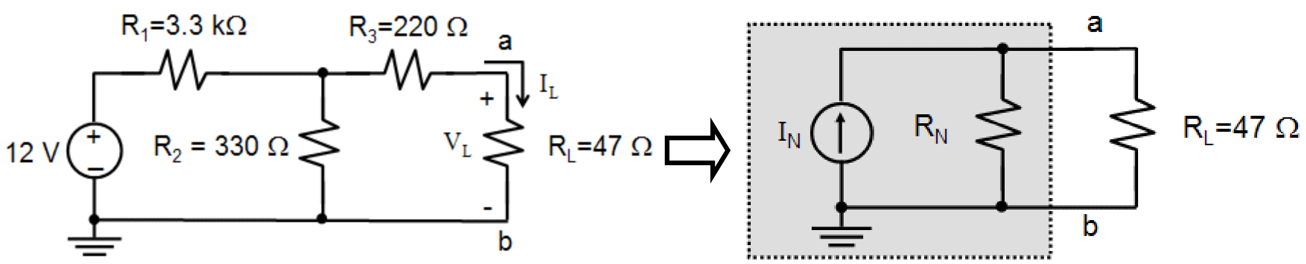
EE 2245 Microelectronics Laboratory

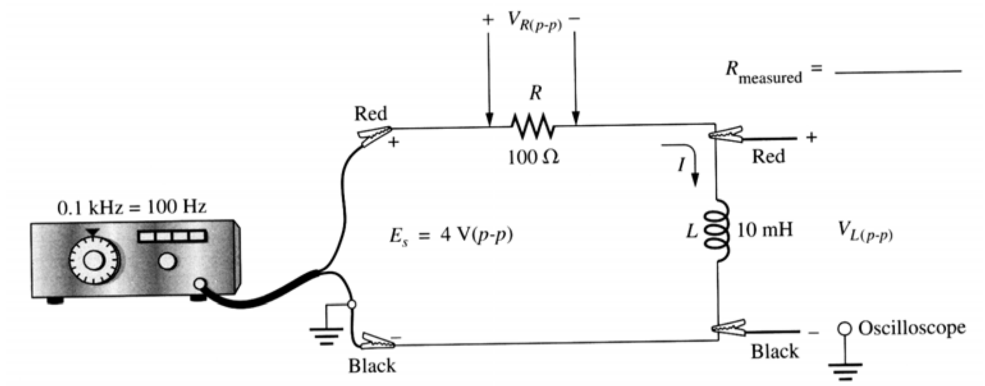
Final Examination (solutions), 6/21/2011

1. (15%)

(a) (8%) For the circuit as shown (left), please calculate the Norton current (I_N) and the Norton resistance (R_N) for the network to the left of the 47-Ω resistor. Please also explain how you would implement the current source I_N?



(b). (7%) For the R-L circuit as shown, there is one thing you must do before measuring the voltage waveform V_R across the resistor by using an oscilloscope. What is that important step? Please describe the reason as well. Is it true that the peak-to-peak voltage values are related to the input voltage by: $V_{R(p\text{-}p)} + V_{L(p\text{-}p)} = E_{(p\text{-}p)}$? If not, please derive the correct relationship.

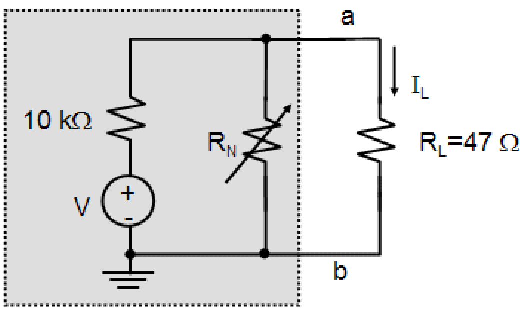


Sol:

(a) $R_{th} = 220 + (3.3k||330) = 520\ \Omega$

$$V_{th} = 12 \cdot \frac{330}{330 + 3.3k} = 1.091\text{V}; I_N = \frac{V_{th}}{R_{th}} = 2.1 \times 10^{-3}\text{A}$$

The current source I_N can be constructed by using a power supply in series with a resistor (the resistance is large) as shown.



(b) The important step: switch the positions of R and L; otherwise the negative end of the measuring probe would short the inductor. R,L 不互換的話，在量測 V_R 時會讓電感兩端短路。

No. It should be: $V_{L(p-p)}^2 + V_{R(p-p)}^2 = E_{p-p}{}^2$.

Reason:

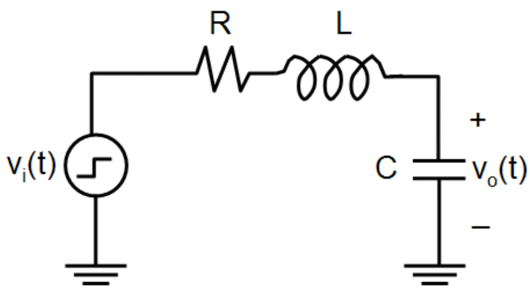
$$V_L(s) = \frac{Ls}{Ls+R}E(s); V_R(s) = \frac{R}{Ls+R}E(s)$$

$$V_L(j\omega) = \frac{jL\omega}{jL\omega+R}E(j\omega); V_R(j\omega) = \frac{R}{jL\omega+R}E(j\omega)$$

$$|V_L(j\omega)| = \frac{\omega L}{\sqrt{\omega^2L^2+R^2}}\cdot|E(j\omega)|; |V_R(j\omega)| = \frac{R}{\sqrt{\omega^2L^2+R^2}}\cdot|E(j\omega)|$$

So $V_{L(p-p)}^2 + V_{R(p-p)}^2 = E_{p-p}{}^2$

2. (15%) For the R-L-C circuit as shown, the input $v_i(t)$ is a unit-step, and R = 5 Ω, L = 1 H, and C = 10 mF. Assume the initial condition $v_o(0) = 0$, please calculate the output response $v_o(t)$. In addition, please calculate the rise time and overshoot in the step response.



Sol:

$$\frac{v_o(s)}{v_i(s)} = \frac{\frac{1}{sC}}{R+sL+\frac{1}{sC}} = \frac{100}{s^2+5s+100}, \text{so } \omega_n = 10, \xi = 0.25$$

$$v_o(s)=\frac{100}{s^2+5s+100}v_i(s)=\frac{100}{s^2+5s+100}\frac{1}{s}=\frac{1}{s}-\frac{s+5}{s^2+5s+100}$$

$$=\frac{1}{s}-\frac{s+\frac{5}{2}}{\left(s+\frac{5}{2}\right)^2+\left(\frac{5\sqrt{15}}{2}\right)^2}-\frac{\frac{5}{2}}{\left(s+\frac{5}{2}\right)^2+\left(\frac{5\sqrt{15}}{2}\right)^2}$$

$$\therefore v_o(t)=u(t)-e^{-2.5t}\cos\left(\frac{5\sqrt{15}}{2}t\right)-\frac{1}{\sqrt{15}}e^{-2.5t}\sin\left(\frac{5\sqrt{15}}{2}t\right)$$

Rise time = $\tan^{-1}\left(-\sqrt{1-\xi^2}/\xi\right)/\left(\omega_n\sqrt{1-\xi^2}\right)=\dfrac{1.82347}{\frac{5\sqrt{15}}{2}}=0.188$ sec.

Overshoot = $\exp\left(-\xi\pi/\sqrt{1-\xi^2}\right)=0.444=44.4\%$

3. (20%)

(a) (8%) Given a 2nd-order RLC circuit whose transfer function is $H(s)=\dfrac{25}{s^2+s+25}$, please calculate the magnitudes and phases at frequencies ω = 0, $\propto$, and the natural frequency.

(b) (6%) Please use one resistor (R), one capacitor (C), and one inductor (L) to implement a passive band-reject filter. You must derive the transfer function between the input and output.

(c) (6%) Please use passive elements to implement a high-pass filter. You must derive the transfer function between the input and output.

Sol.:

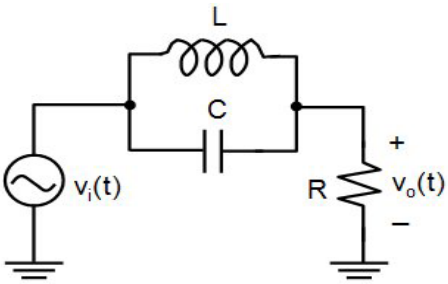
(a) $H(j\omega)=\dfrac{25}{(25-\omega^2)+j\omega}, |H(j\omega)|=\dfrac{25}{\sqrt{(25-\omega^2)^2+\omega^2}}, \angle H(j\omega)=-\tan^{-1}\left(\dfrac{\omega}{25-\omega^2}\right)$

ω= 0, $|H(j\omega)|=1, \angle H(j\omega)=0°$

$\omega=\omega_n$=5, $|H(j\omega)|=5, \angle H(j\omega)=-90°$

$\omega=\infty$, $|H(j\omega)|=0, \angle H(j\omega)=-180°$

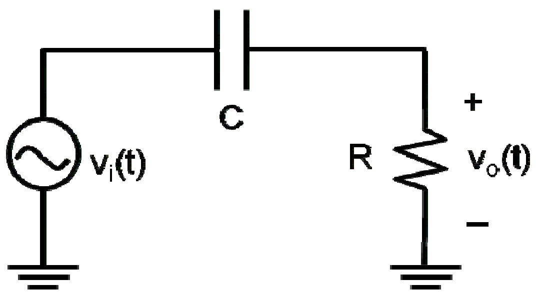
(b)Band-reject:



$$\frac{v_o(s)}{v_i(s)} = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

At ω = 0, $\left|\frac{v_O(j\omega)}{v_i(j\omega)}\right| = 1$. At ω = ∝, $\left|\frac{v_O(j\omega)}{v_i(j\omega)}\right| = 1$. At $\omega = 1/\sqrt{LC}$, $\left|\frac{v_O(j\omega)}{v_i(j\omega)}\right| = 0$. Thus it is a band-reject filter.

(c)

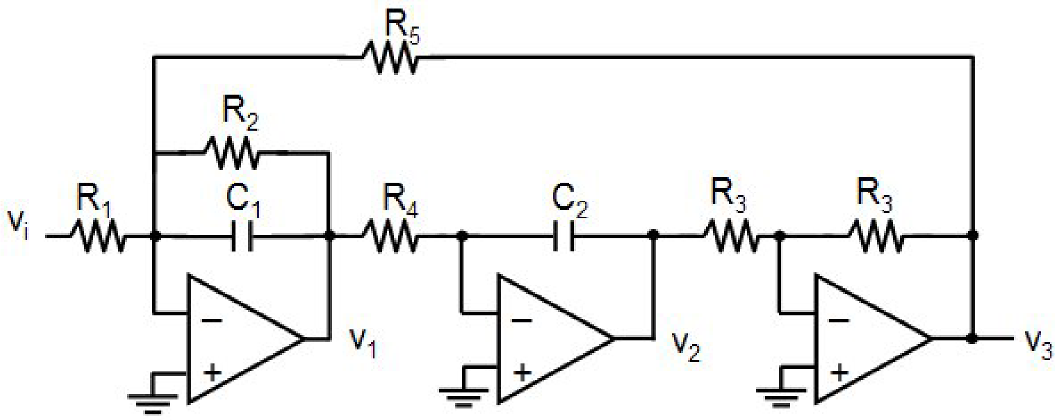


$$\frac{v_o(s)}{v_i(s)} = \frac{R}{R + \frac{1}{sC}} = \frac{sRC}{sRC + 1}$$

4.(15%)

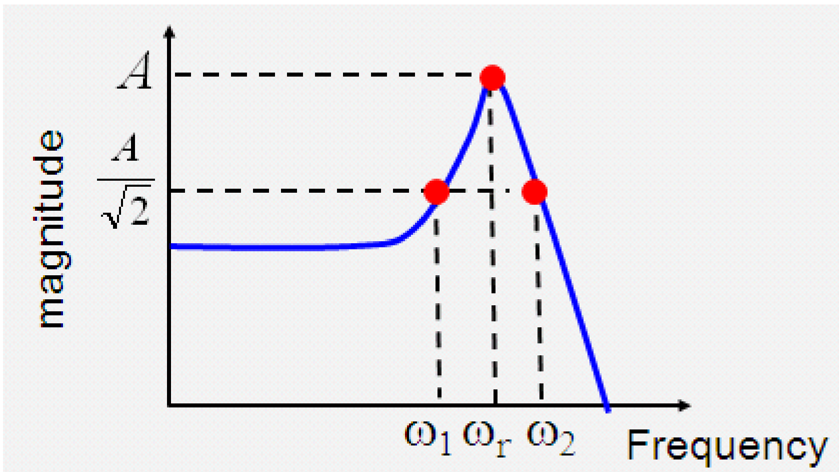
(a) (5%) How to find the quality factor of a circuit from the measured frequency response?

(b) (10%) Find the transfer function $v_2(s)/v_i(s)$ of the biquad filter as shown.



Sol:

(a) From the measured frequency response as shown: $Q = \frac{\omega_r}{\omega_2 - \omega_1}$

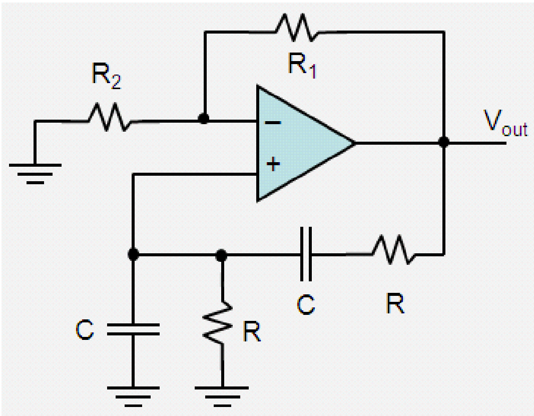


(b) $\frac{v_i}{R_1} + \frac{v_3}{R_5} + \frac{v_1}{R_2} + \frac{v_1}{\frac{1}{sC_1}} = 0$ (1), $v_2 = -\frac{1}{sR_4C_2} v_1$ (2), $v_3 = -v_2$ (3)

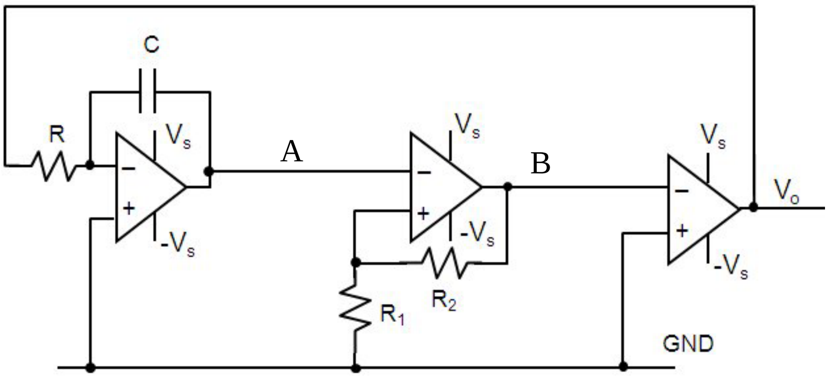
Solve (1),(2) and (3), $\frac{v_2(s)}{v_i(s)} = \frac{1}{C_1C_2R_1R_4} \frac{1}{s^2 + \frac{1}{R_2C_1}s + \frac{1}{C_1C_2R_4R_5}}$

5. (20%)

(a) (10%) For the oscillator circuit as shown, please derive the oscillation frequency and determine the ratio of R_1/R_2 in order to start the oscillation.



(b) (10%) For the triangular, square-wave oscillator as show: $R_1 = 4R_2$, R = 1 kΩ, C = 0.1 μF, and the output V_o oscillates between ±10 V. Please point out where the triangular wave and square wave are produced, respectively (namely, point A or point B). Also, please calculate the oscillation frequency of the circuit.



Sol:

(a) $$\frac{v^+(s)}{v_{out}(s)} = \frac{R \,||\, \frac{1}{sC}}{\left(R + \frac{1}{sC}\right) + \left(R \,||\, \frac{1}{sC}\right)} = \frac{sRC}{R^2C^2s^2 + 3RCs + 1}$$

$$\frac{v^+(j\omega)}{v_{out}(j\omega)} = \frac{j\omega RC}{\left(1 - \omega^2 R^2 C^2\right) + j\omega 3RC}$$. when at resonant frequency $\omega_r = 1/(RC)$

$$\left|\frac{v^+(j\omega)}{v_{out}(j\omega)}\right| = \frac{1}{3}$$, Then $(1 + R_1/R_2) = 3$ to start the oscillation. $R_1/R_2 = 2$.

(b) Point A: triangular wave

Point B: square waveform

The square waveform oscillates between ±10 V, so the triangular wave oscillates between ±8 V (±10 ·R1/(R1+R2)= ±10x0.8 = ±8)

So for the integrator on the left: for Vo = 10 V

$$V_o \frac{1}{RC} \cdot \Delta t = 10 \cdot \frac{1}{1k\Omega \cdot 0.1\mu F} \Delta t = 8\text{ V} - (-8\text{ V}) = 16\text{ V}$$

$\Rightarrow \Delta t = 0.00016$ sec.

The complete period T = 2Δt = 0.00032 (after considering Vo = -10 V)

Frequency = 1/T = 3125 Hz

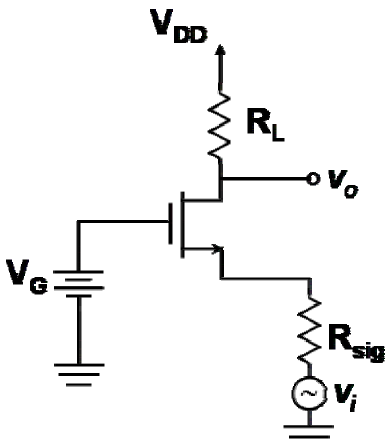
6. (15%)

(a) (6%) Please draw the schematic of a common-gate amplifier and compare its small-signal gain to that of a common-source amplifier. Which gain is larger? Which amplifier has a smaller input impedance? Please explain.

(b) (9%) Please draw the schematic of a cascode amplifier using n-type MOS transistors and a load resistor R_L. Please explain how to make sure the transistors are operated in the saturation region in terms of the gate-to-source voltage (V_{GS}), drain-to-source voltage (V_{DS}), and the threshold voltage (V_T).

Sol.:

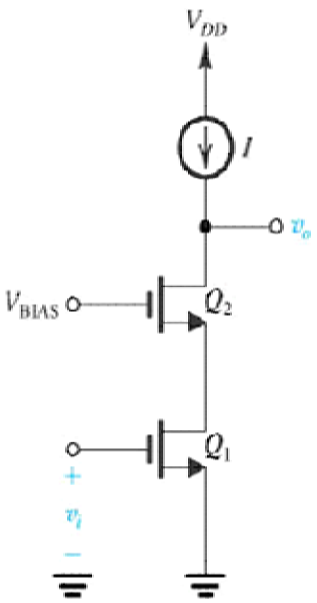
(a) The common-gate amplifier:



Compared to the common-source amplifier, CG's overall voltage gain is smaller by a factor of about $1+g_mR_{sig}$.

The input impedance of CG is around 1/gm, while that of CS is very large (since the gate current is close to zero due to the gate oxide)

(b)



For operation in the saturation region:

$V_{DS,Q1} \geq V_{GS,Q1} - V_T > 0$; $V_{DS,Q2} \geq V_{GS,Q2} - V_T > 0$