EE 2245 Microelectronics Labs

Lab 2: Time-Domain Responses of 1st- and 2nd-Order Circuits (2 Weeks)

實驗室：218 組別：_____ Names and ID Numbers: 吳俊毅 106061125

Objectives：

(1) Validate conclusions regarding the behavior of capacitors in a steady-state dc network.

(2) Validate the behaviors of the 1st-order RC circuits.

(3) Design and implement a *RLC* circuit with the desired specifications.

Equipment Required：

Resistors: 1 kΩ, 5.1 kΩ, 20 kΩ (1.0031K, 5.108K, 19.84K)

Capacitors: 1 μF, 2.2 μF, 100 μF (1.034, 2.2047, 93μ)

Instruments: Multimeter, Oscilloscope, function generator, and dc power supply.

注意：填寫實驗數據時如有『單位』請記得填入。

Procedure :

Part 1: Basic Series *R-C* Circuit

$R_{measured}$ = 19.84 KΩ

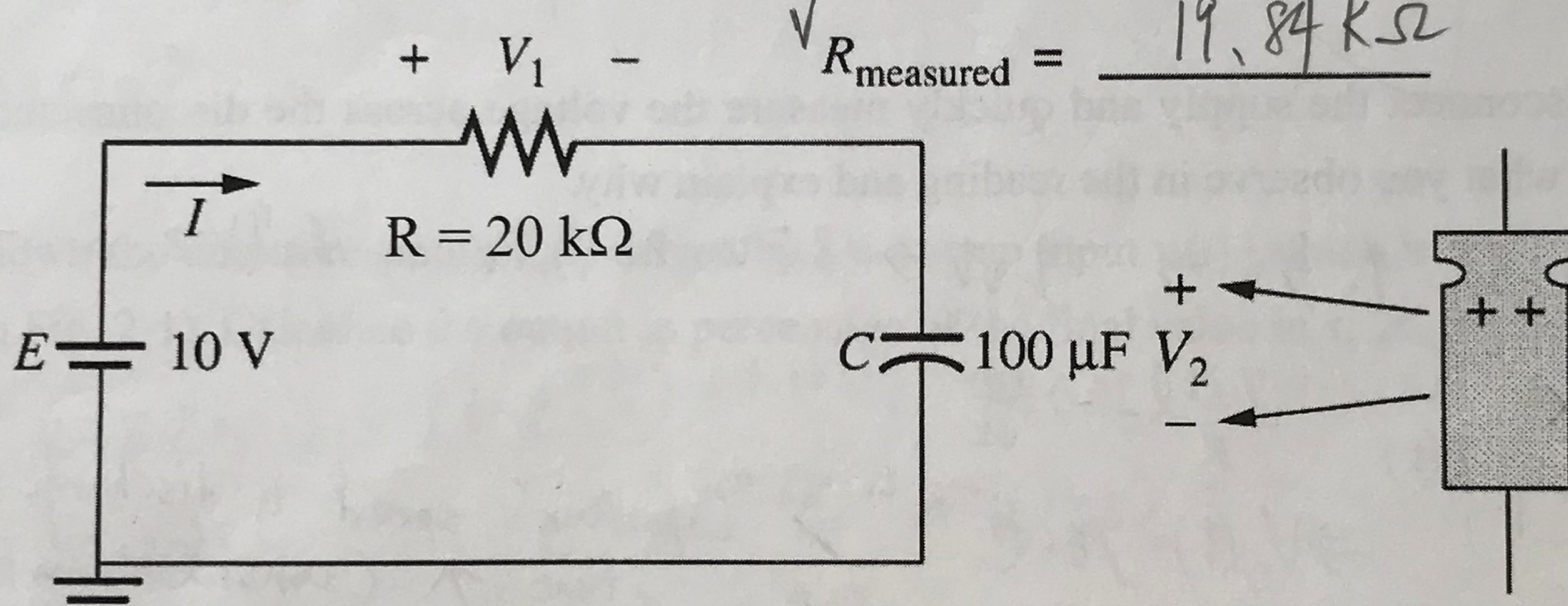


Figure 1-1

(a) Construct the network as shown in Fig. 1-1. Insert the measured resistance value. Be sure to note polarity on electrolytic capacitor as shown in the figure.

(b) Show your calculation of the steady-state value (defined by a period of time greater than five time constants) of the current *I* and the voltages V_1 and V_2.

$t > 5\tau$

$RC = \tau$

Calculation:

$V_1 = V_0 e^{-t/\tau} = 10e^{-5} = 0.0674\ V$

$V_2 = V_0(1-e^{-5}) = 9.9326$

$I = \frac{V_1}{R} = \frac{0.0674}{19840} = 3.397\ \mu A$

$I =$ 3.397 μA, $V_1 =$ 0.0674 V, and $V_2 =$ 9.9326 V.

(c) Measure the steady-state voltages V_1 and V_2 and calculate the current from Ohm's law. Compare with results from part 1(b).

$V_1 =$ 0.0028, $V_2 =$ 9.988, and $I =$ 0.141 μA

Comment:

$I = \frac{0.0028}{19840} = 0.141\ \mu A < I_{\text{calculated}}$ (3.397 μA) ∵ 過太久才去觀察電流值、電流變得太小（$I = e^{-\frac{t}{RC}}$） $(t >> 5\tau)$

(d) Calculate the energy stored in the capacitor.

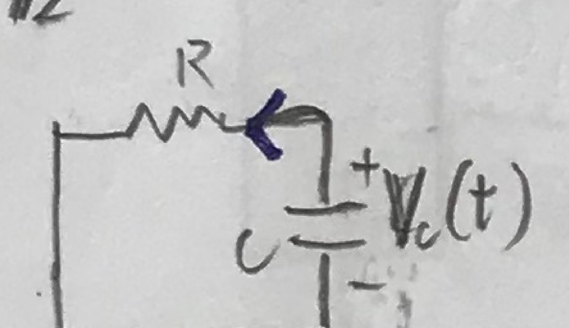
Calculation:

$W = \frac{1}{2} C V_2^2 = \frac{1}{2} \times 93 \times 10^{-6} \times (9.988)^2 = 4.638 \times 10^{-3}$ J

$W =$ 4.638×10^{-3} J

(e) Carefully disconnect the supply and quickly measure the voltage across the disconnected capacitor. Please describe what you observe in the reading and explain why.

Comment: V_2: 9.9V → 9.3V → 9.0V → 8.5V → 7.9V → 6.7V → … → 0 V



$\frac{dV_c(t)}{dt} + \frac{1}{RC} V_c(t) = 0$

$\frac{V_c(t)}{R} = -C \frac{dV_c(t)}{dt}$

$\Rightarrow V_c(t) = 10 \cdot e^{-\frac{1}{RC} t}$ ⇒ Capacitor's speed to discharge got faster as time ↑ (consistent with observation

(f) Short the capacitor by connecting a wire to both terminals and then measure V_C again. What happens after this step?

Comment:

$V_c = 0$, and $Q = CV_c \Rightarrow Q = 0$. Capacitor is discharged due to short circuit

Part 2: Time-Domain Response of the Basic *R-C* Circuit

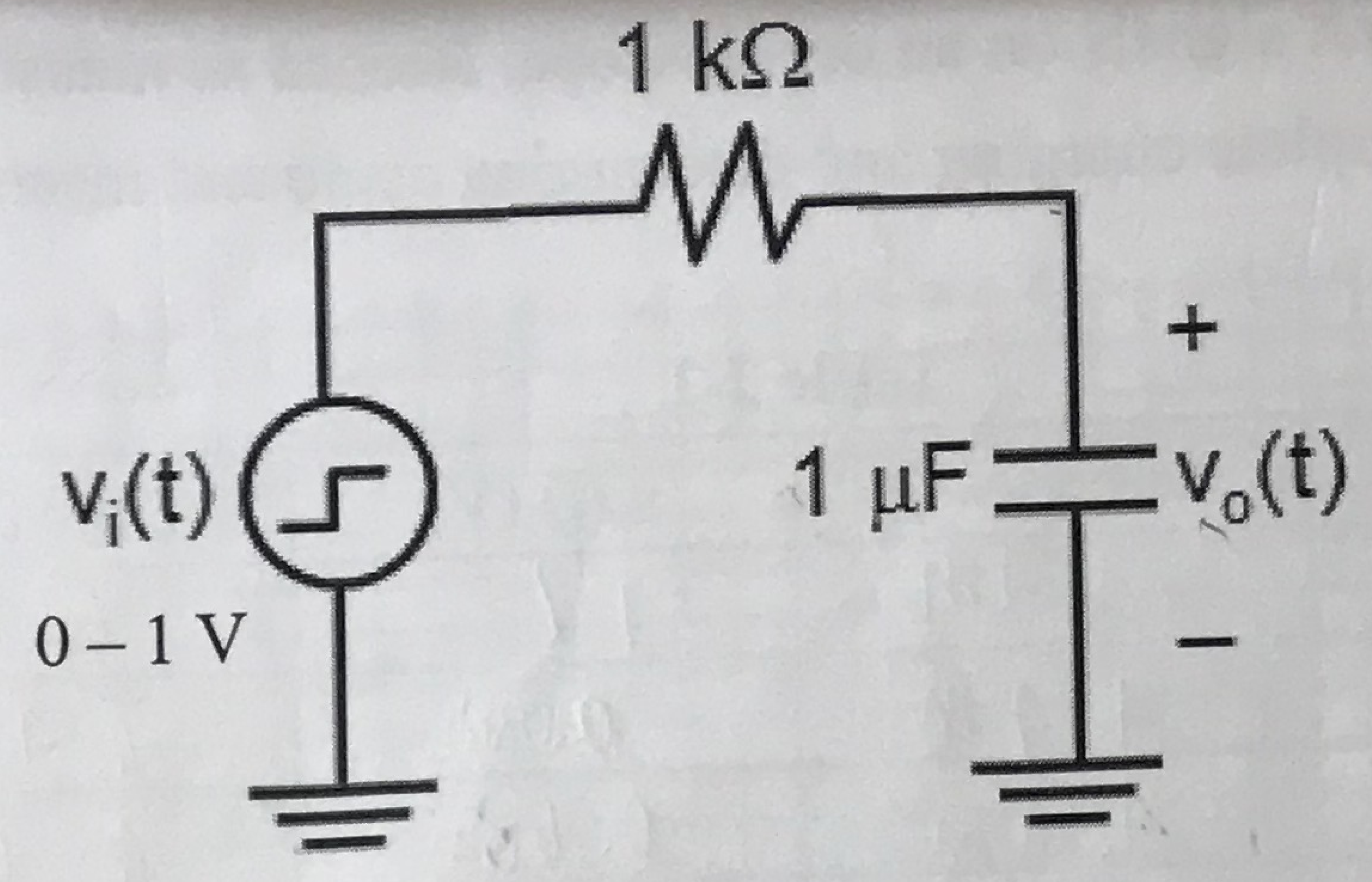


Figure 2-1

(a) Construct the RC circuit of Fig. 2-1. Measure the resistance and capacitance values. Use the function generator to produce the input square waveform $v_i(t)$ with a minimum value at 0 V and a maximum value at 1 V. Check the input waveform on a oscilloscope before you connect it to the circuit.

$R =$ 1.0031 kΩ, $C =$ 1.034 μF

(b) Calculate the RC time constant τ using the measured resistance.

Calculation: $\tau = R \cdot C \doteq 1.037 \times 10^{-3}$

$\tau =$ 1.037×10^{-3} s

(c) Write down the output response $v_o(t)$ driven by a unit-step input $v_i(t)$ (which is replaced by a square waveform in Fig. 2-1). Calculate the output in percentage of the final value in τ, 2τ, 3τ, 4τ, and 5τ.

Calculation:

$V_o(\tau) = (1-e^{-1}) = 0.6321$ $\quad V_o(4\tau) = (1-e^{-4}) = 0.9817$

$V_o(2\tau) = (1-e^{-2}) = 0.8647$ $\quad V_o(5\tau) = (1-e^{-5}) = 0.9933$

$V_o(3\tau) = (1-e^{-3}) = 0.9502$

$v_o(\tau) =$ 63.21 % $v_i(t)$, $v_o(2\tau) =$ 86.47 % $v_i(t)$, $v_o(3\tau) =$ 95.02 % $v_i(t)$, $v_o(4\tau) =$ 98.17 % $v_i(t)$, $v_o(5\tau) =$ 99.33 % $v_i(t)$

(d) Based on the time constant, determine a proper input frequency f_i that would allow you to clearly observe the transient and steady-state responses of the output waveform. Comment on how you pick the frequency.

$f_i =$ 96.4 Hz

Comment:

$f_i \le \frac{1}{5\tau} = \frac{1}{5RC}$

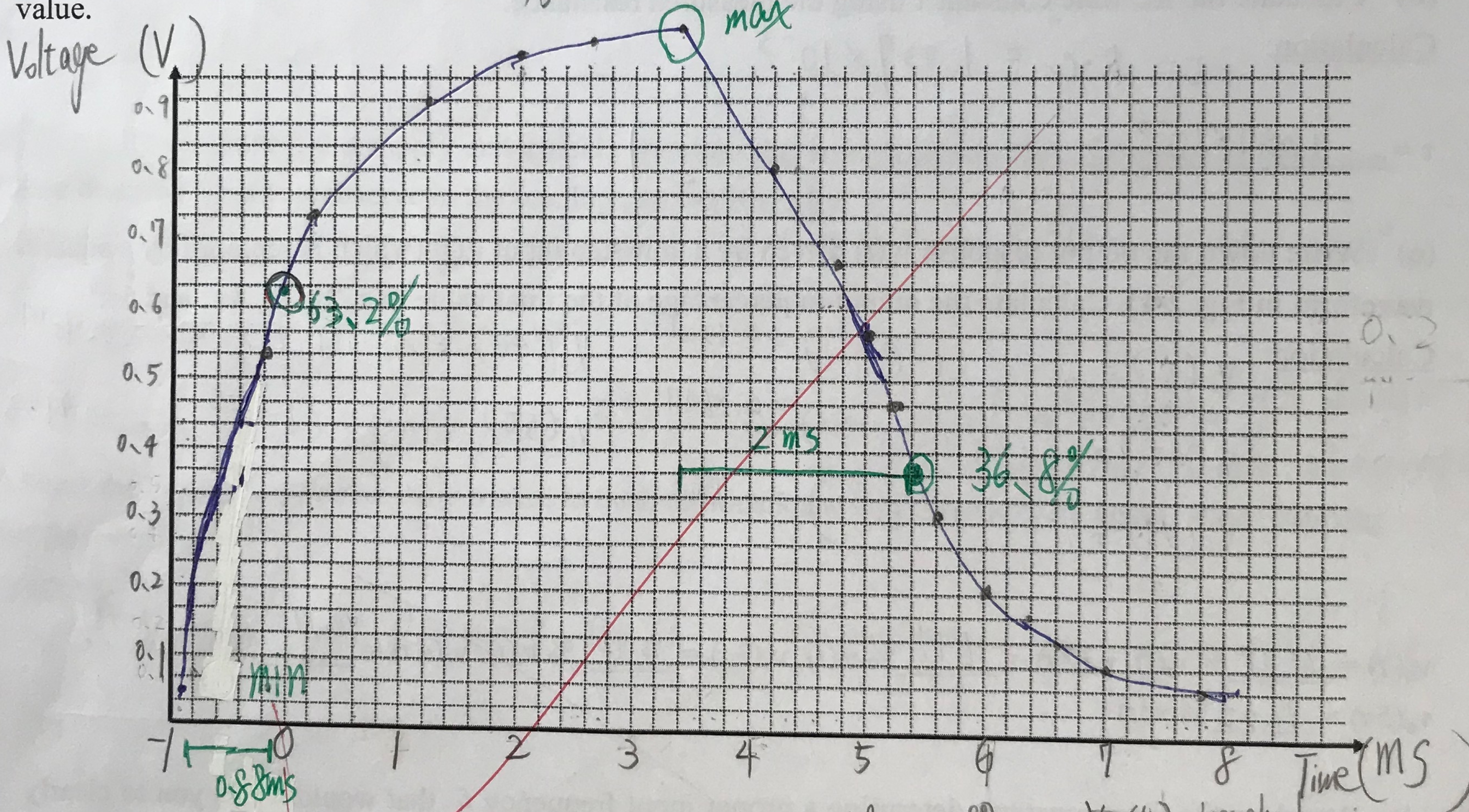
choose $T = 10\tau \Rightarrow f_i = \frac{1}{10\tau} = 96.41$ Hz

(e) Display the input and output signals on an oscilloscope. Record as many data points as possible of the output waveform in one complete charging and discharging cycle and insert them in Table 2-1.

Table 2-1

Time (sec)	$v_o(t)$ (V)	Time (sec)	$v_o(t)$ (V)	Time (sec)	$v_o(t)$ (V)
-950 μ	0.04	3.4 m	1	6 ms	0.22
-400 μ	0.52	4.4 m	0.8	6.4 m	0.18
200 μ	0.74	4.6 m	0.68	6.6 m	0.14
1.2 m	0.90	5 m	0.5	7 m	0.1
2 m	0.98	5.2 m	0.42	7.2 m	0.1 m
2.6 m S	0.99	5.6 m	0.32	7.8 m	0.06

(f) Plot one period of the output response (charging and discharging) on Graph 2-1. Label the voltage and time at 63.2% of the steady-state value when charging the capacitor, and the voltage and the time at 36.8% of the steady-state value when discharging the capacitor. You must label the unit of time on the x axis. Calculate the capacitance value based on the measured time constant and the measured resistor value.



Graph 2-1

After 0.88 ms $V_o(t)$ reached its 63.2% of max value.

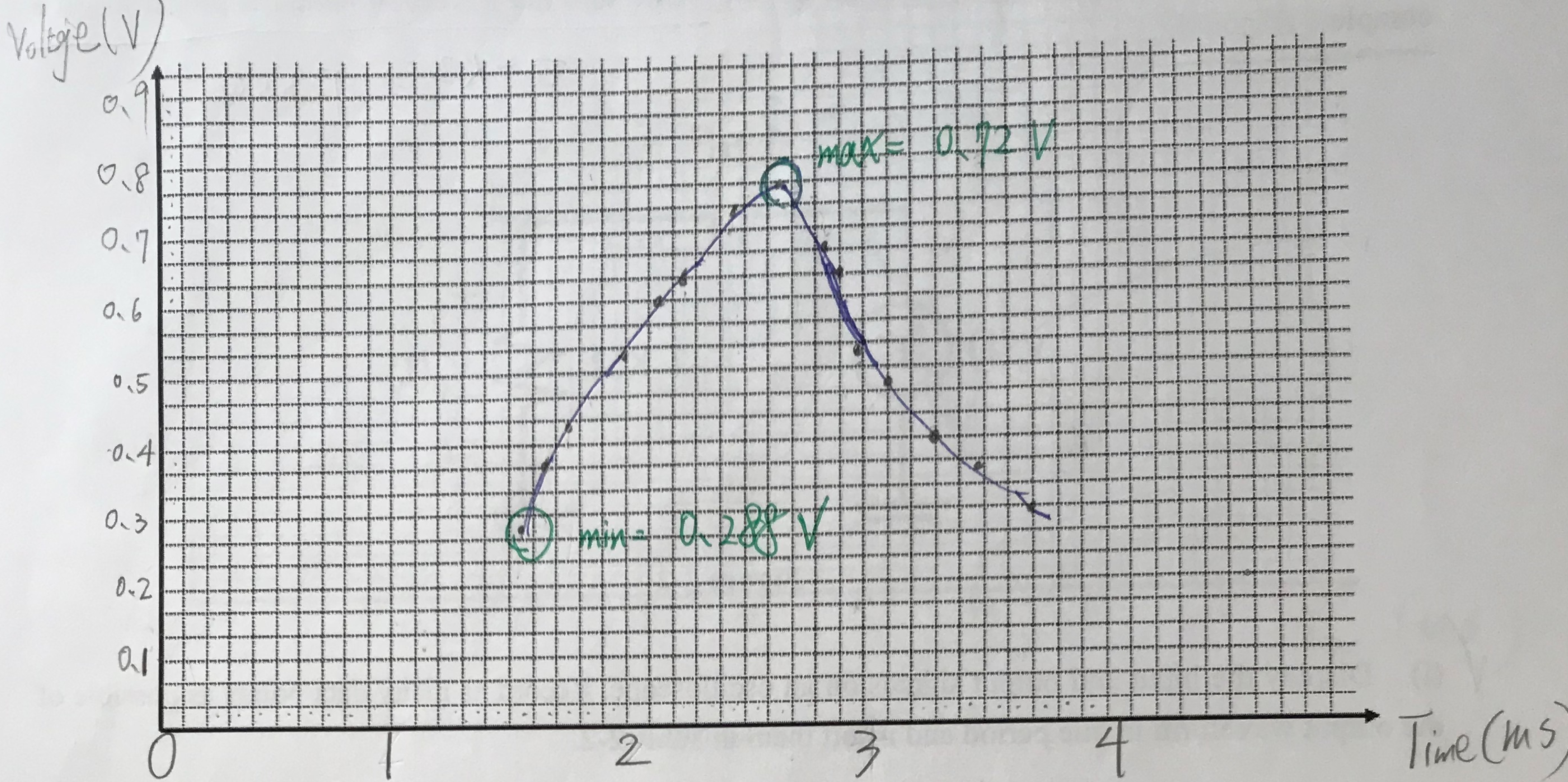
$t = 0.88\,m = 1.0031\,K \cdot C \Rightarrow 0.877\,\mu F$

$C_{measured} =$ 0.877 μF

(g) Repeat part (f) by using an input square waveform with a period of only two RC time constants (T = 2RC). The output would look like a sawtooth waveform. Plot one period of the steady-state output

$f = 482.16$ Hz

waveform on Graph 2-2, and label the maximum and minimum voltage values. Comment on if the voltage levels are reasonable based on your analysis.



Graph 2-2

Analysis & Comment:

√

Time (sec)	$V_o(t)$ (V)
1.56 m	288 m
1.68 m	368 m
1.76 m	416 m
1.8 m	432 m
1.84 m	456 m
2 m	528 m
2.08 m	568 m
2.16 m	600 m
2.24 m	6.24
2.44 m	688 m
2.56 m	720 m
2.72 m	648 m
2.8 m	608 m
2.96 m	520 m
3.12 m	456 m
3.28 m	392 m
3.4 m	360 m
3.6 m	296 m

1. max value is 0.72 V. In theory, $V_c(2\tau)$ = 0.86. It's probably because due to internal resistor inside voltmeter, so there is error

2. Voltage across capacitor didn't reach its original maximum, minimum (1V, 0V). Because period of square wave is too short to charge and discharge capacitor completely

3. min value ≠ 0, reasonable! ∵ Capacitor was not discharged completely

(h) Construct the *RC* circuit of Fig. 2-2. Use the function generator to produce the input square waveform $v_i(t)$ with a minimum value at 0 V and a maximum value at 1 V. Check the input waveform on an oscilloscope before you connect it to the circuit. Make sure the waveform period is proper to get a complete response.

$\tau = RC \Rightarrow T > 5\tau$

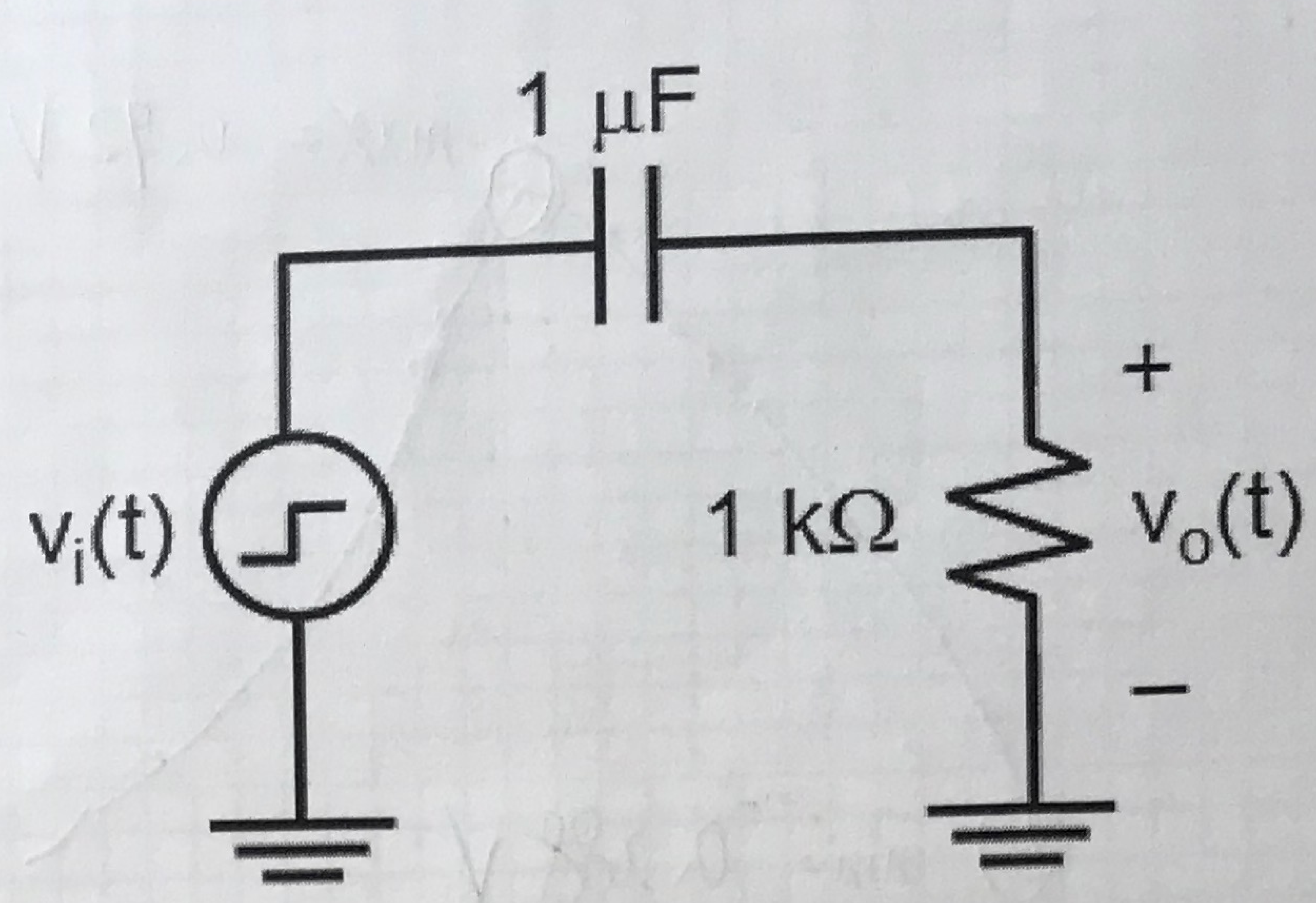


Figure 2-2

(i) Display the input and output signals on an oscilloscope. Record as many data points as possible of the output waveform in one period and insert them in Table 2-2.

Table 2-2

Time (sec)	$v_o(t)$ (V)	Time (sec)	$v_o(t)$ (V)	Time (sec)	$v_o(t)$ (V)
10 ms	0	13.6 m	0.08	16.4 m	−0.4
10.4 m	0.86	13.8	0.06	16.6 m	−0.34
10.6 m	0.72	14.2	0.06	16.8 m	−0.28
10.8 m	0.6	14.4	0.06	17.2 m	−0.18
11.2 m	0.42	15.2	0.04	17.6 m	−0.14
12 m	0.22	15.6	−0.8	18 m	−0.08
13.2 m	0.1	16	−0.58	19.8 m	0

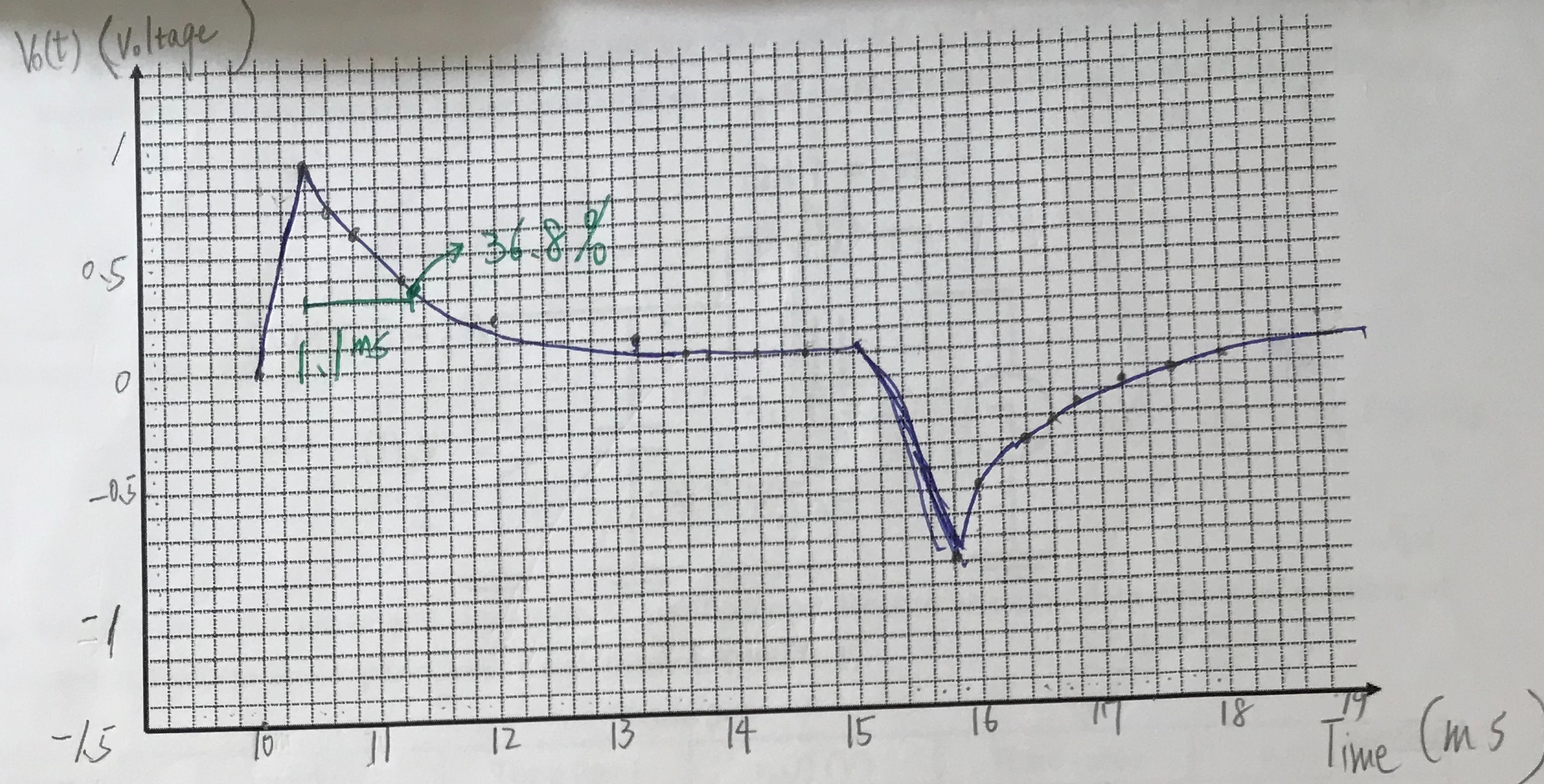
(j) Plot one period of the output response on Graph 2-3 and label the curve. Label the voltage and time at 36.8% of the maximum input value. You must label the unit of time on the *x* axis. Do you think the measured waveform is correct? Is it accurate based on the measured resistance and capacitance values? Comment accordingly.

Comment: $\tau = RC = 1003.1 \times 1.034 \times 10^{-6} = 1.037$

$V_o(t) = e^{-t/\tau} = 0.368 \Rightarrow t_{calculated} = 1.036$ ms

by graph 2.3, $t_{measured} \approx 1.1$ ms 與 $t_{calculated}$ 幾乎相同

Error is $\frac{1.1 - 1.036}{1.036} = 6.18\%$



Graph 2-3

Question: Please derive the analytic expression of the step response for the RC circuit in Fig. 2-3.

$\frac{dV_o(t)}{dt} + \frac{1}{C_2}\left(\frac{1}{R_2}+\frac{1}{R_1}\right)V_o(t) = V_i(t)$

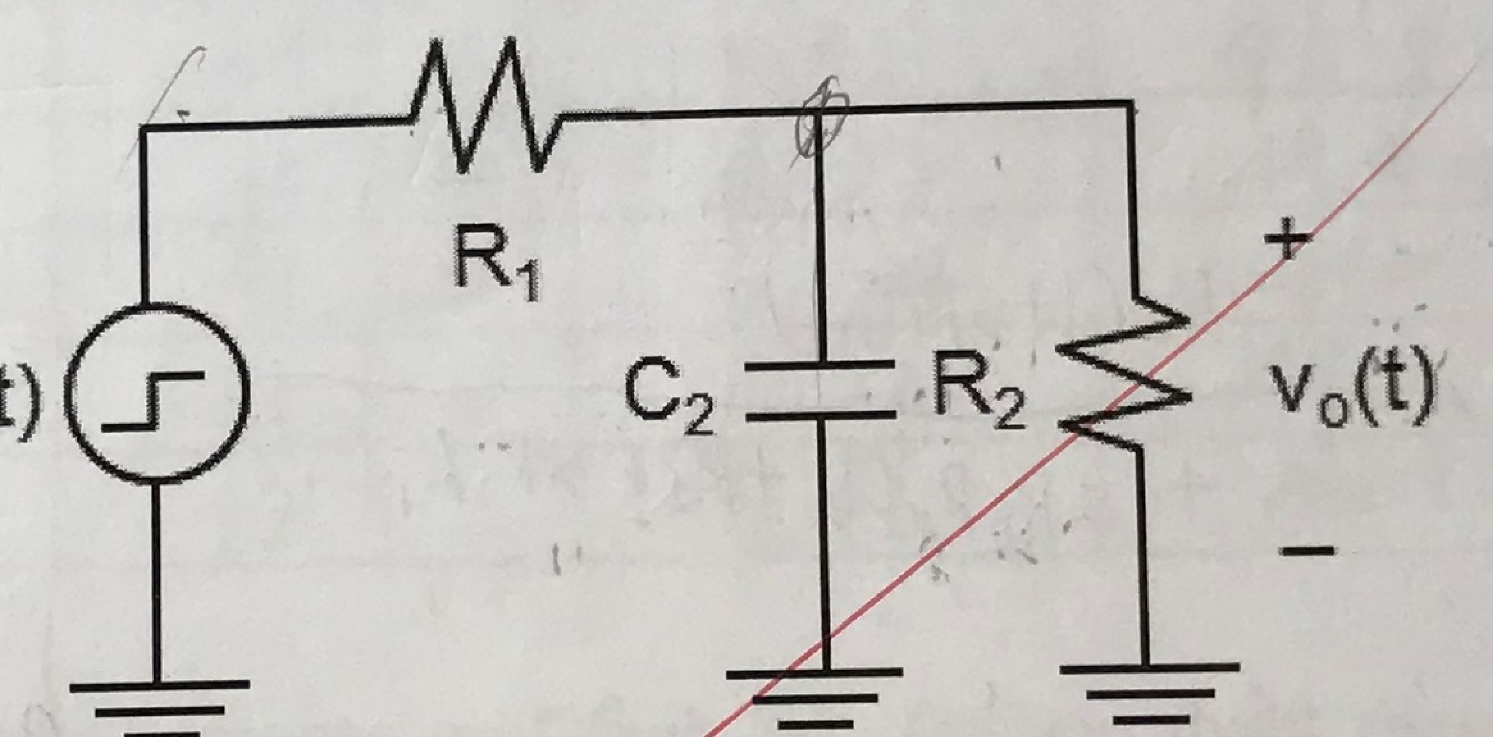


Figure 2-3

Analysis: $\frac{V_i(t)-V_o(t)}{R_1} = C_2\frac{dV_o(t)}{dt} + \frac{V_o(t)}{R_2}$

$\Rightarrow R_1C_2\frac{dV_o(t)}{dt} + \left(\frac{R_1}{R_2}+1\right)V_o(t) = V_i(t)$

$\Rightarrow \frac{dV_o(t)}{dt} + \frac{1}{C_2}\left(\frac{1}{R_2}+\frac{1}{R_1}\right)V_o(t) = \frac{V_i(t)}{C_2}$

$V_{oh}(t)$:

$D + \frac{1}{C_2}\left(\frac{1}{R_2}+\frac{1}{R_1}\right) = 0 \Rightarrow D = -\frac{1}{C_2}\left(\frac{1}{R_2}+\frac{1}{R_1}\right)$

$\therefore V_{oh}(t) = A\cdot e^{-\frac{1}{C_2}\left(\frac{1}{R_2}+\frac{1}{R_1}\right)t}$ (harmogenous)

$V_{op}(t) = \frac{R_2}{R_1+R_2}$ if $V_i(t)$ is a "unit step function"

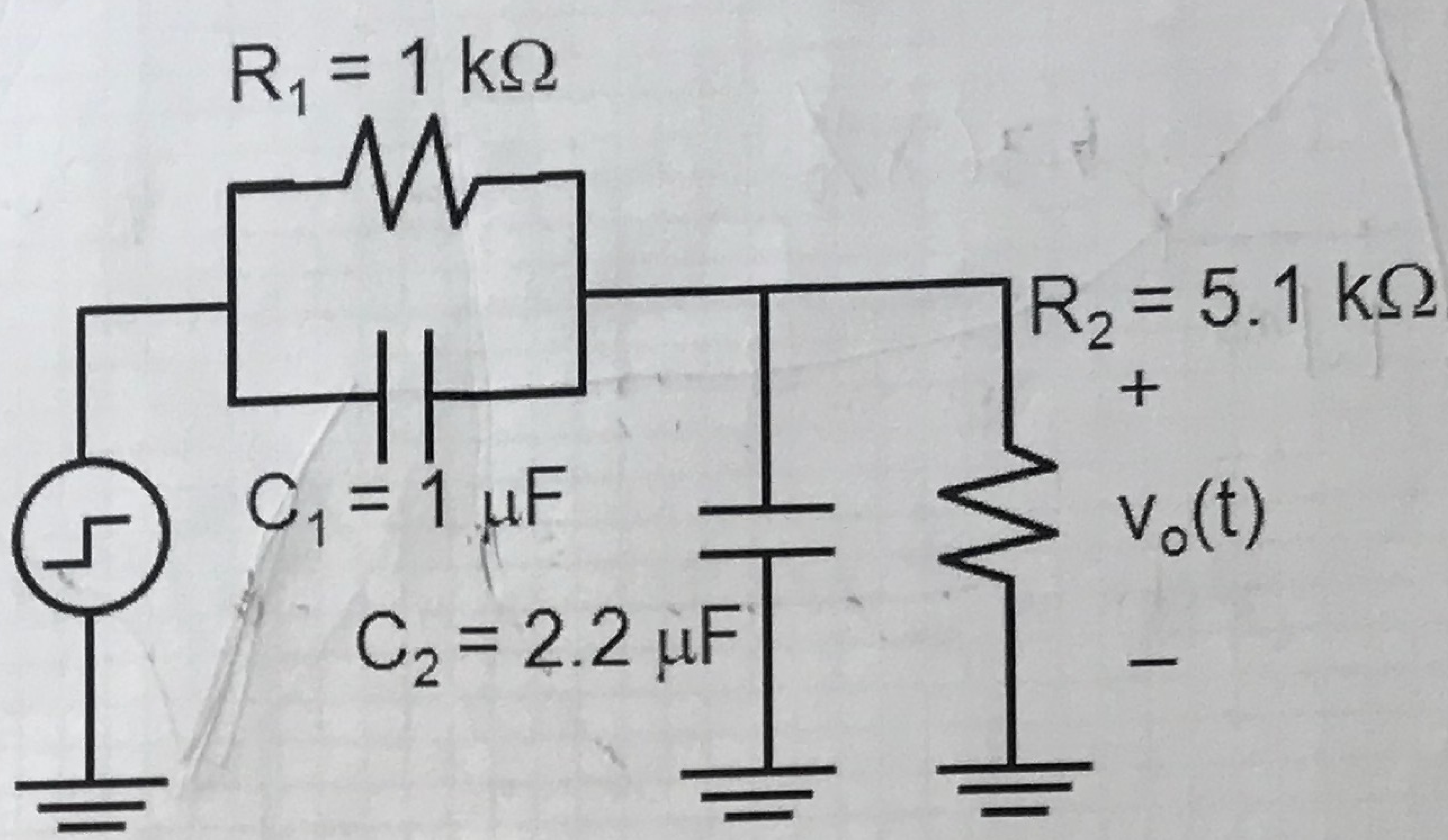
$V_o(t) = A\cdot e^{-\frac{1}{C_2}\left(\frac{1}{R_2}+\frac{1}{R_1}\right)t} + \frac{R_2}{R_1+R_2}$

at $t=0$, $V_o(t)=0 \Rightarrow A = -\frac{R_2}{R_1+R_2}$ $\therefore V_o(t) = \frac{R_2}{R_1+R_2}\left(1-e^{-\frac{1}{C_2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)t}\right)u(t)$

Part 3: Step Response of a RC network

(a) Construct the *RC* circuit of Fig. 3-1. Derive the output waveform $v_o(t)$ due to a unit-step input using values of R_1, C_1, R_2, and C_2 as shown.

$\frac{R_1 +}{R_1 R_2}$



$\frac{1}{R_1} + \frac{SC}{SC_1}$ $\quad \frac{R_1 + SC_1}{R_1 SC_1}$

Figure 3-1

Analysis:

R_1 in parallel with $C_1 \Rightarrow \dfrac{R_1}{1+SR_1C_1}$ (effective resistance)

R_2 in parallel with $C_2 \Rightarrow \dfrac{R_2}{1+SR_2C_2}$ (effective resistance)

$$\mathcal{L}\{V_i(t)\} = \frac{1}{S}$$

$$\Rightarrow V_o(S) = \frac{1}{S} \times \frac{R_2(1+SR_1C_1)}{R_1 + sR_1R_2C_2 + R_2 + sR_2R_1C_1}$$

$$= \frac{5100+5.1S}{6100+16.32S} \times \frac{1}{S} = \frac{a}{6100+16.32S} + \frac{b}{S}$$

$\therefore a = -8.545, \quad b = 0.836$

$$\therefore V_o(S) = \frac{-0.524}{S+374} + \frac{0.836}{S}$$

$$V_o(t) = \mathcal{L}^{-1}\{V_o(s)\} = \frac{51}{61}u(t) - 0.524e^{-374t}u(t)$$

↓ unit step function

$$= (0.836 - 0.524e^{-374t})u(t)$$

通解:

$$V_0(s) = \frac{1}{s} \times \frac{R_2 + R_2R_1C_1S}{(R_1+R_2) + R_1R_2(C_1+C_2)S}$$

$$= \frac{A}{(R_1+R_2) + R_1R_2(C_1+C_2)S} + \frac{B}{S}$$

$$\Rightarrow SA + B(R_1+R_2) + BR_1R_2(C_1+C_2)S = R_2 + R_2R_1C_1S$$

$$\Rightarrow S(A + BR_1R_2(C_1+C_2)) + B(R_1+R_2) = sR_2R_1C_1 + R_2$$

$$\therefore B = \boxed{\frac{R_2}{R_1+R_2}}$$

$$A = R_2R_1C_1 - \frac{R_2}{R_1+R_2} \cdot R_1R_2(C_1+C_2)$$

$$= \frac{R_1^2R_2C_1 - R_2^2R_1C_2}{R_1+R_2}$$

$$= \boxed{\frac{R_1R_2(R_1C_1 - R_2C_2)}{R_1+R_2}}$$

square waveform $v_i(t)$ with a minimum value at 0 V and a maximum value at 1 V. Check the input waveform on a oscilloscope before you connect it to the circuit. What the input frequency f_i of $v_i(t)$ that you pick? Why?

R_1 = 1.0031 kΩ, C_1 = 1.034 μF, R_2 = 5.108 kΩ, C_2= 2.2047 μF

f_i = 36.825 Hz

Comment on the choice of f_i:

$RC = 368.25$

$\tau = \frac{R_1R_2(C_1+C_2)}{R_1+R_2}$ 2.715 ms

$\Rightarrow$ choose $T \geq 5\tau \Rightarrow T = 27.15$ ms $\Rightarrow f = 36.825$ Hz

(enough time to charge capacitor)

(c) Display the input and output signals on an oscilloscope. Record as many data points as possible of the output waveform in one period and insert them in Table 3-1.

Table 3-1

Time (sec)	$v_o(t)$ (V)	Time (sec)	$v_o(t)$ (V)	Time (sec)	$v_o(t)$ (V)
24.8m	0.02	34m	0.84	42.8m	0.14
25.6m	0.08	37.6m	0.86 max	43.6m	0.12
26 m	0.42	38.8m	0.86	44 m	0.1
27.2m	0.58	39.2m	0.5	45.6m	0.06
27.6m	0.62	39.6m 39.91ms	0.42 0.316V	46.8 m	0.04
28.8 m	0.7 m	40.8 m	0.28	47.6m	0.04
29.6m	0.74	41.2m	0.24	48m	0.04
30.8m	0.78	41.6 m	0.22	49.2	0.02
32 m	0.82	42.4	0.16	50.8m	0.02

(d) Plot one period of the output response on Graph 3-1. Do you think the measured waveform is accurate based on the measured resistance and capacitance values? Comment accordingly.

Comment: Use measured R.C 代入通解

$A = -8.572$, $B = 0.836$

$\Rightarrow V_o(s) = \frac{-0.517}{s+368.5} + \frac{0.836}{s}$

$\Rightarrow V_o(t) = (0.836 - 0.517e^{-368.5t})u(t)$

① $V_o(t)_{max} = 0.836$ V

$V_o(t)_{measured,\ max} = 0.86$ V

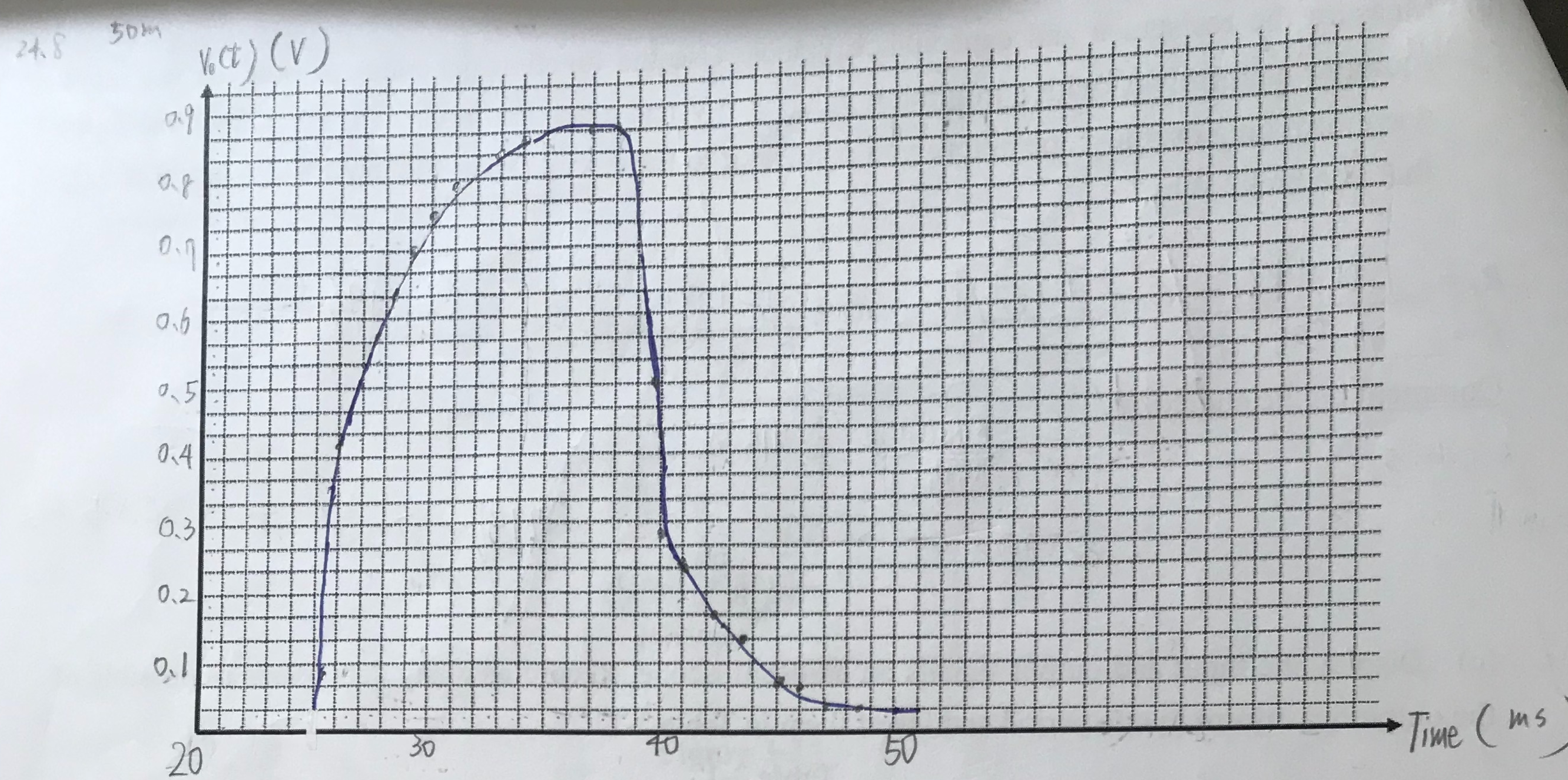
$\Rightarrow$ Error $= \frac{0.86-0.836}{0.86} \approx 2.79\%$

② When $t = \tau = 2.715 \times 10^{-3}$ s

$V_o(\tau) = 0.645$ V $\Rightarrow$ Error $= \frac{0.645-0.62}{0.145} \approx 4.015\%$

measured value = 0.62V

In general, error is pretty small. The results are very consistent!



Graph 3-1

Part 4: Design Problem: Under-Damped Response of the 2nd-Order Series *RLC* Circuit

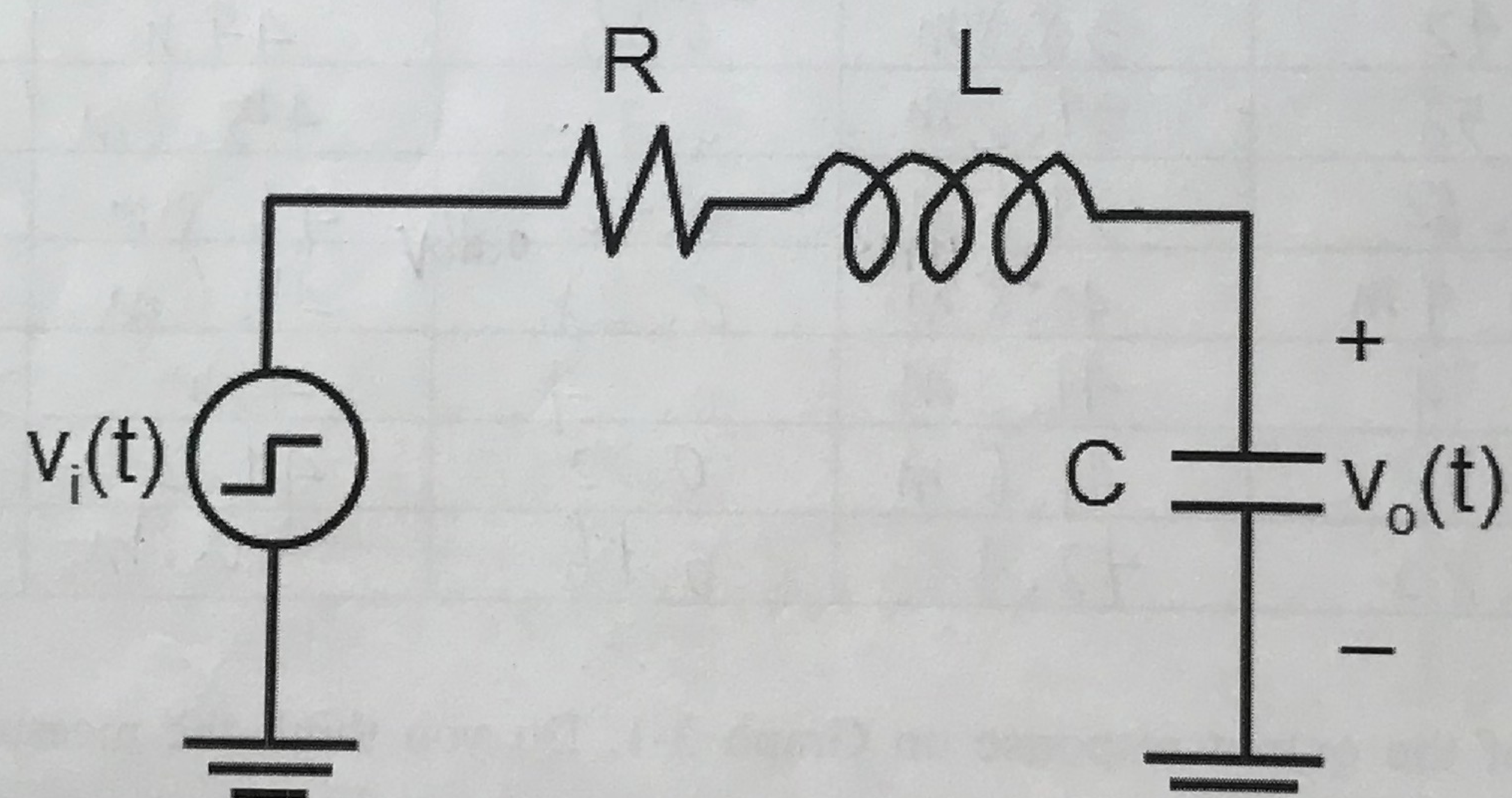


Figure 4-1

(a) A 2nd-order series *RLC* circuit is shown in Fig. 4-1. You are required to select the proper *R*, *L*, and *C* in the circuit to achieve the following design specifications for the output response under a step input (use square waveform with the minimum and maximum values of 0 V and 1 V, respectively).

(1) Rise time = 100 to 110 μs; t_r

(2) Overshoot = 30 to 40% M_p

Show your calculation for the design, and report values of the passive elements. Report the passive elements that you actually use in experiment.

Objective:
$t_r = 100\,\mu s$
$M_p = 35\%$

R_{cal} = 1265 Ω, L_{cal} = 100 mH, and C_{cal} = 25 nF

R_{exp} = 1053 Ω, L_{exp} = 100.1 mH, and C_{exp} = 26.5 nF

Calculation:

$$\boxed{M_p = e^{-\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)\pi}} \qquad \boxed{t_r = \frac{\tan^{-1}\left(\frac{-\sqrt{1-\zeta^2}}{\zeta}\right)}{W_d}}$$

$W_n = \frac{1}{\sqrt{LC}}$

$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$

$W_d = W_n\sqrt{1-\zeta^2}$

Let $M_p = 0.35$

$\Rightarrow -\frac{\zeta}{\sqrt{1-\zeta^2}}\pi = \ln(0.35)$

$\Rightarrow \zeta = \frac{|\ln(0.35)|}{\sqrt{[\ln(0.35)]^2 + \pi^2}}$

$= 0.317$

$f = \frac{R}{2L} = 6280$ Hz

$\Rightarrow$ choose $= 628$ Hz square wave frequency

Let $t_r = 100\,\mu s$

$W_d = \frac{\tan^{-1}\left(\frac{-\sqrt{1-\zeta^2}}{\zeta}\right)}{t_r}$ rad ≈ -1.24 rad $\overset{+\pi}{\approx} 1.89$ rad

$= 18932 = W_n\sqrt{1-\zeta^2}$

$\Rightarrow W_n = 19961.5$

$\Rightarrow LC = 2.5 \times 10^{-9}$

choose $L = 100$ mH, $C = 25$ nF

$R = 1265\,\Omega$

(b) Display the input and output signals on an oscilloscope. **Be aware, the function generator has a source impedance of 50 Ω which will affect the total resistance.** Record the data points of the output waveform when the capacitor is charged in Table 4-1. **You must show TA your waveform with the correct rise time and overshoot.**

Table 4-1

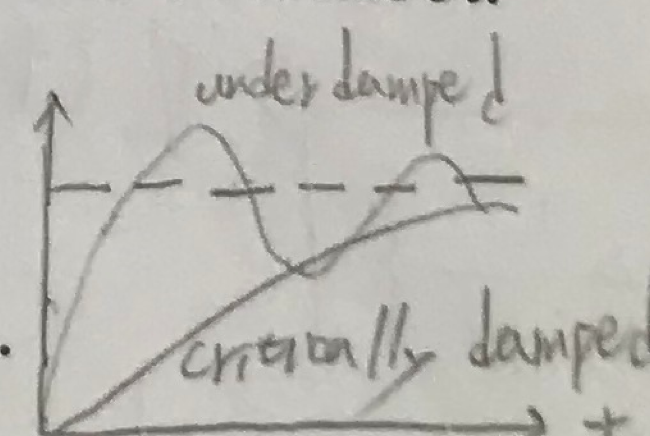
Time (sec)	$v_o(t)$ (V)	Time (sec)	$v_o(t)$ (V)	Time (sec)	$v_o(t)$ (V)
0	0.68	240 μ		960μ	-0.12
10μ	0.8	730 μ		1060 m	0.06
40μ	1.04	750 μ	0.88	1180	0.02
60μ	1.16	790 μ	0.46	1500	0.24
90μ	1.24	820μ	0.14	1540	0.2
150μ	1.18	~~850μ~~	-0.06	1560	0.4
170μ	1.14	920μ	-0.18	1580	0.64
200μ	1.06	950μ	-0.14	1600	0.84

Measured rise time = 101 μs, measured overshoot = 31 %

(c) Change the resistor in the circuit with another value, and record the values of rise time and overshoot. You must make sure the output is underdamped. $\zeta < 1$

R_{exp} = 1.800 kΩ, rise time = 64 μs, overshoot = 18 %

Please analyze whether the values of rise time and overshoot are reasonable based on R_{exp}.



$\frac{0.18}{1}$

Analysis:

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}} = \frac{1800}{2} \times \sqrt{\frac{36.5\times10^{-9}}{100\times10^{-3}}} = 0.463$$

$$M_p = \exp\left(\frac{-\zeta}{\sqrt{1-\zeta^2}}\pi\right) = 19.37\%, \quad \text{Error is } \frac{19.37-18}{18} = 7.61\%$$

$$\omega_n = \frac{1}{\sqrt{LC}} = 19416 \text{ rad/s}, \quad \omega_d = \omega_n\sqrt{1-\zeta^2} = 17209.5$$

$$t_r = \frac{\tan^{-1}\left(\frac{-\sqrt{1-\zeta^2}}{\zeta}\right)}{\omega_d} = 63.3\ \mu s, \quad \text{Error is } \frac{63.3-64}{64} = -1.09\%$$

Error is pretty small. The results are precise!

$\zeta = 1$

(d) Change the resistor in the circuit until you get a critically damped response (overshoot = 0). Record the resistor value and analyze whether resistor value is reasonable.

R_{exp} = 3.95 Ω

Analysis:

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}} = \frac{R_{cal}}{2}\cdot\sqrt{\frac{26.5\times10^{-9}}{100\times10^{-3}}} = 1 \Rightarrow R_{cal} = 3882\ \Omega$$

$$\text{Error} = \frac{3950-3882}{3882} = 1.75\%$$

Question: Please derive the step response of the RLC circuit in Fig. 4-2.

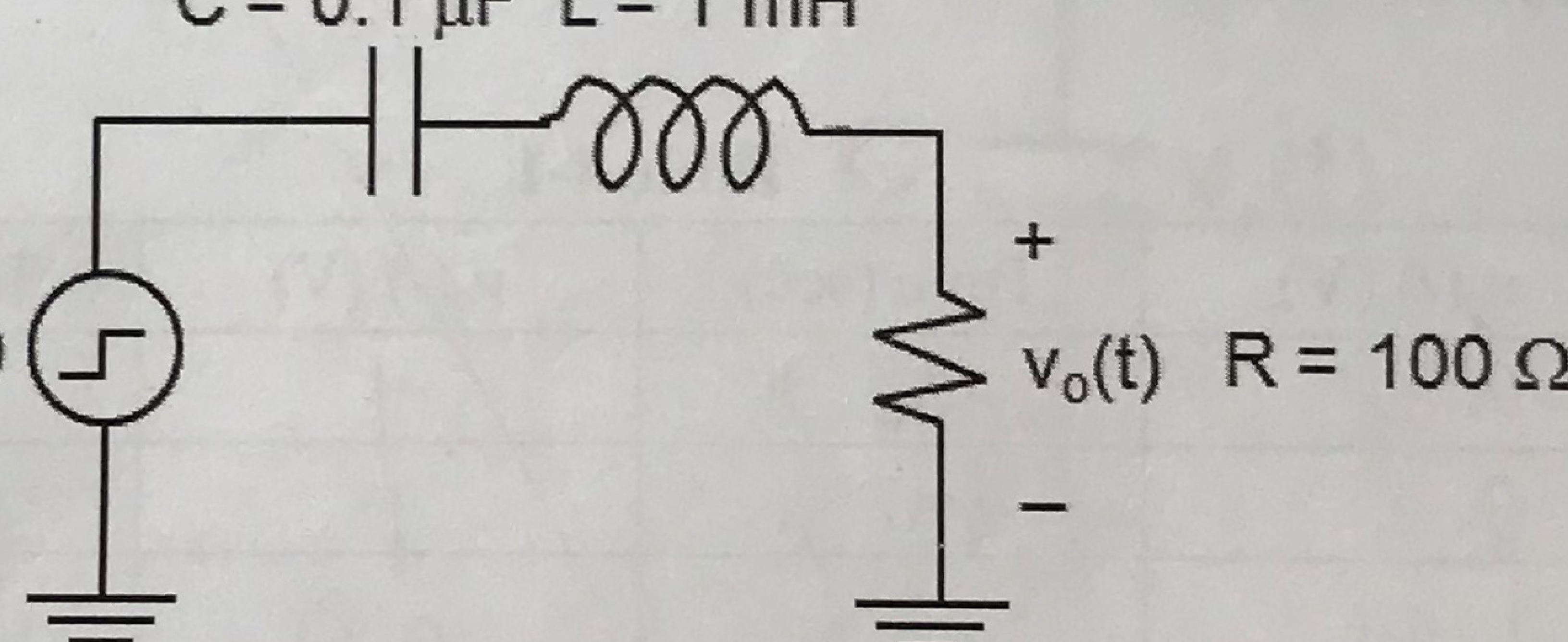


Figure 4-2

Analysis:

$$V_o(s) = \frac{1}{s} \times \frac{R}{R + \frac{1}{sC} + sL} = \frac{R}{s^2L + sR + \frac{1}{C}} = \frac{R}{L} \cdot \frac{1}{s^2 + \frac{sR}{L} + \frac{1}{LC}}$$

$$= 10^5 \cdot \frac{1}{\left(s+\frac{50}{L}\right)^2 + \frac{1}{LC} - \left(\frac{50}{L}\right)^2} = 10^5 \cdot \frac{1}{\sqrt{\frac{1}{LC} - \left(\frac{50}{L}\right)^2}} \cdot \frac{\sqrt{\frac{1}{LC} - \left(\frac{50}{L}\right)^2}}{\left(s+\frac{50}{L}\right)^2 + \left(\sqrt{\frac{1}{LC} - \left(\frac{50}{L}\right)^2}\right)^2}$$

$$\Rightarrow V_o(t) = 10^5 \times \frac{1}{\sqrt{\frac{1}{LC} - \left(\frac{50}{L}\right)^2}}\, e^{-\frac{50}{L}t} \sin\left(\sqrt{\frac{1}{LC} - \left(\frac{50}{L}\right)^2}\, t\right) u(t)$$

$$= 1.15 \times e^{-50000t} \cdot \sin(86602.5t)\, u(t)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-a)^2+b^2}\right\} = \frac{1}{b} e^{at} \sin(bt) \cdot u(t)$$