EE2210 Electric Circuits , Spring 2017

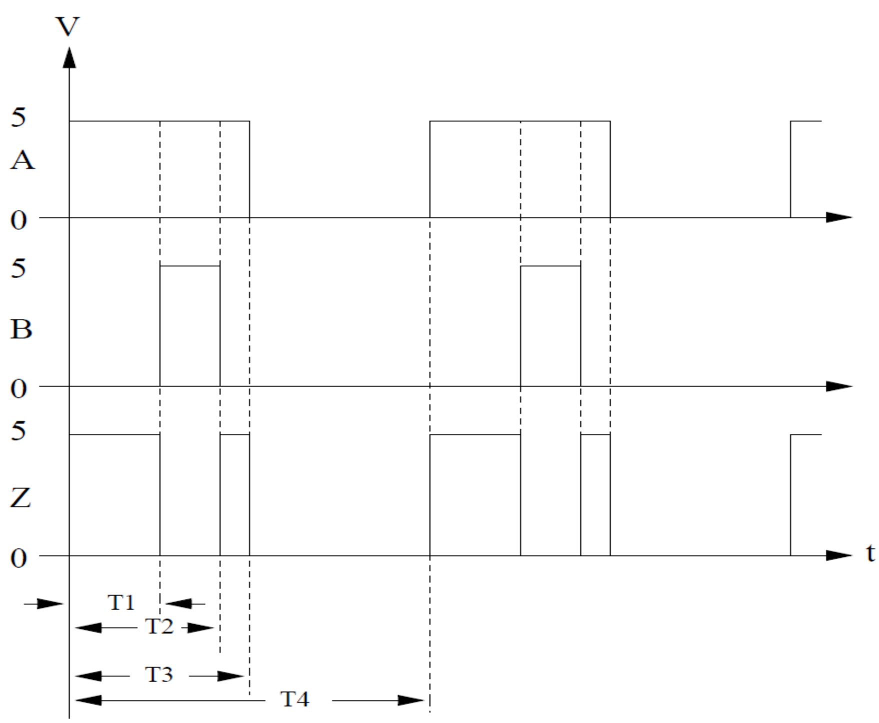
Practice Problems Solutions (Lec8 – Lec10)

1.

a) The waveform for the output Z for $0 \le t \le T_4$ is given .

The truth table:

A	B	Z
0	0	0
0	1	0
1	0	1
1	1	1



b) Assuming $R_{ON} = 0$, then: For $0 \le t \le T_1$ only the first MOSFET is on, i.e.

$$P_{\text{static}} = \frac{V_S^2}{R_L}$$

For $T_1 \le t \le T_2$ the first and the third MOSFET's are on, i.e.

$$P_{\text{static}} = 2\frac{V_S^2}{R_L}$$

For $T_2 \le t \le T_3$ again only the first MOSFET is on, i.e.

$$P_{\text{static}} = \frac{V_S^2}{R_L}$$

For $T_3 \le t \le T_4$ only the second MOSFET is on, i.e.

$$P_{\text{static}} = \frac{V_S^2}{R_L}$$

Therefore, the time-average static power consumed by the circuit is given by

$$\begin{aligned}
P_{\text{static,ave}} &= \frac{T_1}{T_4}\left(\frac{V_S^2}{R_L + R_{on}}\right) + \frac{T_2 - T_1}{T_4}\left(\frac{2V_S^2}{R_L}\right) + \frac{T_3 - T_2}{T_4}\left(\frac{V_S^2}{R_L}\right) \\
&\quad + \frac{T_4 - T_3}{T_4}\left(\frac{V_S^2}{R_L}\right) \\
&= \frac{V_S^2}{R_L}\left(\frac{-T_1 + T_2 + T_4}{T_4}\right)
\end{aligned}$$

c) For $0 \le t \le T_1$ the dynamic dissipation occurs while C_G discharges, and C_L charges, i.e.

$$P_{\text{dynamic}} = \frac{C_G V_S^2 + C_G V_S^2}{2T_1}$$

For $T_1 \le t \le T_2$ the dynamic dissipation occurs while C_L discharges, i.e.

$$P_{\text{dynamic}} = \frac{C_L V_S^2}{2(T_2 - T_1)}$$

For $T_2 \le t \le T_3$ the dynamic dissipation occurs while C_L charges, i.e.

$$P_{\text{dynamic}} = \frac{C_L V_S^2}{2(T_3 - T_2)}$$

For $T_3 \le t \le T_4$ the dynamic dissipation occurs while C_L, C_G charges, and C_L discharges, i.e.

$$P_{\text{dynamic}} = \frac{C_G V_S^2 + C_G V_S^2}{2(T_4 - T_3)}$$

Thus, the time-average dynamic power consumed by the circuit is given by

$$P_{\text{dynamic,ave}} = \frac{T_1}{T_4}\left(\frac{C_G V_S^2 + C_G V_S^2}{2T_1}\right) + \frac{T_2 - T_1}{T_4}\left(\frac{C_L V_S^2}{2(T_2 - T_1)}\right)$$

$$+\frac{T_3 - T_2}{T_4}\left(\frac{C_L V_S^2}{2(T_3 - T_2)}\right) + \frac{T_4 - T_3}{T_4}\left(\frac{C_G V_S^2 + C_G V_S^2}{2(T_4 - T_3)}\right)$$

$$= \frac{V_S^2}{T_4}(C_G + 2C_L)$$

d) Static:

$$P_{\text{static,ave}} = \frac{V_S^2}{R_L + R_{on}}\left(\frac{-T_1 + T_2 + T_4}{T_4}\right)$$
$$= \frac{5^2}{10 \times 10^3 + 0}\left(\frac{-100 + 200 + 600}{600}\right)$$
$$= 2.9\, mW$$

Dynamic:

$$P_{\text{dynamic,ave}} = \frac{1}{T_4}\left(C_G V_S^2 + 2C_L V_S^2\right)$$
$$= \frac{1}{600 \times 10^{-9}}\left(100 \times 10^{-15} \cdot 5^2 + 2 \cdot 1 \times 10^{-12} \cdot 5^2\right)$$
$$= 87.5\, \mu W$$

e)

$$E = Pt$$
$$= (P_{static} + P_{dynamic})(60sec)$$
$$= 0.180J$$

f) Since the power depends linearly on V_S^2, 30% drop in V_S translates to a 51% drop in the total time-average power consumption.

2.

a) $\frac{d^2 v_C}{dt^2} + \frac{1}{RC}\frac{dv_C}{dt} + \frac{1}{LC} = 0$

$2 \cdot \alpha = \frac{1}{RC}$

${\omega_o}^2 = \frac{1}{LC}$

$\alpha < \omega_o \quad \rightarrow$ UNDERDAMPED

b) $v_C = K\ e^{-\alpha t} \cdot \cos(\omega_d \cdot t + \phi)$

$\omega_d = \sqrt{{\omega_o}^2 - \alpha^2}$

$\phi = \tan^{-1}(\frac{\alpha}{\omega_d})$

$\omega_o = 10 \times 10^6$

$\alpha = 3.33 \times 10^6$

c) (1) v_C in RC circuit in zero-input case decays as $e^{-t/\tau} = e^{-t/RC}$.

(2) v_C above in the RLC circuit decays with "envelope" as $e^{-\alpha\tau} = e^{-t/2RC}$.

Therefore, the RC circuit zero-input response decays twice as fast as the RLC response;
i.e. $\tau_{RLC} = 2 \cdot \tau_{RC}$;
RLC takes twice as long to decay.

3.

For the zero-input case, we may treat the circuit as if R_1 were not there.

$$iR_2 + v_C + v_L = 0$$

$$R_2\frac{di}{dt} + \underbrace{\frac{dv_C}{dt}}_{\frac{i}{C}} + \underbrace{\frac{dv_L}{dt}}_{L\cdot\frac{d^2i}{dt^2}} = 0 \quad \rightarrow \quad (s^2 + \frac{R_2}{L}s + \frac{1}{LC})i = 0$$

$$2\alpha = \frac{R_2}{L} \quad \rightarrow \quad \alpha = 7.5 \times 10^6$$

$$\omega_o = \sqrt{1/LC} = 10 \times 10^6$$

$\alpha < \omega_o$, therefore the response is underdamped.

4.

a) Again, only C_1 has any voltage. Thus, the total charge of the system is $Q_i = C_1 V$.

b) We are told that the voltage across A tends to zero. Therefore, $v_1 = v_2$ after a long time. Let's call this voltage v_f. The final charge of the system is

$$Q_f = C_1 v_f + C_2 v_f = (C_1 + C_2) v_f$$

Charge must be conserved since there is no place for charge to go. Thus, $Q_f = Q_i = C_1 V$. Substituting $C_1 V$ for Q_f, we have

$$C_1 V = (C_1 + C_2) v_f \Rightarrow v_f = \frac{C_1}{C_1 + C_2} V$$

c) Since both capacitors have the same voltage, the energy as $t \to \infty$ is

$$E_f = \frac{1}{2} C_1 v_f^2 + \frac{1}{2} C_2 v_f^2 = \frac{1}{2}(C_1 + C_2) v_f^2$$

Substituting the expression we found for v_f, we get

$$E_f = \frac{1}{2}(C_1 + C_2)\frac{C_1^2}{(C_1 + C_2)^2} V^2 = \frac{1}{2}\frac{C_1^2}{C_1 + C_2} V^2$$

d)

$$\frac{E_f}{E_i} = \frac{\frac{1}{2}\frac{C_1^2}{C_1+C_2} V^2}{\frac{1}{2} C_1 V^2} = \frac{C_1}{C_1 + C_2}$$

The rest of the energy, namely $\frac{C_2}{C_1+C_2}$, must be dissipated in element A.

e) If A is a resistor R, then the system is first order with $\tau = R\frac{C_1 C_2}{C_1+C_2}$, since C_1 and C_2 are in series as seen from the resistor. We also know that the initial and final voltage across R is the difference in voltage of the two capacitors:

$$v_R(0) = V, \quad v_R(t \to \infty) = 0$$

From this information, we can obtain the voltage across R:

$$v_R(t) = V e^{-t/\tau}, \quad \tau = R\frac{C_1 C_2}{C_1 + C_2}$$

The power lost across R is

$$P_R = \frac{v_R^2}{R} = \frac{V^2}{R}e^{-t/\tau}, \quad \tau = R\frac{C_1C_2}{C_1+C_2}$$

The energy lost in R is

$$E_R = \int_0^\infty \frac{V^2}{R}e^{-t/\tau}dt, \quad \tau = R\frac{C_1C_2}{C_1+C_2}$$

This integral yields

$$E_R = \frac{V^2}{2}\frac{C_1C_2}{C_1+C_2}$$

f)

$$\frac{E_R}{E_i} = \frac{\frac{V^2}{2}\frac{C_1C_2}{C_1+C_2}}{\frac{1}{2}C_1V^2} = \frac{C_2}{C_1+C_2}$$

The ratio can be checked by noting that $E_R/E_i + E_f/E_i = 1$, thus accounting for all the energy in the system. surprisingly, the energy lost in the resistor is independent from the value of its resistance.

g) If an inductor was placed in series with R, the charge would oscillate between the two capacitors until it reached equilibrium. At that point the current through the inductor and the resistor would be zero. The energy lost in the resistor would be the same as before, since our assumption about element A (in this case, a series combination of an inductor and a resistor) still holds.

5.

a) $i_\mathrm{R}(0) = \dfrac{15}{200} = 75\,\mathrm{mA}$

$$i_\mathrm{L}(0) = -45\,\mathrm{mA}$$

$$i_\mathrm{C}(0) = -i_\mathrm{L}(0) - i_\mathrm{R}(0) = 45 - 75 = -30\,\mathrm{mA}$$

b) $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(200)(0.2\times10^{-6})} = 12{,}500$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(50\times10^{-3})(0.2\times10^{-6})} = 10^8$$

$$s_{1,2} = -12{,}500 \pm \sqrt{1.5625\times10^8 - 10^8} = -12{,}500 \pm 7500$$

$$s_1 = -5000 \text{ rad/s}; \qquad s_2 = -20{,}000 \text{ rad/s}$$

$$v = A_1 e^{-5000t} + A_2 e^{-20{,}000t}$$

$$v(0) = A_1 + A_2 = 15$$

$$\frac{dv}{dt}(0) = -5000A_1 - 20{,}000A_2 = \frac{-30 \times 10^{-3}}{0.2 \times 10^{-6}} = -15 \times 10^4 \text{V/s}$$

$$\text{Solving,} \quad A_1 = 10; \quad A_2 = 5$$

$$v = 10e^{-5000t} + 5e^{-20{,}000t}\,\text{V}, \qquad t \geq 0$$

c)

$$\begin{aligned} i_C &= C\frac{dv}{dt} \\ &= 0.2 \times 10^{-6}[-50{,}000e^{-5000t} - 100{,}000e^{-20{,}000t}] \\ &= -10e^{-5000t} - 20e^{-20{,}000t}\,\text{mA} \end{aligned}$$

$$i_R = 50e^{-5000t} + 25e^{-20{,}000t}\,\text{mA}$$

$$i_L = -i_C - i_R = -40e^{-5000t} - 5e^{-20{,}000t}\,\text{mA}, \quad t \geq 0$$

6.

a)

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\frac{10^6}{(8)(0.125)}} = 10^3 \text{ rad/s}$$

Therefore for critical damping,

$$\alpha = 10^3 = \frac{1}{2RC},$$

then ,

$$R = \frac{10^6}{(2000)(0.125)} = 4000\ \Omega.$$

b)

Because v is the voltage across the terminals of a capacitor, we have

$$v(0) = v(0^+) = V_0 = 0.$$

Because $v(0^+) = 0$, the current in the resistive branch is zero at $t = 0^+$. Hence the current in the capacitor at $t = 0^+$ is the negative of the inductor current:

$$i_C(0^+) = -(-12.25) = 12.25 \text{ mA}.$$

Therefore the initial value of the derivative is

$$\frac{dv(0^+)}{dt} = \frac{(12.25)(10^{-3})}{(0.125)(10^{-6})} = 98{,}000 \text{ V/s}.$$

For ,

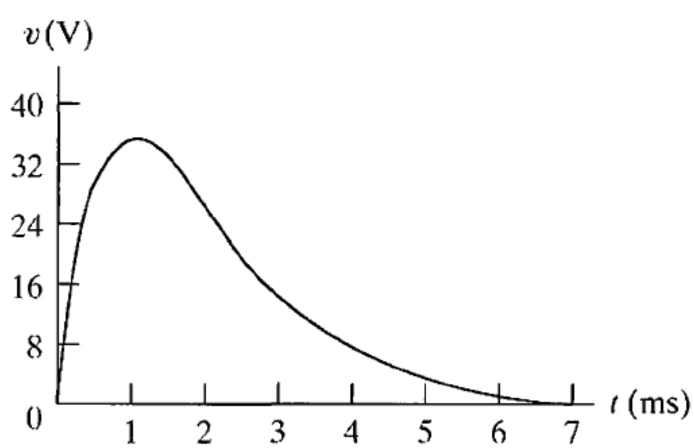
$$v(t) = D_1te^{-\alpha t} + D_2e^{-\alpha t}$$

$$v(0^+) = V_0 = D_2,$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = D_1 - \alpha D_2.$$

$$v(t) = 98{,}000te^{-1000t} \text{ V}, \quad t \geq 0$$

c)



7.

a) The first step to finding $i(t)$ is to calculate the roots of the characteristic equation. For the given element values,

$$\omega_0^2 = \frac{1}{LC}$$

$$= \frac{(10^3)(10^6)}{(100)(0.1)} = 10^8,$$

$$\alpha = \frac{R}{2L}$$

$$= \frac{560}{2(100)} \times 10^3$$

$$= 2800 \text{ rad/s}.$$

Next, we compare ω_0^2 to α^2 and note that $\omega_0^2 > \alpha^2$, because

$$\alpha^2 = 7.84 \times 10^6$$

$$= 0.0784 \times 10^8.$$

At this point, we know that the response is underdamped and that the solution for $i(t)$ is of the form

$$i(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t,$$

where $\alpha = 2800$ rad/s and $\omega_d = 9600$ rad/s. The numerical values of B_1 and B_2 come from the initial conditions. The inductor current is zero before the switch has been closed, and hence it is zero immediately after. Therefore

$$i(0) = 0 = B_1.$$

To find B_2, we evaluate $di(0^+)/dt$. From the circuit, we note that, because $i(0) = 0$ immediately after the switch has been closed, there will be no voltage drop across the resistor. Thus the initial voltage on the capacitor appears across the terminals of the inductor, which leads to the expression,

$$L\frac{di(0^+)}{dt} = V_0,$$

or

$$\frac{di(0^+)}{dt} = \frac{V_0}{L} = \frac{100}{100} \times 10^3$$
$$= 1000 \text{ A/s}.$$

Because $B_1 = 0$,

$$\frac{di}{dt} = 400B_2e^{-2800t}(24 \cos 9600t - 7 \sin 9600t).$$

Thus

$$\frac{di(0^+)}{dt} = 9600B_2,$$

$$B_2 = \frac{1000}{9600} \approx 0.1042 \text{ A}.$$

The solution for $i(t)$ is

$$i(t) = 0.1042e^{-2800t} \sin 9600t \text{ A}, \quad t \geq 0.$$

b) To find $v_C(t)$, we can use either of the following relationships:

$$v_C = -\frac{1}{C}\int_0^t i\,d\tau + 100 \text{ or}$$

$$v_C = iR + L\frac{di}{dt}.$$

Whichever expression is used (the second is recommended), the result is

$$v_C(t) = (100 \cos 9600t + 29.17 \sin 9600t)e^{-2800t} \text{ V}, \quad t \geq 0.$$