電路學(EE2210)第八次隨堂考

2013年11月27日
時間：15 分鐘
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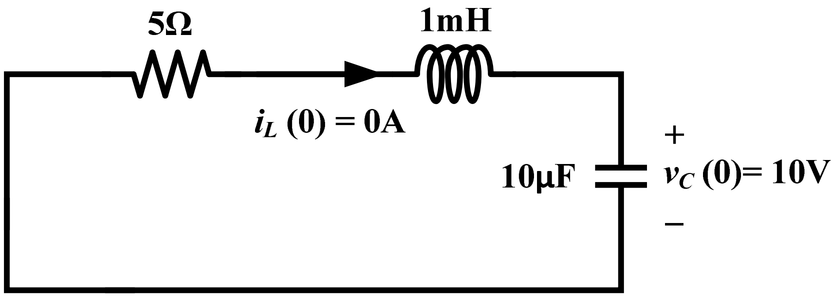
學號：________________________________

姓名：________________________________

For the following circuit as shown, assume the initial state of the capacitor $v_C(0)$ is 10V and that of inductor $i_L(0)$ = 0 A, answer that following questions.



(1) Find the undamped natural frequence, ω_0. (10%)
(2) Find the damping factor, α. (10%)
(3) Find the approximate damped-natural frequence, ω_d. (10%)
(4) Find the approximate period of the ringing, T. (10%)
(5) Find the quality factor, Q. (10%)
(6) Find $v_C(0^+)$. (10%)
(7) Find $\frac{dv_C(0^+)}{dt}$. (10%)
(8) Sketch the zero-input response $v_C(t)$ for $t \geq 0$. (30%)

Solution:
(1), (2), (3), (4) and (5)
To analysis the response for the undriven RLC circuit, we write KVL equation for its loop.
KVL: $v_R + v_L + v_C = 0$

$$\Rightarrow i_C R + L\frac{di_L}{dt} + v_C = 0 \text{, where } i_L = i_C = C\frac{dv_C}{dt}$$

$$\Rightarrow RC\frac{dv_C}{dt} + LC\frac{d^2v_C}{dt^2} + v_C = 0$$

$$\Rightarrow \frac{d^2v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C = 0$$

Thus, the characteristic equation can be written as

$$s^2 + 2\alpha s + \omega_0{}^2 = s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

where

$$\alpha = \frac{R}{2L} = \frac{5}{2\times 1\text{m}} = 2500\text{rads/sec}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\text{mH}\times 10\mu\text{F}}} = 10000\text{rads/sec}$$

$$\because \alpha < \omega_0 \Rightarrow \text{ under-damped dynamics}$$

ω_d is approximate equal to ω_0, or the accurate value is $\omega_d = \sqrt{\omega_0^{\ 2} - \alpha^2} \approx 9682.5\text{rads/sec}$.

Thus, the period is $T \approx \dfrac{2\pi}{\omega_0} = 0.628\text{msec}$, or the accurate value is $T = \dfrac{2\pi}{\omega_d} = 0.65\text{msec}$

Finally, the quality factor is

$$Q = \frac{\omega_0}{2\alpha} = 2.$$

(6)

For a voltage across the capacitor,

$v_C(0^+) = v_C(0^-) = 10\text{V}$.

(7)

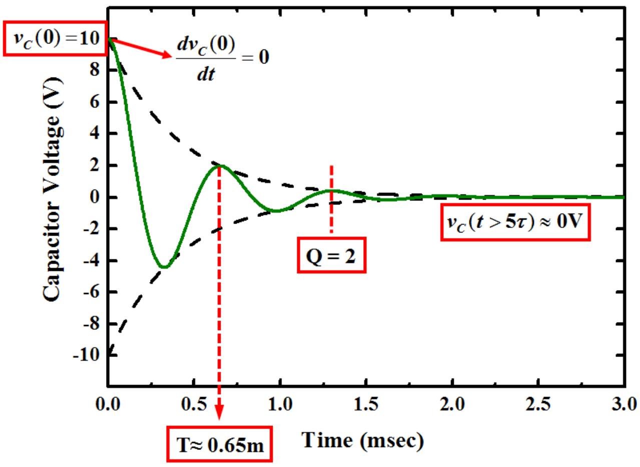
The initial trajectory of the curve need to be found before we sketch the response of $v_C(t)$.

$$\frac{dv_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{i_L(0^+)}{C} = 0\text{V/sec}$$

(8)

From the condition in (7), we know that the initial value is maximum or minimum in the oscillation.

For this undriven RLC circuit, we see that the capacitor voltage is nearly maximum when the inductor current is zero, and the capacitor voltage will be discharged after t = 0.



ω_0 = 10000rads/sec, α = 2500rads/sec, $\omega_d \approx$ ω_0 or 9682.5rads/sec, $T \approx$ 0.628 or 0.65 msec,

Q = 2, $v_C(0^+)$ = 10V, $\dfrac{dv_C(0^+)}{dt}$ = 0V/sec,