100

EE 2245 Microelectronics Labs

Lab 3: Passive Filters

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Objectives：

(1) Become familiar with the characteristics of passive low-pass and high-pass filters.

(2) Plot the phase response for low-pass and high-pass filters.

(3) Analyze the frequency response of tuned band-pass and band-stop filters.

(4) Become adept in the use of semilog graph paper.

Equipment Required :

Resistors: 300 Ω, 1 kΩ

Capacitors: 0.1 μF

Inductors: 10 mH

Instruments: Digital multimeter (DMM), digital oscilloscope, function generator.

注意：填寫實驗數據時如有『單位』請記得填入。 在這個實驗你必須 demo low-pass filter 及 band-stop filter 的設計。

Procedure :

Part 1: Low-Pass R-C Filter Design

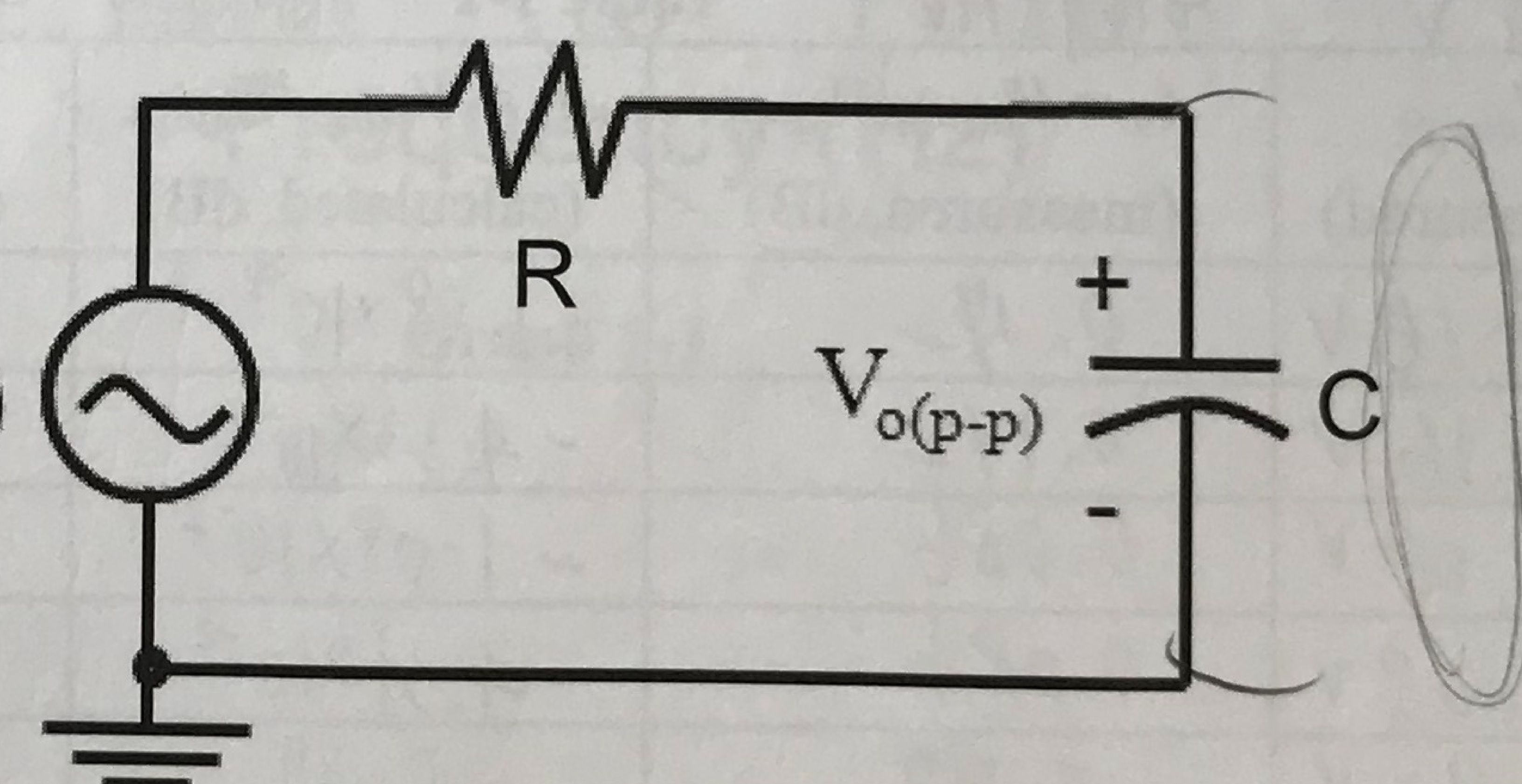


Figure 1-1

You are required to design a low-pass filter, as shown in Fig. 1-1, with a −3-dB frequency at 10 kHz (with tolerance of ±2%, i.e. 9.8 – 10.2 kHz). Please demonstrate your result to a teaching assistant or the teacher.

(a) Please show your analysis, and write down the resistor and capacitor values of your choice. Please also record the measured values.

Analysis: (背面)

$H(S) = \frac{V_o(S)}{V_i(S)} = \frac{\frac{1}{SC}}{R+\frac{1}{SC}} = \frac{1}{1+SRC}$ choose $R = 1.59k$, $C = 10nF$

$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{2}}$ (-3 dB) ∴ $\omega RC = 1 \Rightarrow \omega = \frac{1}{RC}$

$\omega = \frac{1}{RC} = 2\pi f_{10k} \Rightarrow RC = 1.59\times10^{-5}$

R = 1.59KΩ; C= 10nF

✓ $R_{measured}$ = 1.596kΩ; $C_{measured}$ = 9.98nF;

(b) Write down the magnitude and phase of $V_o(j\omega)/V_i(j\omega)$ in terms of R and C.

Analysis: $s = j\omega$

$$\frac{V_o(S)}{V_i(S)} = \frac{\frac{1}{SC}}{\frac{1}{SC}+R} = \frac{1}{1+SRC} = \frac{1}{1+j\omega RC}$$

$|V_o(j\omega)/V_i(j\omega)|$ = $\frac{1}{\sqrt{1+(\omega RC)^2}}$; $\angle(V_o(j\omega)/V_i(j\omega))$ = $-\tan^{-1}(\omega RC)$

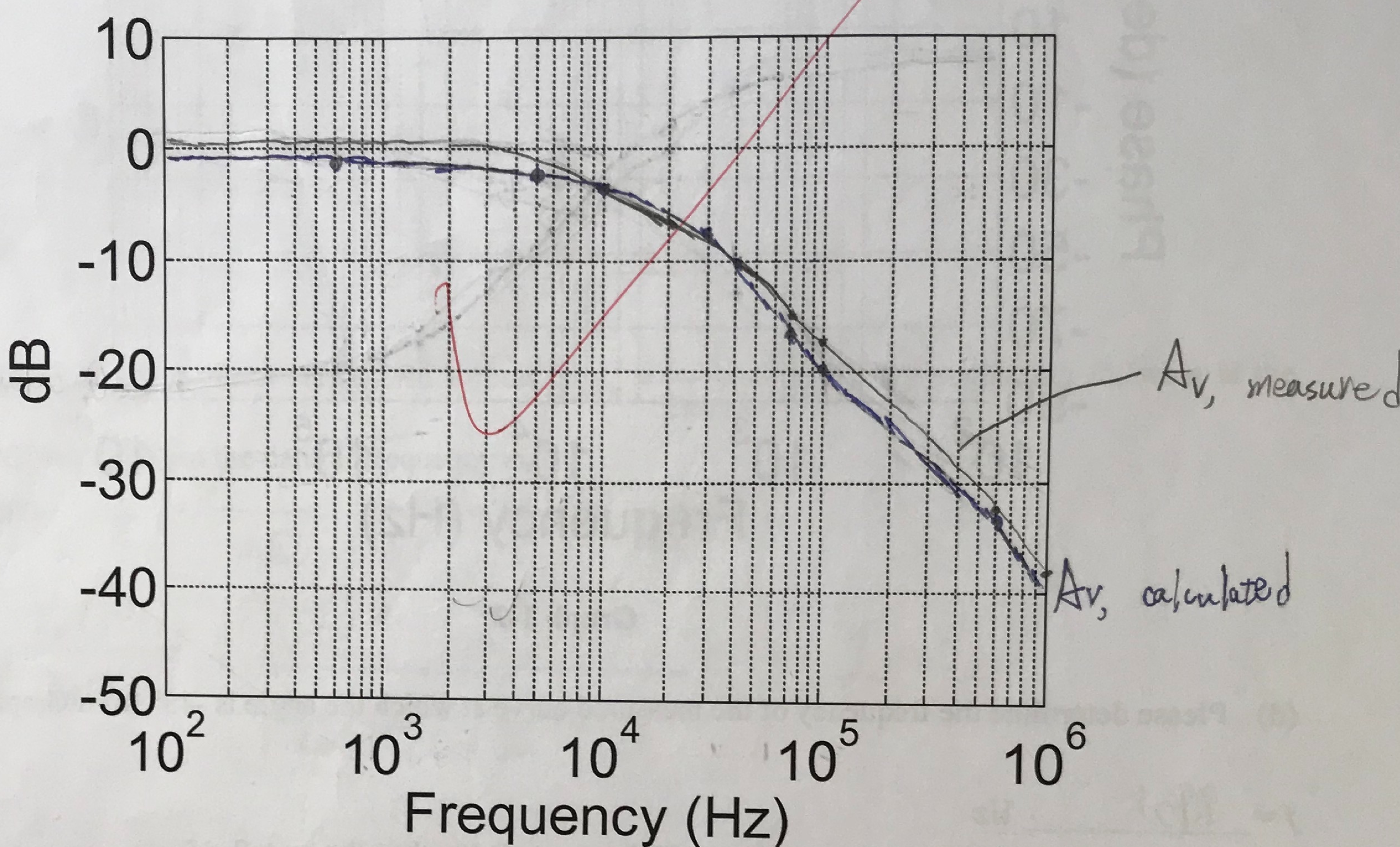
Note: For those of you who use a small resistor or a small capacitor in the circuit, you may not successfully get the desired gain or observe some undesired phenomena in the input.

(c) Please complete Table 1-1. The angle θ is the angle by which V_o leads the input voltage V_i. The sign of θ is minus when V_o lags the input voltage V_i. Please use the measured and calculated gains and phases in Table 1-1 to complete Graph 1-1 and 1-2.

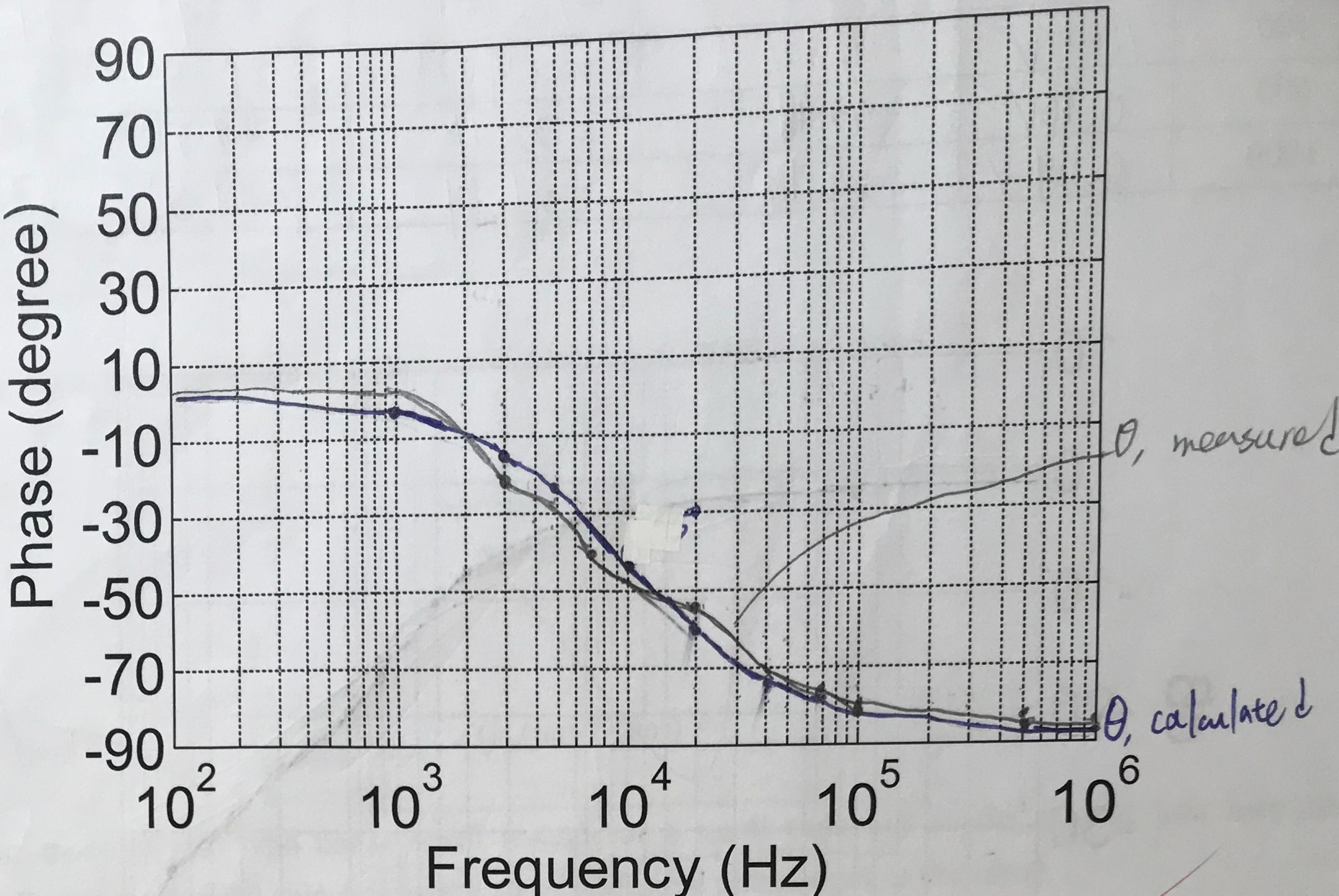
Table 1-1   ✓ $20\log|A_v|$ $20\log\left|\frac{1}{\sqrt{1+(\omega RC)^2}}\right|$ $\theta = \Delta t\cdot f\cdot 360^\circ$

Frequency (kHz)	$V_{o(p\text{-}p)}$ (measured)	$A_v = V_{o(p\text{-}p)}/V_{i(p\text{-}p)}$ (measured, dB)	$A_v = V_{o(p\text{-}p)}/V_{i(p\text{-}p)}$ (calculated, dB)	θ (degree) (measured)	θ (degree) (calculated)
0.05	8.16V	0.172	-1.08×10^{-4}	0	-0.2862°
0.1	8.16V	0.172	-4.33×10^{-4}	0	-0.572°
0.5	8.08V	0.086	-1.08×10^{-2}	0	-2.859°
1.0	8.08V	0.086	-4.31×10^{-2}	0	-5.705°
3.0	7.92V	-0.087	-0.37	-20.52°	-16.6839°
5.0	7.28V	-0.820	-0.96	-27°	-26.5427°
8.0	6.16V	-2.270	-2.145	-39.32°	-38.63°
8.5	6.04V	-2.441	-2.358	-42.13°	-40.33°
9	5.88V	-2.674	-2.573	-43.32°	-41.96°
9.5	5.76V	-2.853	-2.789	-43.6°	-43.50°
10	5.6V	-3.098	-3.006	-46.1°	-44.97°
10.5	5.48V	-3.286	-3.223	-47.30°	-46.37°
11	5.36V	-3.478	-3.439	-49.3°	-47.69°
20	3.64V	-6.839	-6.983	-60.48°	-63.41°

40	2.16V	-11.37	-12.297	-72.6°	-75.95°
70	1.40V	-15.14	-16.981	-80.4°	-81.86°
100	1.04V	-17.72	-20.034	-86.5°	-84.283°
500	0.16V	-33.98	-33.97	-84.31°	-88.8531°
1000	0.09V	-38.98	-39.99	-89.5°	-89.4265°



Graph 1-1



Graph 1-2

(d) Please determine the frequency of the measured curve at which the angle is -45° from Graph 1-2.

f= 9.73k Hz

How does it compare with the measured −3-dB frequency? Are they the same?

Comment: Error = $\frac{9.73-10}{10}$ = 2.66%, They are pretty close!

Part 2: Band-Pass Filter

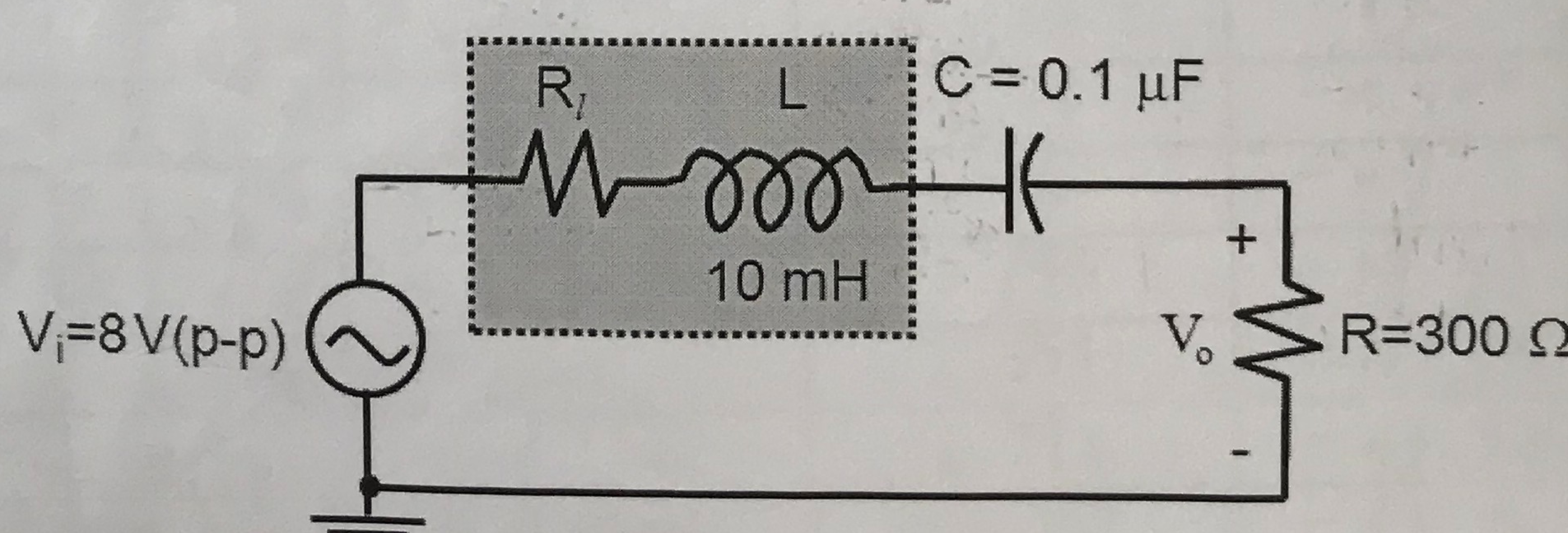


Figure 2-1

√ (a) Construct the network of Fig. 2-1. Fill in the measured resistor value and R_l for the inductor.

$R_{measured}$ = 296 Ω, $R_{l\ measured}$ = 85 Ω, $L_{measured}$ = 9.97 mH, $C_{measured}$ = 98.72 nF

(b) Write down the transfer function of $V_o(s)/V_i(s)$ without considering R_l. By replacing s = $j\omega$ into $V_o(s)/V_i(s)$, what is the dc gain (at $\omega = 0$) of $|V_o(j\omega)/V_i(j\omega)|$?

Calculation: $\frac{V_o(s)}{V_i(s)} = \frac{R}{R + sL + \frac{1}{sC}} = \frac{R}{j\omega L + \frac{1}{j\omega C} + R}$. $\omega \to$, $\frac{V_o(j\omega)}{V_i(j\omega)} \to 0$

$= \frac{sRC}{sRC + s^2LC + 1}$

$V_o(s)/V_i(s)$ = $\frac{sRC}{s^2LC + sRC + 1}$, $|V_o(j\omega)/V_i(j\omega)|_{\omega=0}$ = 0

The natural frequency of the series *LRC* circuit is $f_n = \frac{1}{2\pi\sqrt{LC}}$ (Hz). By neglecting R_l, what is the gain of $|V_o(j\omega)/V_i(j\omega)|$ at the natural frequency ω_n?

Calculation: $f_n = \frac{1}{2\pi\sqrt{LC}} \Rightarrow \omega_n = 2\pi f = \frac{1}{\sqrt{LC}}$

$\frac{V_o(j\omega_n)}{V_i(j\omega_n)} = \left| \frac{j\frac{1}{\sqrt{LC}}RC}{-\frac{1}{LC}LC + j\frac{1}{\sqrt{LC}}RC + 1} \right| = 1$

$f_n = \frac{1}{2\pi\sqrt{LC}}$ = 5073.06 Hz, $|V_o(j\omega_n)/V_i(j\omega_n)|$ = 1

(c) Please calculate the ratio of V_o/V_i at the natural frequency using the measured resistor values (including that of the inductor). Please also calculate the gain at other frequencies listed in Table 2-1 and fill in the values.

Calculation: $H(j\omega) = \frac{R}{R + R_l + j\omega L + \frac{1}{j\omega C}} = \frac{j\omega RC}{j\omega RC + j\omega R_lC - \omega^2LC + 1}$

$\Rightarrow |H(j\omega)|_{\omega=\frac{1}{\sqrt{LC}}} = \left| \frac{RC}{RC + R_lC} \right| = \frac{R}{R + R_l} = 0.776$

$|V_o(j\omega_n)/V_i(j\omega_n)|$ = 0.776

(d) Energize the network of Fig. 2-1 and set the function generator to each of the frequencies appearing in Table 2-1. Make sure $V_i = 8\ V_{(p\text{-}p)}$ on the scope before you apply to the circuit. **You may observe the loading effect of the 300-Ω resistor at some frequencies such that V_i will not be 8 $V_{(p\text{-}p)}$, since the function generator has an impedance of 50 Ω.** Please complete Table 2-1.

(e) Plot the measured and calculated A_v vs. frequency on Graph 2-1. Find the maximum value of A_v and compare to the calculated value in part (c). Please comment accordingly.

$A_{v,\ max}$ = 0.719 (V/V)   In theory, Av is 0.776

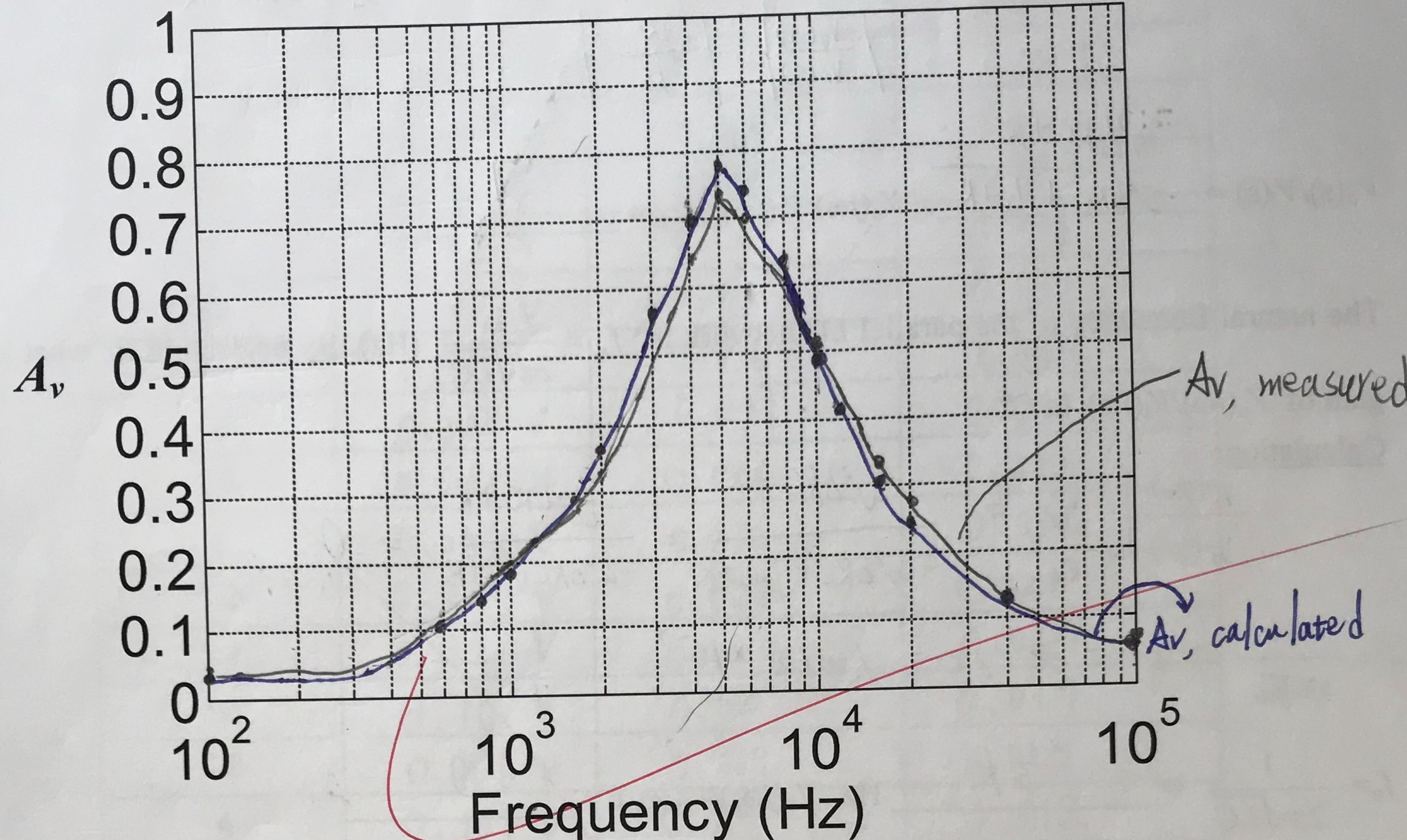
Comment: Av has the maximum at f = 5kHz, Av (measured) = 0.719, Av (calculated) = 0.781

Errors are $\frac{0.719 - 0.776}{0.776} \approx -7\%$ and $\frac{0.781 - 0.776}{0.776} = 0.644\%$ respectively!

Table 2-1

Frequency (kHz)	$V_{o(p\text{-}p)}$ (measured)	$A_v = V_{o(p\text{-}p)}/V_{i(p\text{-}p)}$ (measured)	$A_v = V_{o(p\text{-}p)}/V_{i(p\text{-}p)}$ (calculated)	$V_{i(p,p)}$ (measured)
0.1	0.158 V	0.019	0.017	8.16
0.2	0.308 V	0.037	0.035	8.16
0.4	0.6 V	0.073	0.070	8.16
0.6	0.896 V	0.111	0.106	8.08
0.8	1.17 V	0.143	0.142	8.16
1.0	1.45 V	0.179	0.178	8.08
1.2	1.76 V	0.218	0.215	8.08
1.4	2.00 V	0.248	0.252	8.08
1.6	2.26 V	0.283	0.290	8
1.8	2.50 V	0.313	0.328	8
2.0	2.72 V	0.343	0.366	7.92
3.0	4.14 V	0.539	0.558	7.68
4.0	4.99 V	0.672	0.714	7.44
5.0	5.29 V	0.719	0.781	7.36
6.0	5.44 V	0.708	0.758	7.68
8.0	4.64 V	0.598	0.620	7.76
10.0	3.88 V	0.490	0.498	7.92
12.0	3.32 V	0.411	0.411	8.08
14.0	2.88 V	0.353	0.350	8.16
16.0	2.56 V	0.314	0.304	8.16
18.0	2.28 V	0.277	0.269	8.24
20.0	2.08 V	0.252	0.241	8.24
40.0	1 V	0.120	0.119	8.32

60.0	0.64 V	0.077	0.079	8.32
100.0	0.38 V	0.046	0.047	8.32



Graph 2-1

Part 3: Band-Stop Filter Design

You are required to design a band-stop filter, as shown in Fig. 3-1, with a band-stop frequency (with the lowest gain) at 5 kHz (with a tolerance of ±10%; i.e. 4.5 – 5.5 kHz) and a corresponding gain less than -15 dB (i.e. 0.178) at that frequency. <u>Please demonstrate your result to a teaching assistant or the teacher</u>.

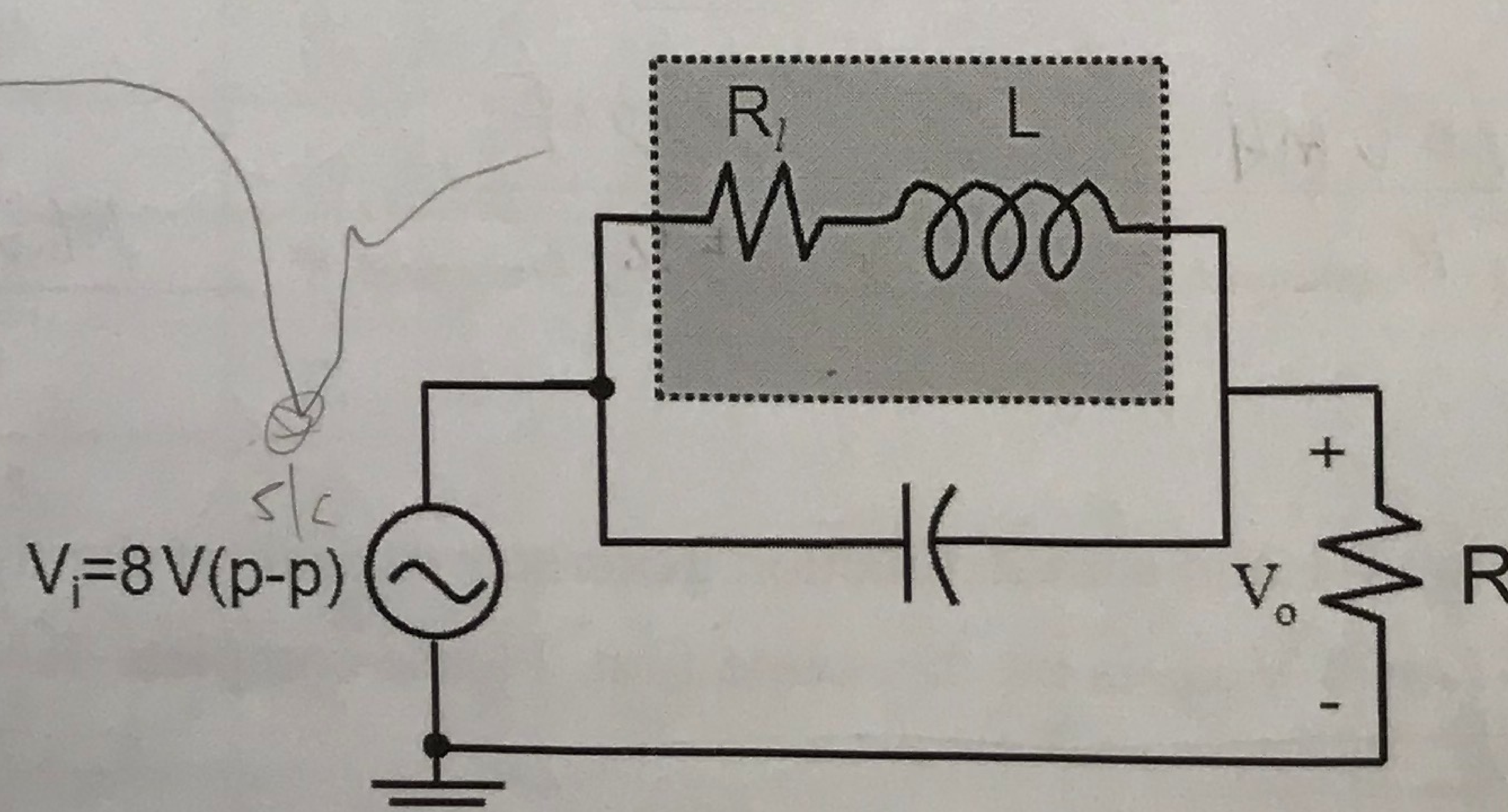


Figure 3-1

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(a) Write down the transfer function of $V_o(s)/V_i(s)$ without considering R_l. By replacing s = $j\omega$ into $V_o(s)/V_i(s)$, what is the dc gain (at $\omega = 0$) of $|V_o(j\omega)/V_i(j\omega)|$?

Calculation: $\frac{V_o(S)}{V_i(S)} = \frac{R}{\frac{L/C}{sL+\frac{1}{sC}} + R} = \frac{sLR + \frac{R}{sC}}{\frac{L}{C} + sLR + \frac{R}{sC}} = \frac{s^2LRC + R}{sL + s^2LRC + R}$

$\left|\frac{V_o(S)}{V_i(S)}\right| = \left|\frac{R}{R}\right| = 1$

$V_o(s)/V_i(s) = \frac{s^2LRC+R}{s^2LRC+sL+R}$, $|V_o(j\omega)/V_i(j\omega)|_{\omega=0} =$ 1

The natural frequency of the parallel LC network is $f_n = \frac{1}{2\pi\sqrt{LC}}$ (Hz). By neglecting R_l, what is the gain of $|V_o(j\omega)/V_i(j\omega)|$ at f_n?

Calculation: $\left.\frac{V_o(j\omega)}{V_i(j\omega)}\right|_{\omega=\frac{1}{\sqrt{LC}}} = \frac{-\omega^2LRC+R}{-\omega^2LRC+j\omega L+R} = \frac{-\frac{1}{LC}\cdot LRC+R}{-\omega^2LRC+j\omega L+R} = 0$

$f = \frac{1}{2\pi\sqrt{LC}} = 5000 \Rightarrow LC = 1.013 \times 10^{-9}$

$f_n = \frac{1}{2\pi\sqrt{LC}} =$ 5k Hz, $|V_o(j\omega_n)/V_i(j\omega_n)| =$ 0

(b) By considering R_l, please determine the ratio $|V_o(j\omega_n)/V_i(j\omega_n)|$.

Calculation: $\frac{V_o(j\omega_n)}{V_i(j\omega_n)} = \frac{R}{(sL+R_l)//\frac{1}{sC}+R} = \frac{R}{\frac{\frac{1}{sC}(sL+R_l)}{sL+R_l+\frac{1}{sC}}+R} = \frac{R(sL+R_l+\frac{1}{sC})}{\frac{1}{sC}(sL+R_l)+R(sL+R_l+\frac{1}{sC})} = \frac{R(s^2LC+sR_lC+1)}{(sL+R_l)+R(s^2LC+sR_lC+1)}$

$\omega_n = \frac{1}{\sqrt{LC}}$

$= \frac{sRR_lC}{sL+R_l+sRR_lC} = \frac{j\frac{1}{\sqrt{LC}}RR_lC}{j\frac{1}{\sqrt{LC}}(L+RR_lC)+R_l} \Rightarrow \left|\frac{V_o(j\omega_n)}{V_i(j\omega_n)}\right| = \left(\frac{R^2R_l^2\frac{C}{L}}{R_l^2+\frac{L}{C}+\frac{R^2R_l^2C}{L}+2RR_l}\right)^{1/2}$

$\frac{1}{LC}(L^2+2RR_lCL+R^2R_l^2C^2)$

$|V_o(j\omega_n)/V_i(j\omega_n)| = \left(\frac{R^2R_l^2\frac{C}{L}}{R_l^2+\frac{L}{C}+\frac{R^2R_l^2C}{L}+2RR_l}\right)^{1/2}$

✓ (c) Please write down the inductor, capacitor, and resistor values of your choice, and the measured values.

R = 100 Ω, L = 100 mH, C = 10.13 nF

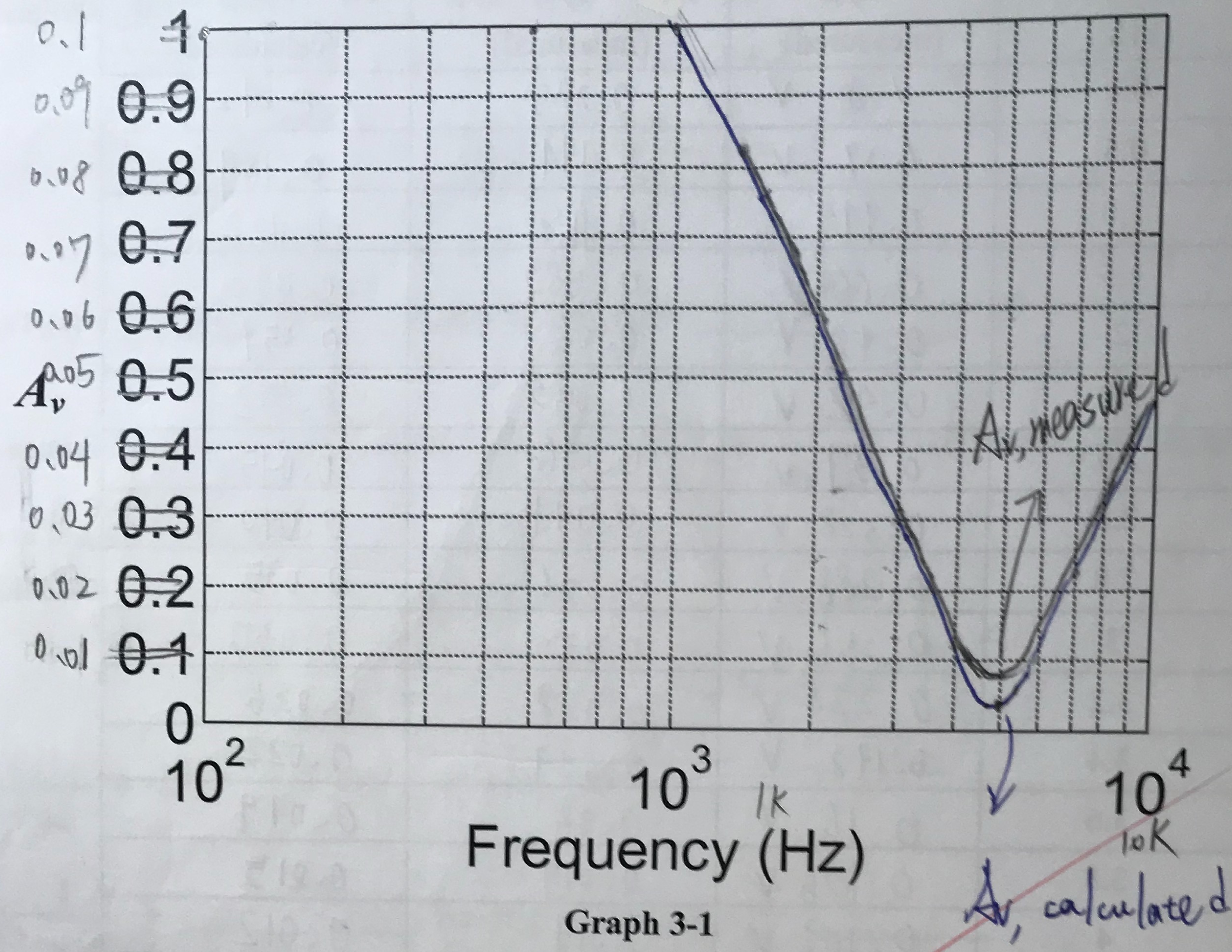
$R_{measured}$ = 99 Ω, $R_{l\ measured}$ = 304 Ω, $L_{measured}$ = 106.5 mH, $C_{measured}$ = 9.98 nF

(d) Energize the network of Fig. 3-1 and set the function generator to each of the frequencies appearing in Table 3-1. Make sure that V_i = 8 $V_{(p-p)}$ is on the scope first. Please complete Table 3-1. R_l should be considered when calculating A_v.

(e) Plot the measured and calculated A_v vs. frequency on Graph 3-1.

Table 3-1

Frequency (kHz)	$V_{o(p-p)}$ (measured)	$A_v = V_{o(p-p)}/V_{i(p-p)}$ (measured)	$A_v = V_{o(p-p)}/V_{i(p-p)}$ (calculated)
0.1	1.8 V	0.225	0.242
0.5	1.39 V	0.174	0.187
1.0	0.944 V	0.118	0.122
1.5	0.664 V	0.083	0.083
2	0.48 V	0.06	0.059
2.2	0.42 V	0.053	0.052
2.4	0.37 V	0.046	0.045
2.6	0.328 V	0.041	0.040
2.8	0.288 V	0.036	0.035
3	0.256 V	0.032	0.030
3.2	0.224 V	0.028	0.026
3.4	0.192 V	0.024	0.022
3.6	0.16 V	0.02	0.019
3.8	0.136 V	0.017	0.015
4	0.109 V	0.014	0.012
4.2	0.088 V	0.011	0.009
4.4	0.067 V	0.008	0.006
4.6	0.0528 V	0.007	0.005
4.8	0.046 V	0.006	0.003
5	0.052 V	0.006	0.003
5.2	0.062 V	0.008	0.005
5.4	0.078 V	0.01	0.008
5.6	0.090 V	0.011	0.009
5.8	0.104 V	0.013	0.011
6	0.119 V	0.015	0.013
6.5	0.16 V	0.02	0.018
7	0.196 V	0.025	0.022
7.5	0.232 V	0.029	0.027
8	0.268 V	0.034	0.031
8.5	0.298 V	0.037	0.035
9	0.33 V	0.041	0.039
10	0.404 V	0.051	0.047



0.1
1
0.09
0.9
0.08
0.8
0.07
0.7
0.06
0.6
0.05
0.5
A_v
0.04
0.4
0.03
0.3
0.02
0.2
0.01
0.1
0
10^2
10^3
1k
10^4
10k
Frequency (Hz)
Av, measured
Av, calculated

Graph 3-1