2) - \omega^2 C_1 C_2 L\right] < 0$ are must satisfied simultaneously,

$\Rightarrow 0 < \omega < \sqrt{\frac{1}{LC_1}}$ and $\omega > \sqrt{\frac{1}{L\frac{C_1C_2}{C_1 + C_2}}}$

Thus in this case, the completed region is none.

By case (1) and (2), the total region is $\boxed{\sqrt{\frac{1}{LC_1}} < \omega < \sqrt{\frac{1}{L\frac{C_1C_2}{C_1 + C_2}}}}$.

(d)

If the impedance $Z(j\omega)$ is like a capacitor $-j\frac{1}{\omega C'}$,

Case (3):

The conditions of $\omega LC_1 - \frac{1}{\omega} > 0$ and $\left[(C_1 + C_2) - \omega^2 C_1 C_2 L\right] < 0$ are must satisfied simultaneously,

$\Rightarrow \omega > \sqrt{\frac{1}{LC_1}}$ and $\omega > \sqrt{\frac{1}{L\frac{C_1C_2}{C_1 + C_2}}}$

In this case, the completed region is $\omega > \sqrt{\frac{1}{L\frac{C_1C_2}{C_1 + C_2}}}$.

Case (4):

The conditions of $\omega LC_1 - \frac{1}{\omega} < 0$ and $\left[(C_1 + C_2) - \omega^2 C_1 C_2 L\right] > 0$ are must satisfied simultaneously,

$\Rightarrow 0 < \omega < \sqrt{\frac{1}{LC_1}}$ and $0 < \omega < \sqrt{\frac{1}{L\frac{C_1C_2}{C_1 + C_2}}}$

In this case, the completed region is $0<\omega<\sqrt{\dfrac{1}{LC_1}}$.

By case (3) and (4),, the total regions are $\boxed{0<\omega<\sqrt{\dfrac{1}{LC_1}} \text{ and } \omega>\sqrt{\dfrac{1}{L\dfrac{C_1C_2}{C_1+C_2}}}}$.

(1) $\mathbf{Z_{AB}}$ = ____, (2) ω_0 = ____,

(3) ____, (4) ____.