Quiz 3 solutions

1.

At time t > 0, the circuit becomes an LC oscillator.

The natural frequency ω_o is equal to $\sqrt{\frac{1}{LC}}$. Since the capacitor starts out charged, initially, the voltage across the capacitor is a cosine function with maximum amplitude of 2 V. The current through the inductor is the same as the current through the capacitor, and it is characterized by the capacitor I-V relation : $i_c = C\frac{dv_C}{dt}$. Taking the derivative, we get a negative sinusoidal relation.

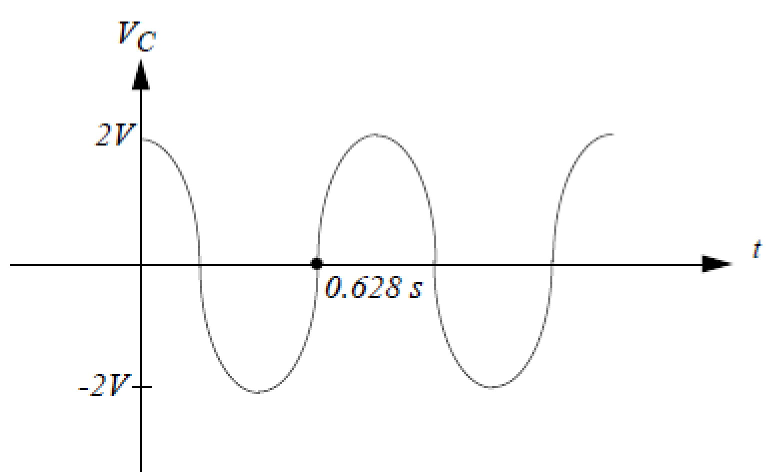
$$v_C(t) = v_C(0)\cos(\omega_o t)$$

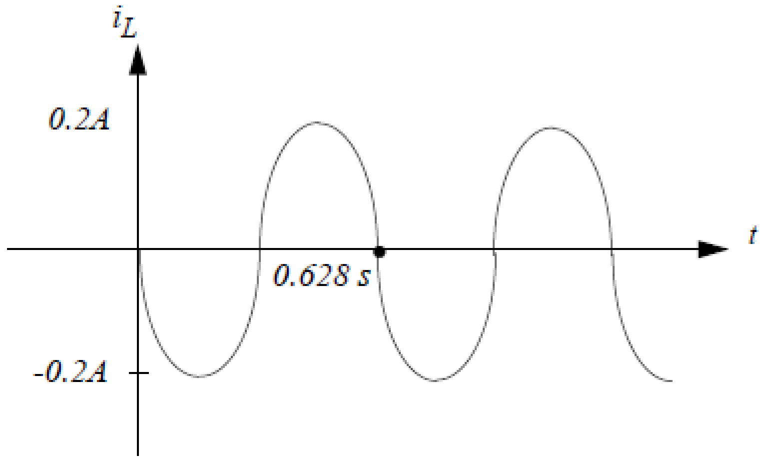
$$i_L(t) = -\sqrt{\frac{C}{L}}v_C(0)sin(\omega_o t)$$

ANS :

$$v_C = 2\cos\left(\sqrt{\frac{1}{LC}}t\right) = 2\cos(10\text{t})$$

$$i_L(t) = -0.2\sin(10\text{t})$$





2.

$$\frac{v_2}{v_i} = \frac{L_s\,||R_2}{R_1 + L_s\,||R_2} = |H(j\omega)|e^{j\emptyset}$$

where

$$|H(j\omega)| = \frac{\omega R_2 L}{\sqrt{\omega^2 L^2 (R_1 + R_2)^2 + R_1{}^2 R_2{}^2}}$$

And

$$\emptyset = \frac{\pi}{2} - tan^{-1}[\frac{\omega L(R_1 + R_2)}{R_1 R_2}]$$

For ω = 4, $\emptyset = \frac{\pi}{4}$, and $|H(j\omega)| = \frac{1}{3\sqrt{2}}$,

$$v_2(t) = \frac{1}{\sqrt{2}}\cos(4t + \frac{\pi}{4})$$

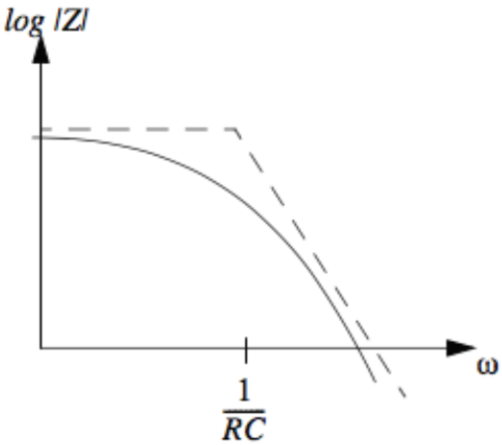
ANS :

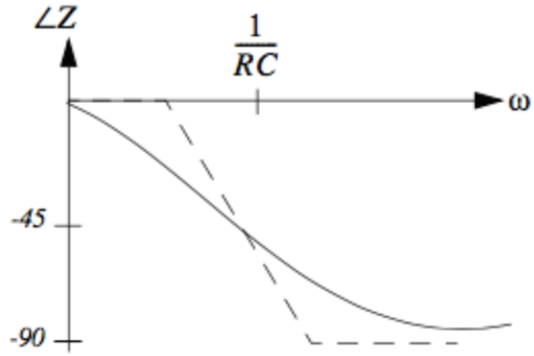
$$v_2(t) = \frac{1}{\sqrt{2}}\cos(4t + \frac{\pi}{4})$$

3.

ANS:

$$Z = \frac{R}{1 + jwRC}$$





4.

$i_L + i_R + i_C = 0$

$\frac{1}{sL} + \frac{1}{R} + sC = 0$

$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0\ (s^2 + 2\alpha s + {\omega_0}^2 = 0)$

$\alpha = \frac{1}{2RC} = \frac{1}{2 * \frac{25}{3} * 0.001} = 60$

$\omega_0 = \frac{1}{\sqrt{LC}} = 100$

$\omega_d = \sqrt{{\omega_0}^2 - \alpha^2} = 80$

$\alpha < \omega_0\ , underdamped$

Assume v(t) $= A_1 e^{s_1 t} + A_2 e^{s_2 t}$

For underdamped ,v(t) can be expressed as

v(t) $= e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$

$$= e^{-60t}(B_1 \cos 80t + B_2 \sin 80t)$$

The initial condition v (t = 0) = 10 V ,

$B_1 = 10$

$$i_L = \frac{1}{L}\int v(t)\, dt$$

$$= \frac{1}{L}\left\{\left(-\frac{1}{500}e^{-60t}\right)[(3B_1 + 4B_2)\cos 80t + (-4B_1 + 3B_2)\sin 80t] + C\right\}$$

The initial condition i_L (t = 0) = -0.6 A ,

$$\frac{1}{0.1}\left(-\frac{1}{500}\right)(3B_1 + 4B_2 + C) = -0.6$$

Because

$$\frac{1}{0.1}\left(-\frac{1}{500}\right)3B_1 = \frac{1}{0.1}\left(-\frac{1}{500}\right)30 = -0.6$$

$4B_2 + C = 0$

Assume $C = 0$, $then\ B_2 = 0$

From above equation ,

v(t) $= 10e^{-60t}\cos 80t\ \ (V)$

the number of cycles v(t) takes to settle = Q ,

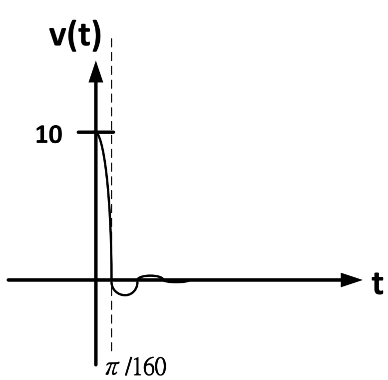
Q $= \frac{\omega_0}{2\alpha} = \frac{100}{2 * 60} = \frac{5}{6}$

the period of v(t) $= \frac{2\pi}{\omega_d} = \frac{2\pi}{80}$

Ans:

a. v(t) $= 10e^{-60t}\cos 80t\ \ (V)$

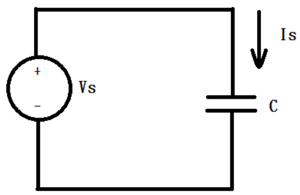
b.

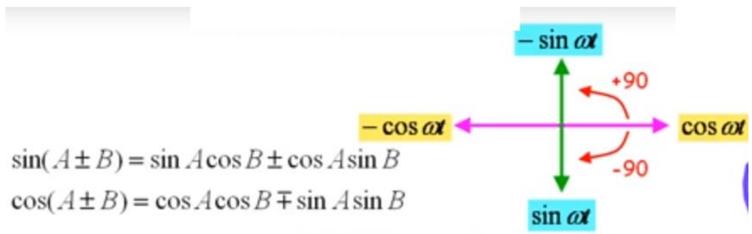


c. the period of v(t) $= \frac{2\pi}{80}$

d. the number of cycles v(t) takes to settle = $\frac{5}{6}$

5.





$$\text{Vs(t)} = 25\ mV * \cos(\omega t)$$

$$\text{Is(t)} = C * \frac{dVs}{dt} = C * (25\ mV)(-\sin(\omega t)) * \omega = C * (25\ mV)(\cos(\omega t + 90^{o})) * \omega$$

Steady state current: $C * (25\ mV) * \omega = 628.32\ \mu A$

$$\rightarrow C = \frac{628.32\ \mu A}{(25\ mV) * \omega} = \frac{628.32\ \mu A}{(25\ mV) * (2\pi * 80\ kHz)} = 5 * 10^{-8} F = 50\ nF$$

Impedance:

$$Z_c = \frac{1}{j\omega C} = -j * \frac{1}{(2\pi * 80\ kHz)(5 * 10^{-8} F)} = -j39.79\ \Omega$$

Ans:

a. f=80 kHz

b. +90°

c. C=$5 * 10^{8} F = 50\ nF$

d. $Z_c = -j39.79\ \Omega$