EE 2240 Basic Circuit Laboratory

Final Examination Solutions

1. (Total 25%)

(a) (8%) For the circuit as shown in Fig. 1(a), please first calculate the value of Thévenin impedance Z_{th} at 10 kHz. Then, please describe the experimental steps for measuring this impedance at 10 kHz.

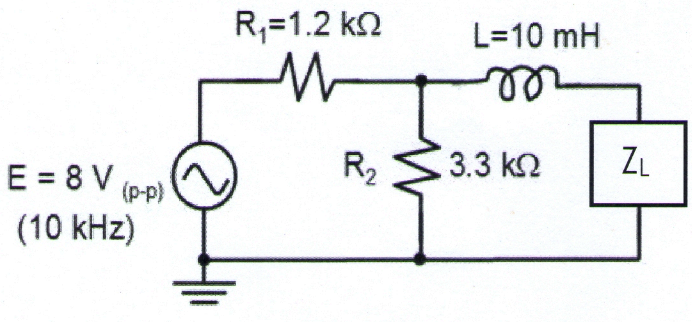


Figure 1(a)

(b) (17%) For the circuit in Fig. 1(b):

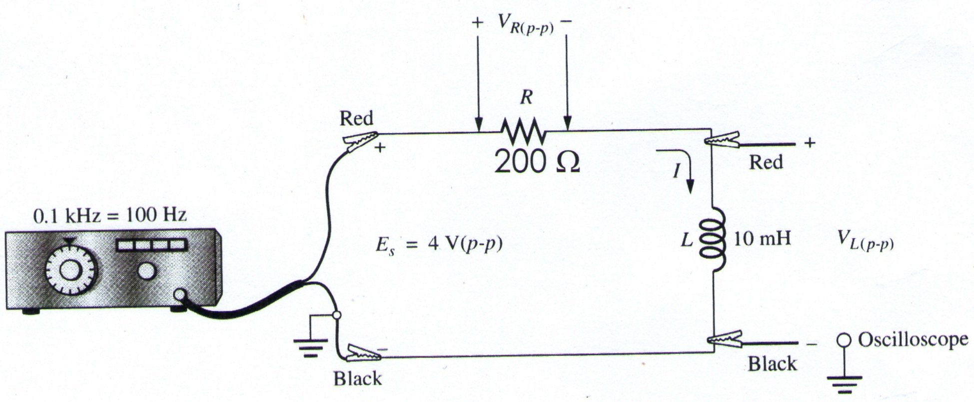


Fig. 1(b)

(b-1) (2%) Explain briefly (just one or two sentences) how you should measure the waveform V_R experimentally, and why.

(b-2) (3%) The measured V_L amplitude at 100 Hz is less than the theory predicts. Please indicate the factor that you should add in your analysis in order to explain the phenomenon.

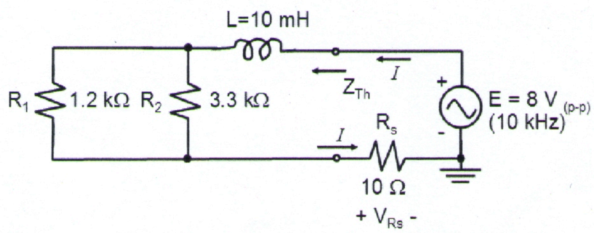
(b-3) (4%) Will the peak-to-peak voltage values satisfy the equation $V_{L(p\text{-}p)} + V_{R(p\text{-}p)} = E_{(p\text{-}p)}$? If not, please give the correct expression. If yes, please give your reasoning.

(b-4) (8%) For the input frequency at 100 Hz and the RL values as shown, please draw the waveforms of V_L and V_R (one period is enough) with respect to time. Please mark their amplitudes, and specifically you should indicate the time difference between the peaks of these two waveforms. What is the phase angle (must specify "+" or "-") of V_L with respect to the input signal at 100 Hz?

Solution:

(a) Zth=(R1//R2)+L=880+628.32j

量測 Zth 的方法可以將電路串聯一個小電阻(此電阻相對 Zth 可忽略)，如下圖所示，如此可由此小電阻得知電路的電流 I，進而由 E/I 求出 Zth.



(b-1) 要量測電阻電壓圖形需將電阻與電感的位置互換因為示波器探棒需與波形產生器之Ground同一點.

(b-2) 未考慮波形產生器內阻.

(b-3) should be $V_{L(p\text{-}p)}^2 + V_{R(p\text{-}p)}^2 = E_{(p\text{-}p)}^2$

(b-4) 輸入訊號為正弦波, Z_T=R+L=200+(2π100)*10*10^{-3}j=200+6.28j

$$V_L = 4\frac{6.28j}{200+j6.28} = 0.062\angle 88.2° \quad \text{(peak-to-peak)}$$

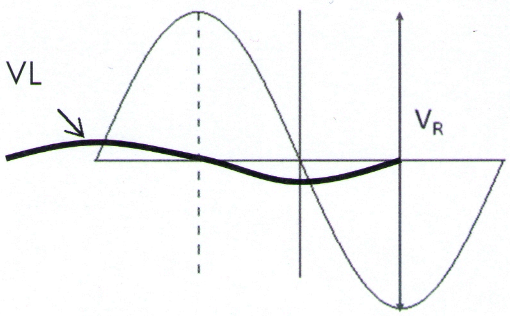
0,1259

$$V_R = 4\frac{200}{200+j6.28} = 3.998\angle -1.8° \quad \text{(peak-to-peak)}$$

$$\frac{4\times 6.28j(200-j6.28)}{40039}$$

$$= \frac{158+5024j}{40039}$$

$$= \frac{5026}{40039}\cdot\frac{158+5024j}{5026} = 0.125\angle 88.2°$$



$$\frac{4\times 200(200-6.28j)}{40039}$$

$$= \frac{160000-5024j}{40039}$$

$$= \frac{160078}{40039}\cdot\frac{160000-5024j}{160078}$$

$$= -1.79\angle -1.8°$$

電感上波形 V_L 領先 V_R 90°， $\Delta t = \frac{90°}{360°}\cdot T = \frac{90°}{360°}\cdot\frac{1}{100} = 2.5\ ms$

2. (Total 20%)

(a) (14%) For the R-L-C circuit as shown in Fig. 2, $v_i(t)$ is a unit-step input, and the value of R and L are 100 Ω and 10 mH, respectively. The output response $v_o(t)$ has an overshoot of 44.4%. Please determine the capacitance value, damping ratio, natural frequency (must specify the unit), and rise time of the output.

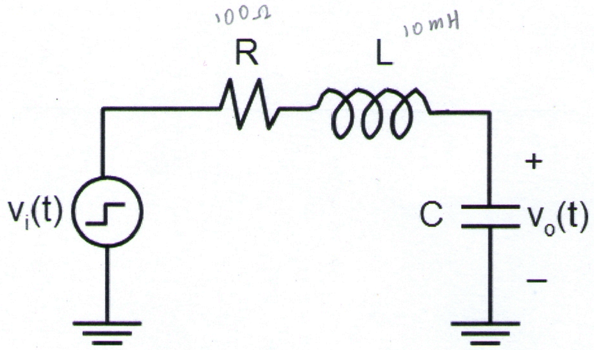


Figure 2

Q
(b) (6%) Express the quality factor of this RLC resonant circuit simply by using the symbols R, L, and C. Explain what is the meaning of quality factor from the energy perspective.

Solution:

(a)

$$M_p = e^{-\left[\frac{\varsigma}{\sqrt{1-\varsigma^2}}\right]\pi} \qquad \text{取 } \ln \qquad (1)$$

將 M_p=44.4%帶入(1)式得：ζ=0.25

$$\varsigma = \frac{R}{2}\sqrt{\frac{C}{L}} \qquad (2)$$

將 R=100Ω、L=10mH、ζ=0.25 帶入(2)式得：C=250 nF

$$\omega_n = \frac{1}{\sqrt{LC}} = 20000 \text{ rad/sec} = 3183 \text{ Hz}$$

$$t_r = \frac{\tan^{-1}\left(-\sqrt{1-\varsigma^2}/\varsigma\right)}{\omega_d}, \quad \omega_d = \omega_n\sqrt{1-\varsigma^2}$$

得：t_r=94 μsec

Ans : The Capacitance Value = <u>250 nF</u>

Damping Ratio = <u>0.25</u>

Natural Frequency = <u>3183 Hz</u>

Rise Time = 94 μsec

(b)(6%)

The Quality Factor $Q = \dfrac{1}{2\varsigma} = \dfrac{1}{2\dfrac{R}{2}\sqrt{\dfrac{C}{L}}} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

The meaning of quality factor from the energy perspective is

$$Q \equiv \frac{Energy \quad Stored}{Average \quad Energy \quad Disspated \quad per \quad Radian}$$

3. (Total 30%)

(a) Fig. 3(a) shows the schematic of an active filter which can produce low-pass, band-pass, and high-pass filter.

(a-1) (6%) Please link v_1, v_2, and v_3 correctly with these three different outputs. Briefly explain your reasoning in a few sentences to get full credit.

(a-2) (6%) By using superposition, v_1 can be expressed as $v_1 = Av_i + Bv_2 + Cv_3$. Please just write down A, B, and C.

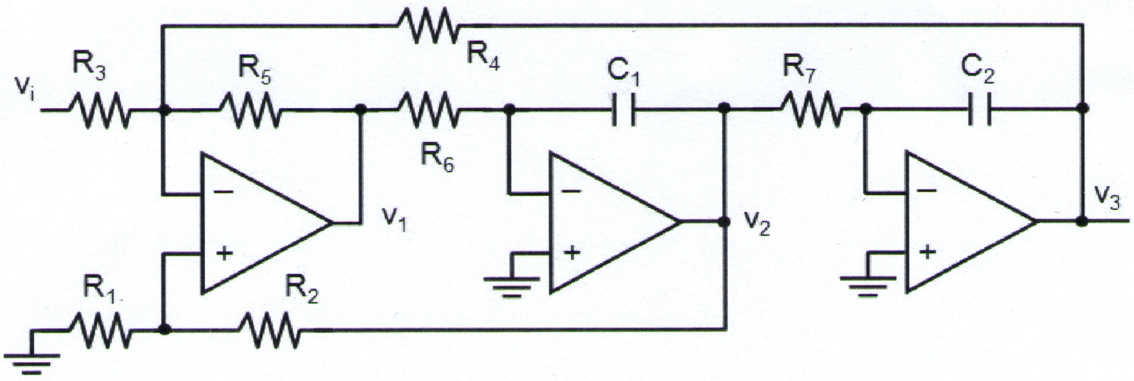


Figure 3(a)

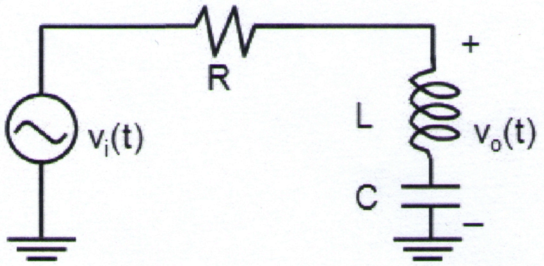
(b) Butterworth filter

(b-1) (5%) The denominator of a third-order Butterworth filter contains a root at the location of

-500 + j 866. Please determine the other two roots.

(b-2) (8%) Determine the order of a low-pass Butterworth filter that has a dc gain of 1, a cut-off frequency of 5 kHz, and a gain of no more than -60 dB at 40 kHz.

(c) (5%) Please determine the filter type (low-pass, high-pass, band-pass, or band-reject) for the circuit shown below. You have to give the correct reasoning to receive full credit.



s^2 s^1 s^0

(a-1) V1 → High-pass, V2 → Band-pass, V3 → Low-pass

V1/Vi has a s^2 term in the numerator, and after one integrator, it becomes s^1 and band-pass; and after the final integrator, it becomes s^0 and low-pass.

(a-2) $A=-\frac{R_5}{R_3}$ $B=-\frac{R_5}{R_4}$ $C=\left(\frac{R_1}{R_1+R_2}\right)\left(1+\frac{R_5}{R_3 // R_4}\right)$

(b-1) The two other poles should be on the left-half s plane; -500-j866、-1000

(b-2) $n=Log_{10}\left[\frac{\sqrt{10^{-0.1A_s}-1}}{\sqrt{10^{-0.1A_p}-1}}\right]\Big/ Log_{10}\left(\frac{\omega_s}{\omega_p}\right), n=Log_{10}\left[\frac{\sqrt{10^{-0.1\times(-60)}-1}}{\sqrt{10^{-0.1\times(-3)}-1}}\right]\Big/ Log_{10}\left(\frac{40}{5}\right)\cong 3.231=4$

(c) vo/vi = 1 at ω = 0 and ∝ since the load impedance of L and C is infinite. vo/vi = 0 at $\omega = 1/(LC)^{0.5}$ because their impedances cancel off.

4. (Total 25%)

(a) For the oscillator as shown in Fig. 4(a):

$\omega = \frac{1}{RC}$

(a-1) (10%) Please derive the oscillation frequency (Note: No credit if only the answer is given). Also, please give the ratio of R_1/R_2 in order to start the oscillation.

(a-2) (3%) Please describe (and draw) the additional circuit technique that we used in our experiment in order to start the oscillation easier and control the oscillation amplitude.

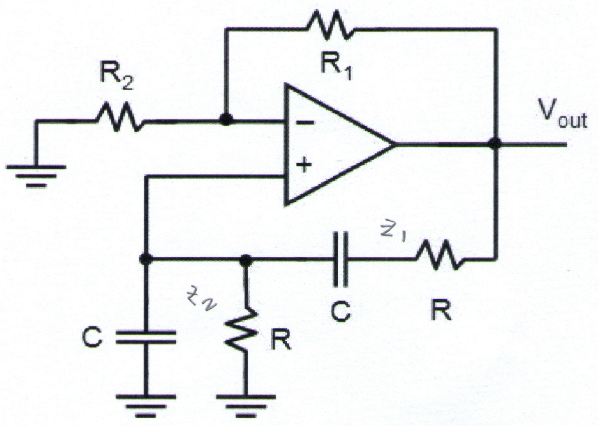


Figure 4(a)

(b) For the triangular and square-wave generator as shown in Fig. 4(b):

(b-1) (7%) Explain the working principles of the circuit. You must plot the waveforms of v_1, v_2, and v_o to receive full credit.

(b-2) (5%) Now given that R = 1 kΩ, C = 1 μF, R_1 = 20 kΩ, R_2 = 10 kΩ, and the output waveform v_o has the maximum and minimum values at 12 V and -12 V, please calculate the oscillation frequency of the output waveform.

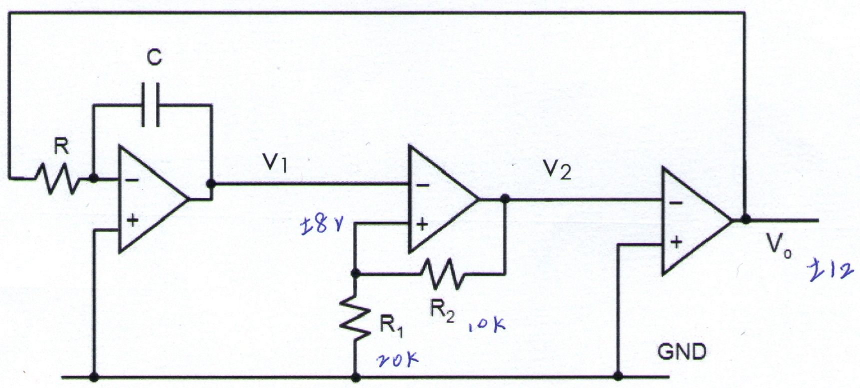
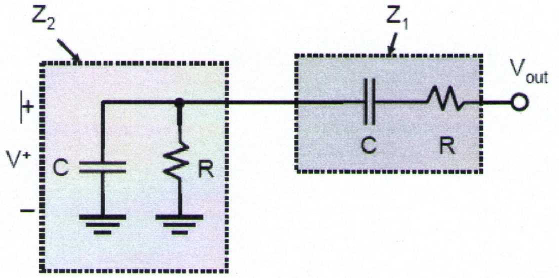


Figure 4(b)

Solution:

(a-1)



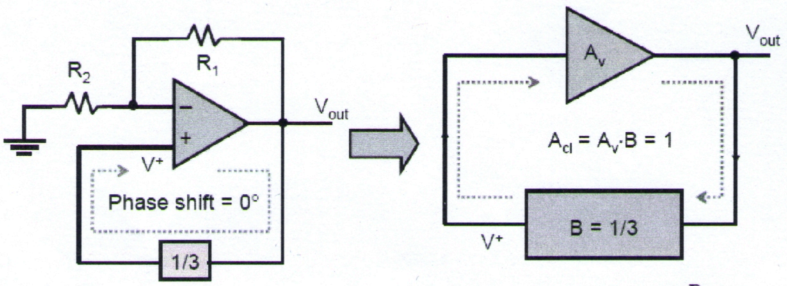
$$\frac{V^+(s)}{V_{out}(s)} = \frac{Z_2}{Z_1 + Z_2} = \frac{sRC}{R^2C^2s^2 + 3RCs + 1}$$

Replacing s = jω: $\dfrac{V^+(j\omega)}{V_{out}(j\omega)} = \dfrac{j\omega RC}{\left(1-\omega^2R^2C^2\right) + j\omega 3RC}$

Remember that the phase shift between V^+ and V_{out} must be zero, therefore:

$$1-\omega^2R^2C^2 = 0, \quad \omega = \frac{1}{RC}$$

$$\frac{V^+}{V_{out}} = \frac{j\omega RC}{j\omega 3RC} = \frac{1}{3}$$

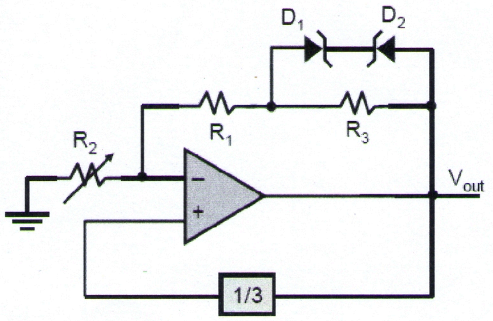


Therefore: $A_v = 3 = 1 + \dfrac{R_1}{R_2}$

$$\therefore R_1 = 2R_2$$

(a-2)

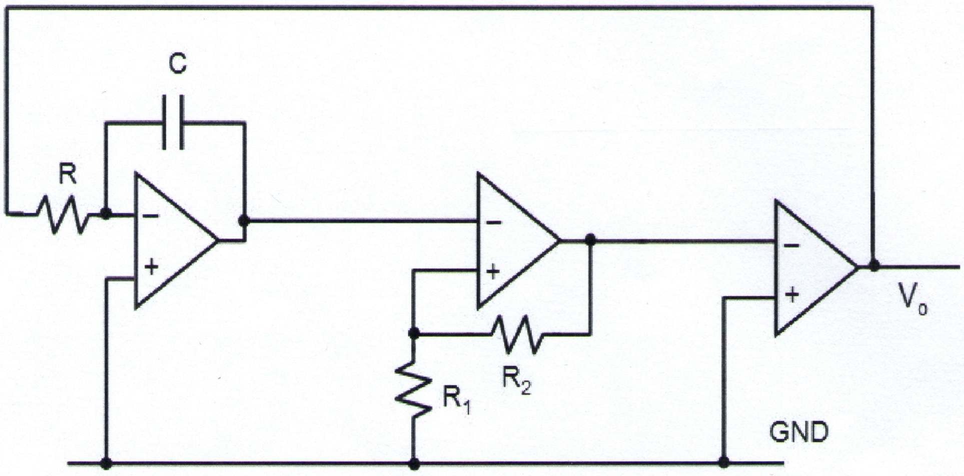
When the output grows and reaches the zener breakdown voltage, the zener diodes conduct and effectively short out R_3, reducing the loop gain



(b) For the triangular and square-wave generator as shown in Fig. 4(b):

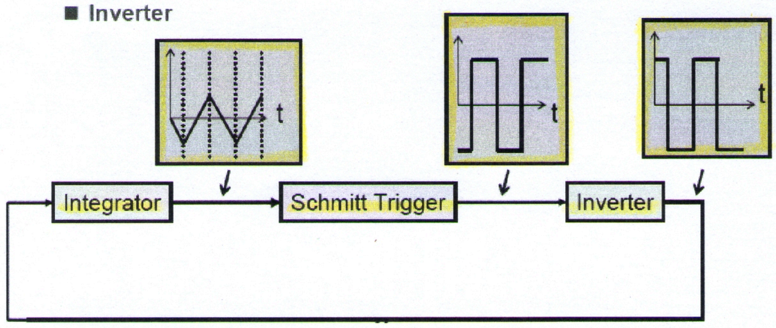
(b-1) (7%) Explain the working principles of the circuit. You must plot the waveforms of v_1, v_2, and v_o to receive full credit.

(b-2) (5%) Now given that R = 1 kΩ, C = 1 μF, R_1 = 20 kΩ, R_2 = 10 kΩ, and the output waveform v_o has the maximum and minimum values at 12 V and -12 V, please calculate the oscillation frequency of the output waveform.



(b-1) The integrator produces the triangular waveform and sends it to the Schmitt trigger for comparison with the values defined by the ratio of R1 over R1+R2. Thus the Schmitt trigger produces a square waveform, which is inverted afterward and sent back to the integrator. The inversion is necessary to establish a sustained oscillation.

- **Integrator**
- **Schmitt trigger**
- **Inverter**



(b-2)

Vo=±12 (v)

In the Schmitt trigger, $V^+ = \dfrac{R1}{R1+R2} Vo = \dfrac{20k}{20k+10k} \pm 12 = \pm 8$ (v)

The output Vo(t) of integrator is confined between +8 and -8 V, so

Vo(t)= $-\frac{1}{RC}Vi(t)$ *t

8x2 = 16 = $-\frac{1}{1kohm*1uF}*-12$ *t , t is one half of the period (Vi = -12 V here)

t=$\frac{16}{12}$ms

f=$\frac{1}{T}=\frac{1}{2t}$= 375 Hz

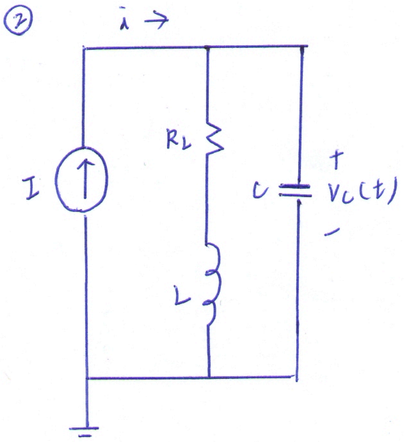
①

(1) Find ΔT of input and output

(2) v_C is lagging

(3) $\theta = -\frac{\Delta T}{T} \cdot 360°$

②



$Q = w_r \cdot \frac{L}{R_L}$ $R_L \downarrow, Q \uparrow$

③

Max power transfer

$R_L = R$, $\omega L = \frac{1}{\omega C}$

④

what type?

(1) transfer function

(2) at $w = 0$

at $w = \infty$

at $w = \frac{1}{\sqrt{LC}}$

⑤

$$\frac{v_2}{v_i} = \frac{1}{C_1 C_2 R_1 R_4} \cdot \frac{1}{s^2 + \frac{1}{C_1 R_2} s + \frac{1}{C_1 C_2 R_4 R_5}}$$

⑥

$$V_{max} = -\frac{R_1}{R_1+R_F} V_{max} + \left[V_{max} - \left(-\frac{R_1}{R_1+R_F} V_{max} \right) \right] \left(1 - e^{-\frac{t}{RC}} \right)$$

T/2

↑

$\frac{R_1}{R_1+R_F} V_{max}$

求T → f.