

Practice Problems Solutions (Lecture4-Lecture7)

1.

Solution:

$$(1) \frac{1}{C_{total}} = \frac{1}{1\mu F} + \frac{1}{10\mu F} \rightarrow \rightarrow \rightarrow C_{total} = \frac{1\mu F * 10\mu F}{1\mu F + 10\mu F} = \frac{10}{11} \mu F$$

$$(2) \frac{1}{L_p} = \frac{1}{2mH} + \frac{1}{1mH} \rightarrow \rightarrow \rightarrow L_p = \frac{2mH * 1mH}{1mH + 2mH} = \frac{2}{3} mH$$

$$L_{total} = L_p + 1\mu H = 666.67\mu H + 1\mu H = 667.67\mu H$$

2.

Solution:

$$v(t) = L * \frac{di(t)}{dt} \rightarrow \rightarrow \rightarrow v(t)dt = L * di(t)$$

$$\int v(t)dt = L * i(t) \rightarrow \rightarrow \rightarrow i(t) = \frac{\int v(t)dt}{L} \rightarrow \rightarrow \rightarrow i(t) = \frac{1}{L}$$

3.

Solution:

$$v = L_2 * \frac{di(t)}{dt} \rightarrow \rightarrow \rightarrow 1 = L_2 * 100 \rightarrow \rightarrow \rightarrow L_2 = \frac{1}{100} H$$

$$\frac{L_1/100}{L_1 + 100} * 400 = 1 \rightarrow \rightarrow \rightarrow 4L_1 = L_1 + \frac{1}{100} \rightarrow \rightarrow \rightarrow L_1 = \frac{1}{300} H$$

4.

Solution:

$$v(t) = \frac{d(L(t) * I)}{dt} = I \frac{dL(t)}{dt} = IL_1 \omega \cos \omega t$$

ANS: $v(t) = IL_1 \omega \cos \omega t$

5.

Solution:

(a)

$$i_D = K_1 v_B = K_1 v_I$$
$$v_o = i_D R_V = K_1 v_I R_V$$

(b)

$$i_D = K_1 v_B = K_2 i_B = K_2 \frac{v_I}{R_I}$$
$$v_o = i_D R_V = K_2 \frac{v_I R_V}{R_I}$$

ANS:(a) $v_o = K_1 v_I R_V$ (b) $v_o = K_2 \frac{v_I R_V}{R_I}$

6.

Solution:

The inductor first acts as an open circuit and eventually becomes a wire:

$$t \geq 0 \left\{ \begin{array}{l} \text{initially: } i_1(t) = 0 \text{ (open circuit)} \\ \text{finally: } i_1(t) = \frac{V_S(t)}{3k} + i_S(t) = \frac{4}{3} \text{ mA} \end{array} \right.$$

Assume $i_S(t)$ source points down.

$$i_1(t) = (\text{Final Value}) + (\text{Initial Value} - \text{Final Value}) e^{-t/\tau}$$

$$i_1(t) = 4/3 (1 - e^{-t/\tau}) [mA]$$

$$\tau = L/R = 1/3ms$$

7.

Solution:

$$i_c = C \cdot \frac{dv_c}{dt}$$

$$v_c = \frac{\int i_c}{C} = 10t \text{ for } 0 < t < 1 \text{ second}$$

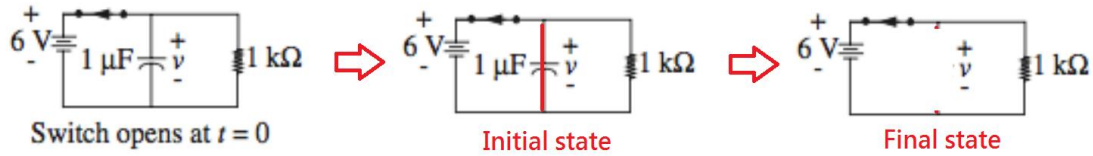
$$v_c = \frac{\int i_c}{C} = \text{a constant, otherwise, when } i_c = 0$$

Therefore,

$$v_c(t = -1 \text{ second}) = -5V$$

8.

Solution:



For $t > 0$, the capacitor just like short circuit at initial state

$$\rightarrow V = 0 \text{ V}$$

At final state, the capacitor just like open circuit.

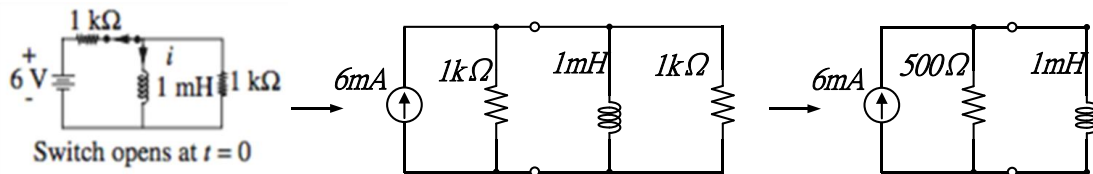
$$\rightarrow V = 6 \text{ V}$$

Thus,

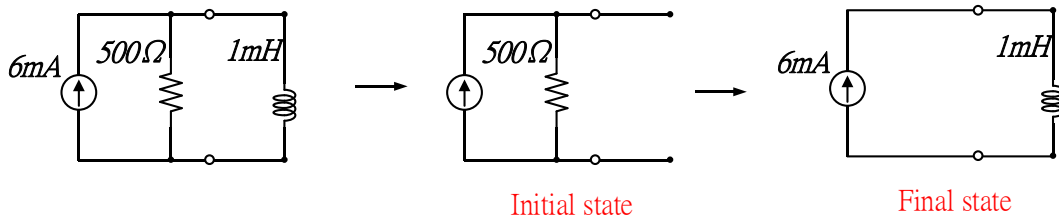
$$V(t) = \text{final state} + (\text{initial state} - \text{final state})e^{-\frac{t}{\tau}}$$

$$\rightarrow V(t) = 6 \text{ V} + (0\text{V} - 6\text{V})e^{-1000t} = 6(1 - e^{-1000t}) \text{ V}$$

Solution:



For an easy way to calculate, we may transfer the circuit into Norton equivalent.



For $t > 0$, the inductor just like open circuit at initial state

$$\rightarrow i = 0 \text{ A}$$

At final state, the inductor just like short circuit.

$$\rightarrow i = 6 \text{ mA}$$

Thus,

$$i(t) = \text{final state} + (\text{initial state} - \text{final state})e^{-\frac{t}{\tau}}$$

$$\rightarrow i(t) = 6 \text{ mA} + (0\text{V} - 6\text{mA})e^{-500000t} = 6(1 - e^{-500000t}) \text{ mA}$$

9.

Solution:

$$[\mathbf{a}] \quad v_o(t) = v_o(0^+)e^{-t/\tau}$$

$$\therefore v_o(0^+)e^{-10^{-3}/\tau} = 0.5v_o(0^+)$$

$$\therefore e^{10^{-3}/\tau} = 2$$

$$\therefore \tau = \frac{L}{R} = \frac{10^{-3}}{\ln 2}$$

$$\therefore L = \frac{10 \times 10^{-3}}{\ln 2} = 14.43 \text{ mH}$$

$$[\mathbf{b}] \quad v_o(0^+) = -10i_L(0^+) = -10(1/10)(30 \times 10^{-3}) = -30 \text{ mV}$$

$$v_o(t) = -0.03e^{-t/\tau} \text{ V}$$

$$p_{10\Omega} = \frac{v_o^2}{10} = 9 \times 10^{-5}e^{-2t/\tau}$$

$$w_{10\Omega} = \int_0^{10^{-3}} 9 \times 10^{-5}e^{-2t/\tau} dt = 4.5\tau \times 10^{-5}(1 - e^{-2 \times 10^{-3}/\tau})$$

$$\tau = \frac{1}{1000 \ln 2} \quad \therefore \quad w_{10\Omega} = 48.69 \text{ nJ}$$