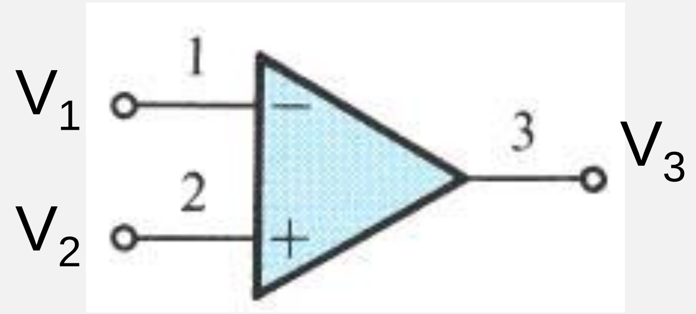
Lab 4: Active Filters

- Operational amplifier (Op-amp)
- Quality factor
- First-order active filters
- Second-order active filters
- Design problem for Electrocardiogram (ECG)
- Implementation using the LM348 and TLC2264 chips

The Ideal Op-amp

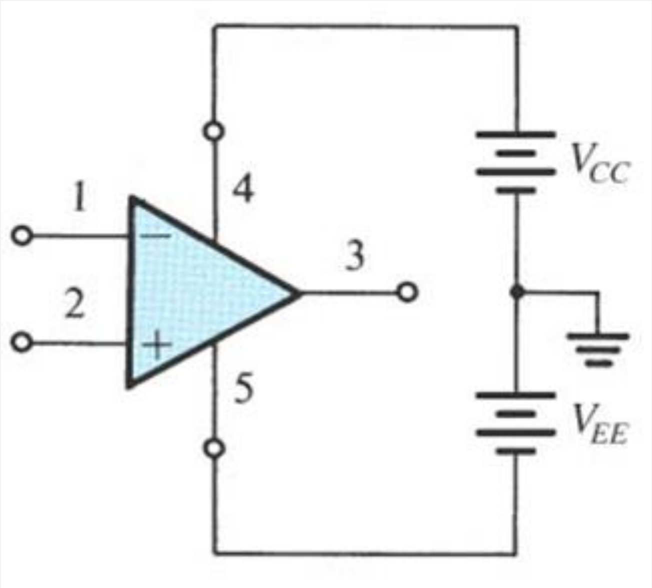
- An op-amp has
  - Differential inputs: V_1 and V_2
  - Single-ended output : V_3
  - So:

$$V_3 = A \cdot (V_2 - V_1)$$



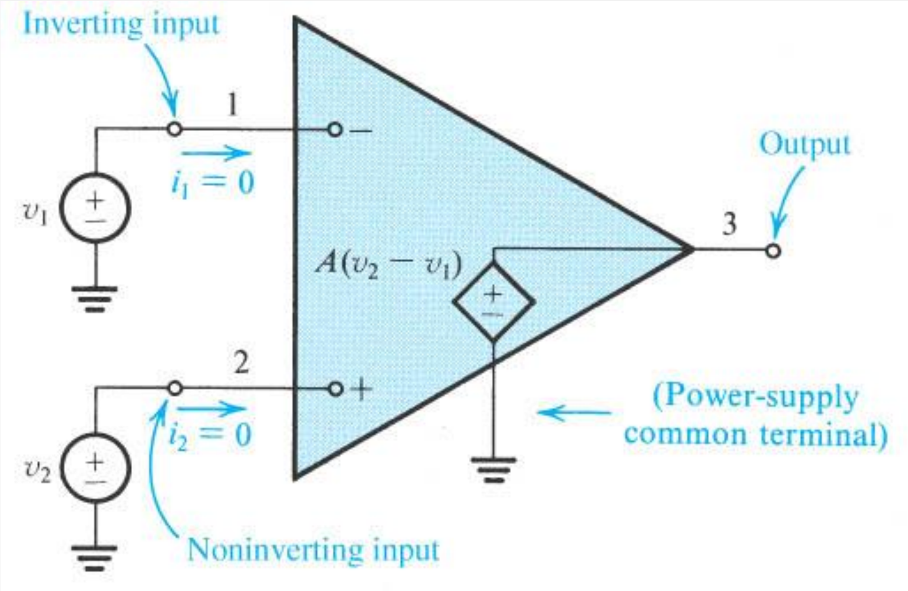
Gain = *A*

- For an ideal op-amp: $A = \infty$ ($A = 10^3$ to 10^5 for typical op-amps)
  - What is the large gain for?
- An op-amp needs DC power supplies, either positive V_{CC} and negative V_{EE}, or V_{dd} and 0

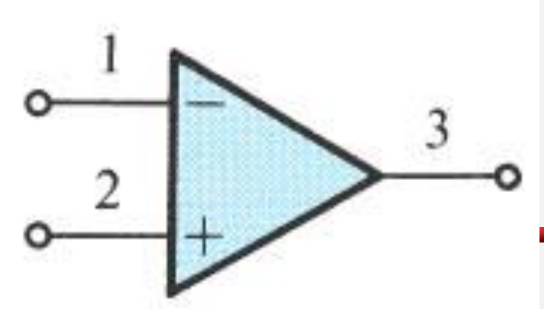


The Ideal Op-amp

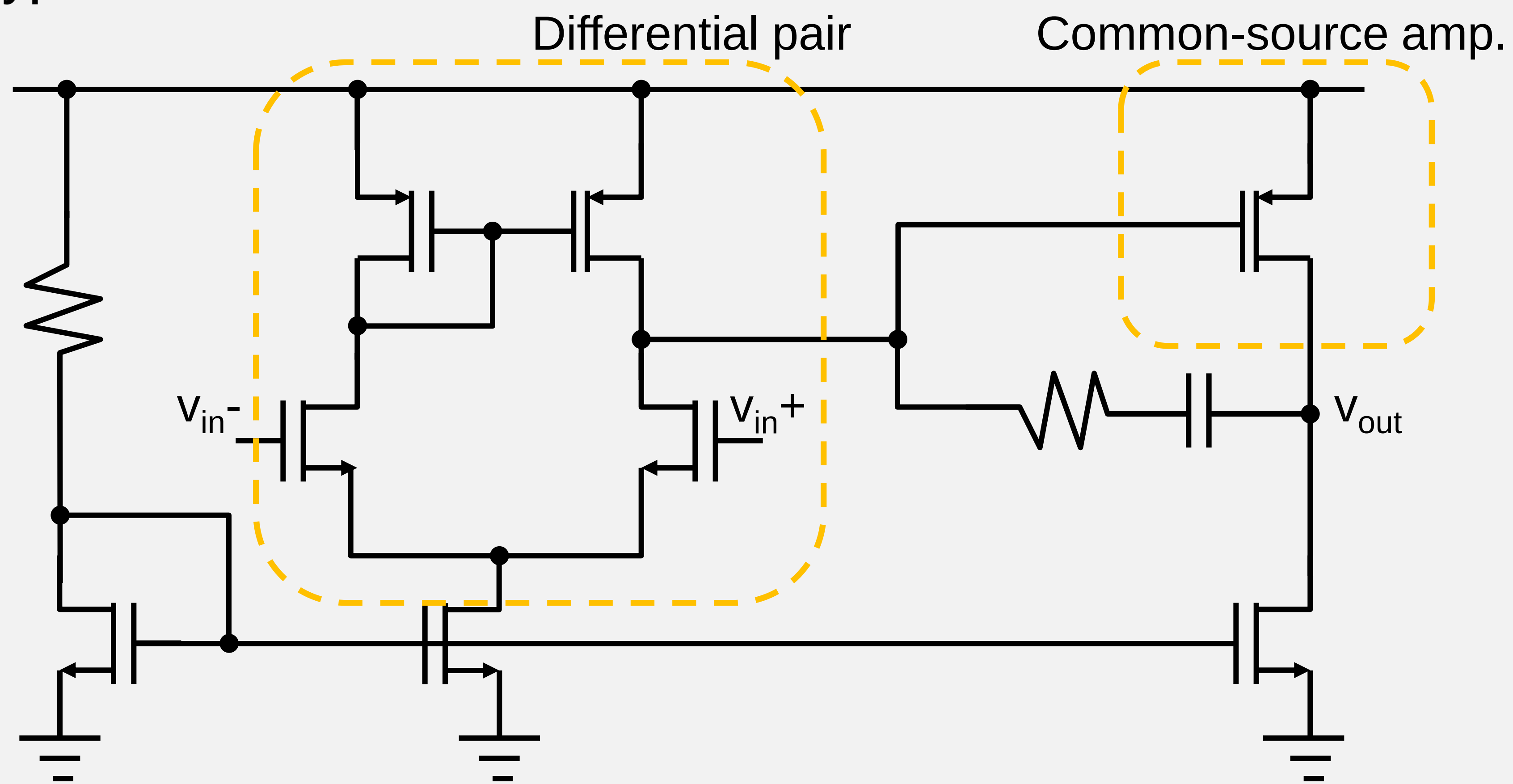
- An ideal op-amp can be used conveniently to implement both linear and nonlinear circuits. It features:
  - Infinite input impedance
    - » No current flowing in
  - Zero output impedance
  - Common-mode inputs produce no output
    - » Infinite common-mode rejection ratio (CMRR)
  - Infinite open-loop gain, *A*
  - Infinite bandwidth



Example of Op-amp Design (類比電路設計)

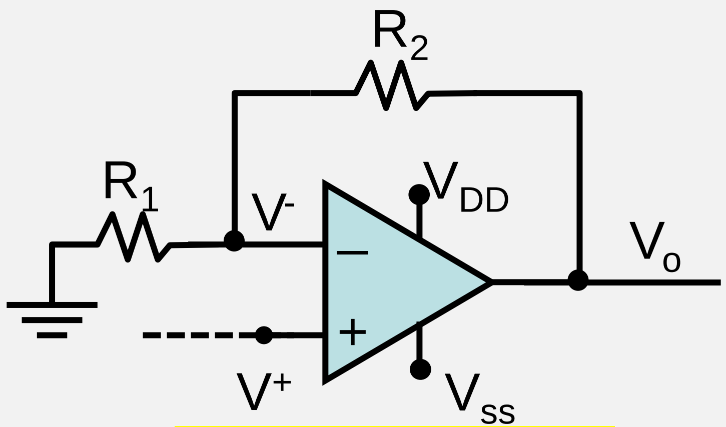


- This is a typical "Two-stage Op-amp", and there are other types.

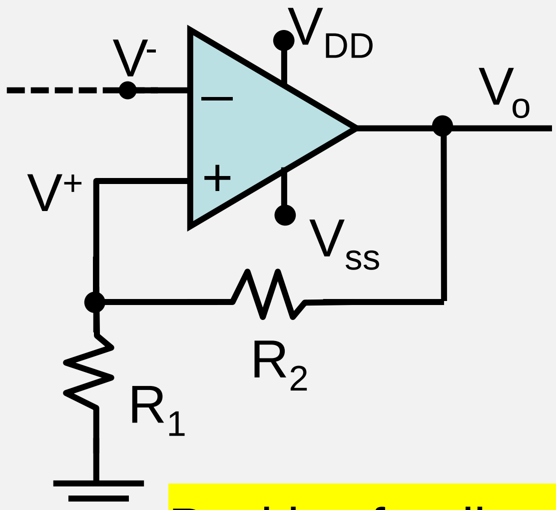


Negative or Positive Feedback around an Opamp

- 請注意：Op-amp的增益太大，不能直接拿來作為放大器
- An op-amp is typically operated by "negative feedback"（負迴授）to build various circuits
  - 負迴授可達到在輸入端$V^+ = V^-$, 即所謂virtual short（虛擬短路），是 建構電路的重要性質
- Positive feedback can be used for making oscillators while virtual short does not necessarily exist



Negative feedback



Positive feedback

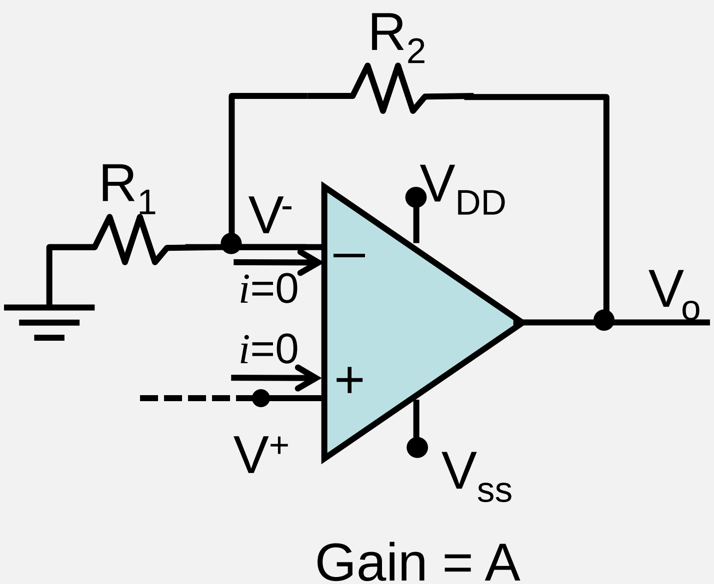
How is Virtual Short ($V^+ = V^-$) achieved using Negative Feedback?

- By derivation，we have $V^+ = V^-$ when op-amp's gain is ∝
  - 實際上op-amp的增益一般在60 dB (1,000) 到 ~100 dB
  - 合理假設op-amp輸入電阻夠大，電流無法流入

$$V_o = A \cdot \left(V^+ - V^-\right)$$

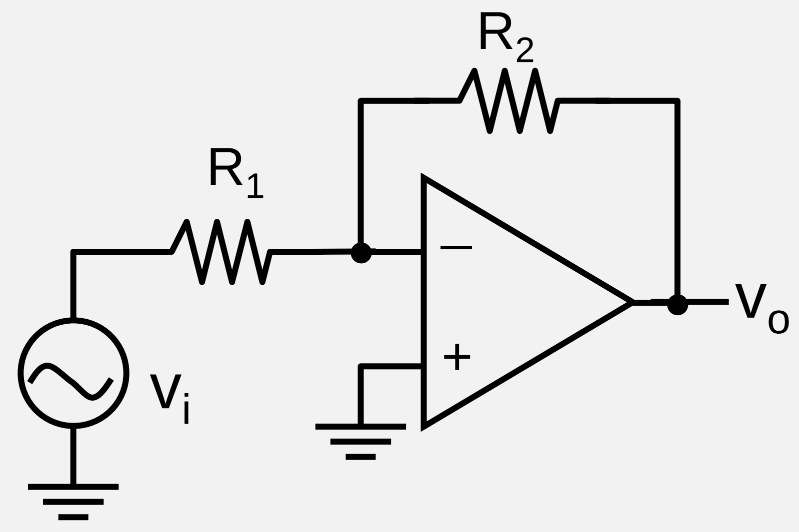
$$V^- = \frac{R_1}{R_1 + R_2} V_o$$

$$\Rightarrow V^+ = \left(\frac{1 + \frac{R_2}{R_1}}{\underbrace{A}_{\approx 0}} + 1 \right) \cdot V^-$$



Op-amp電路（一）：Inverting Amplifier

- It is not easy to make an amplifier with accurate gain using single transistors (for example, a common-source amplifier)
- 以op-amp製作放大器的基本觀念：op-amp的gain很大，使得能以負迴授達到”虛擬短路”；在此基礎下，只要電阻值（R_1, R_2）準確，便能有準確的增益（v_o/v_i）
- “Inverting” means the achieved gain has a negative sign



Cont'd：Inverting Amplifier

- Virtual short, so:

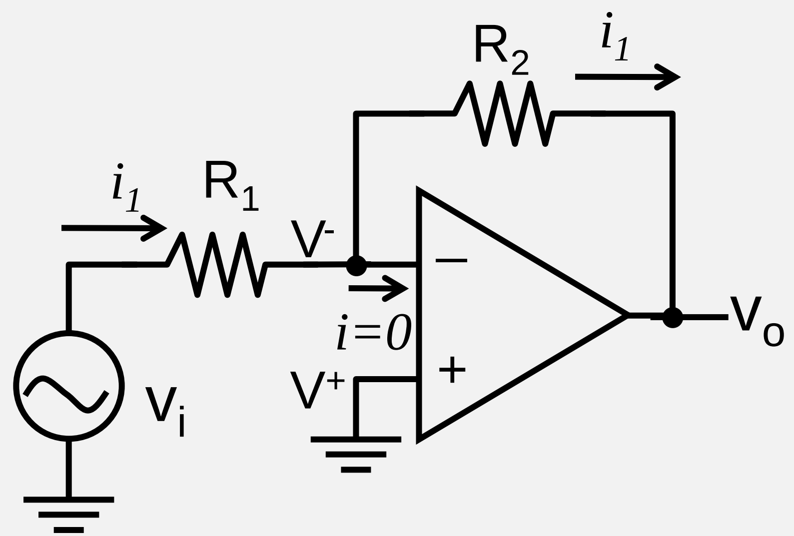
$$V^- = V^+ = 0$$

$$i_1 = \frac{V_i - V^-}{R_1} = \frac{V_i}{R_1}$$

$$V_o = V^- - i_1 R_2 = -\frac{R_2}{R_1} V_i$$

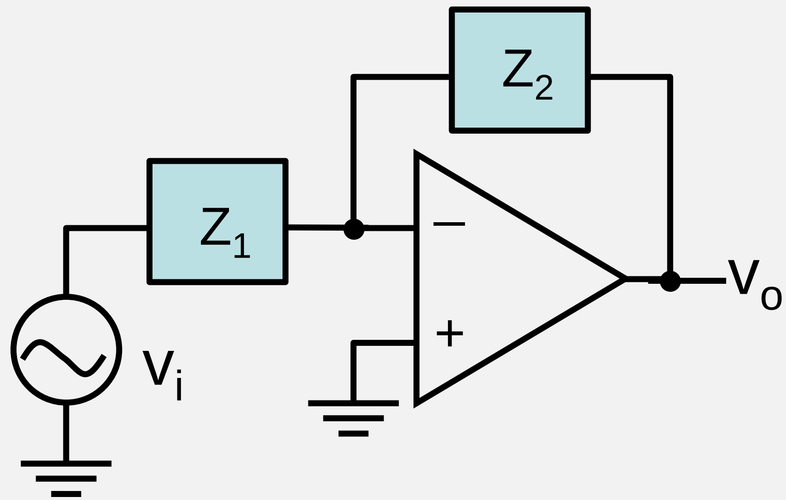
$$\Rightarrow \frac{V_o}{V_i} = -\frac{R_2}{R_1}$$

$$R_{in} = R_1$$



（重要）Inverting Configuration

- In general: $$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2}{Z_1}$$



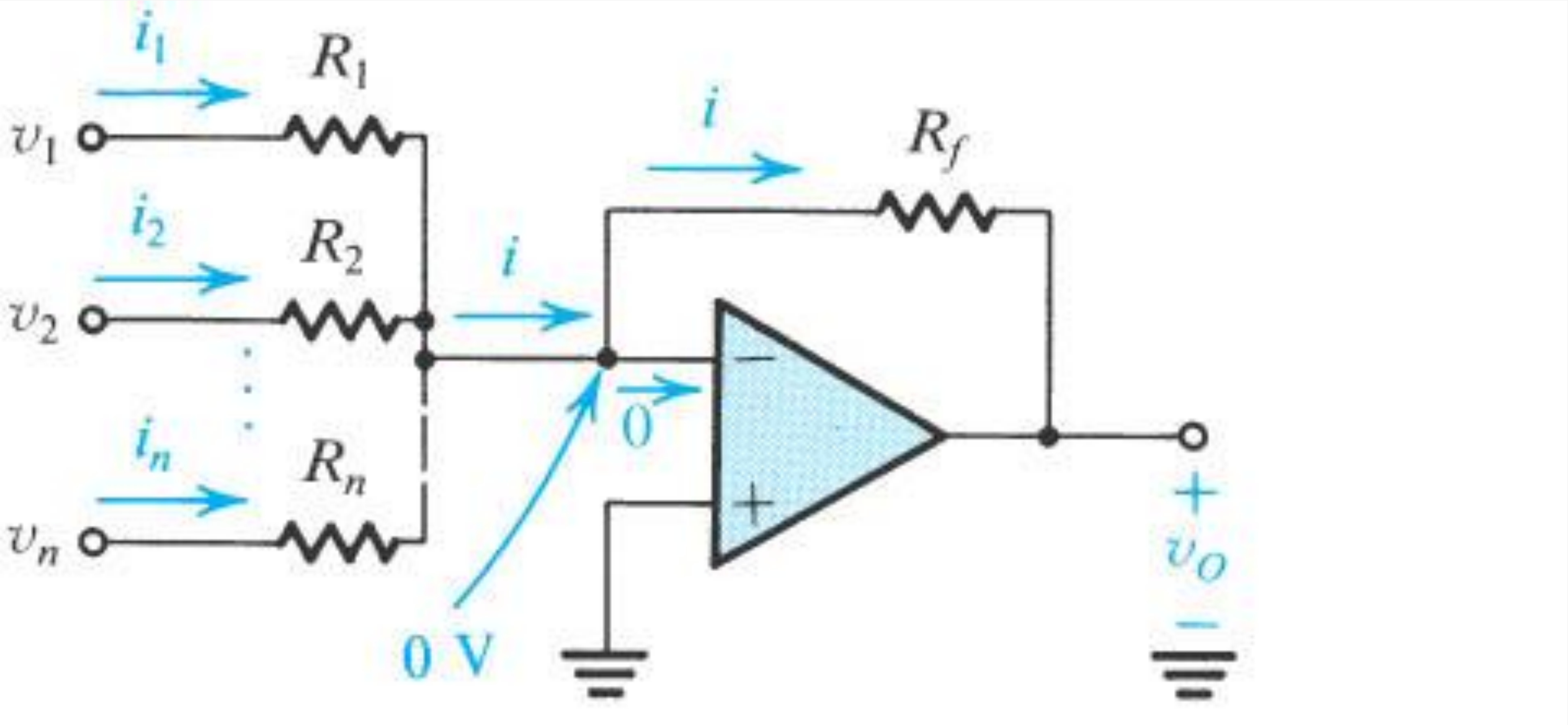
- For example: Z_1 = R, Z_2 = 1/(sC):

$$\frac{V_o(s)}{V_i(s)} = -\frac{1}{sRC}$$ **(integrator)**

$$\frac{V_o(s)}{V_i(s)} = -\frac{1}{sRC} \Rightarrow V_o(s) = -\frac{1}{sRC} V_i(s)$$

$$\Rightarrow V_o(t) = -\frac{1}{RC}\int V_i(t)\cdot dt$$

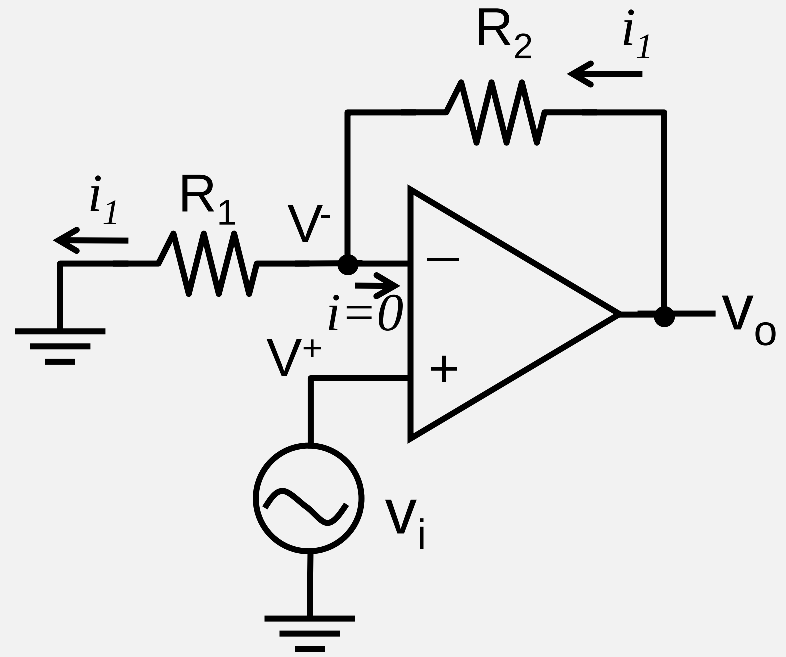
Example: Weighted Summer



$$v_o = -\left(\frac{R_f}{R_1} \cdot v_1 + \frac{R_f}{R_2} \cdot v_2 + .. + \frac{R_f}{R_n} \cdot v_n \right)$$

Opamp電路（二）：Non-Inverting Amplifier

- "Non-inverting" means the gain is positive
- Virtual short, so:



$$V^- = V^+ = V_i$$

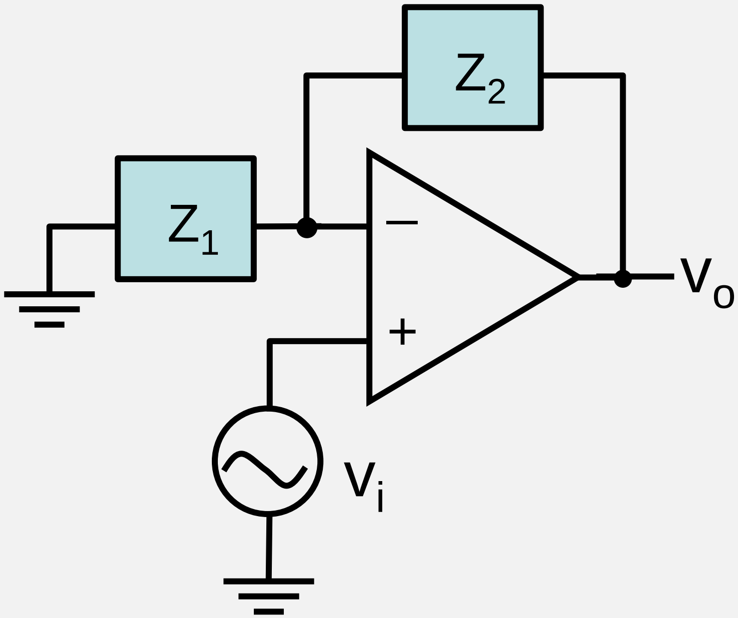
$$i_1 = \frac{V^-}{R_1} = \frac{V_i}{R_1}$$

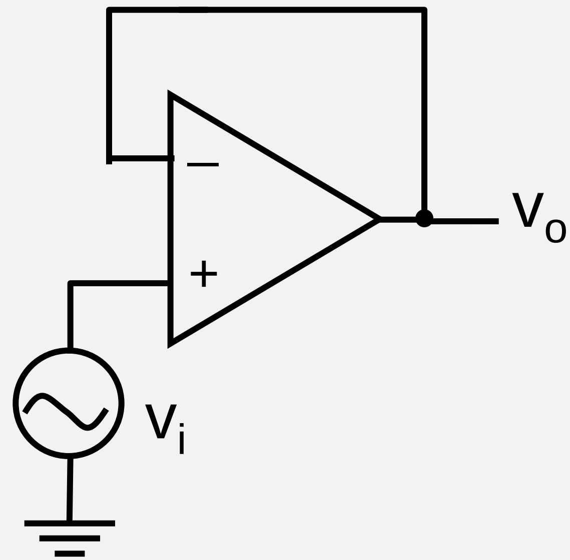
$$V_o = V^- + i_1 \cdot R_2 = V_i + \frac{V_i}{R_1} \cdot R_2$$

$$\Rightarrow \frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}$$

R_{in} is very large, which is good

（重要）Non-Inverting Configuration





- Similarly:

$$\frac{V_o(s)}{V_i(s)} = 1 + \frac{Z_2}{Z_1}$$

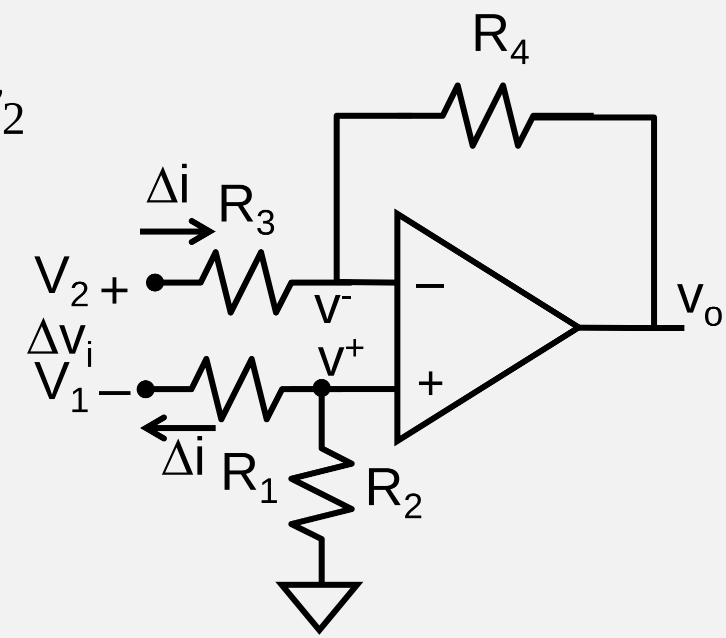
- Voltage follower ($Z_1 = \infty$, $Z_2 = 0$) with a gain of 1 and better driving capability

$$\frac{V_o(s)}{V_i(s)} = 1$$

Difference Amplifier

- It can remove the common-mode signal, and subtract and amplify the differential signal
- By superposition:

$$v_o = \left(1 + \frac{R_4}{R_3}\right) \cdot \left(\frac{R_2}{R_1 + R_2}\right) \cdot v_1 - \frac{R_4}{R_3} \cdot v_2$$



  - For $R_1 = R_2 = R_3 = R_4$

$$v_o = v_1 - v_2$$

$$\Delta v_i - (R_1 + R_3) \cdot \Delta i + \underbrace{(v^+ - v^-)}_{\approx 0} = 0$$

$$R_{in} = \frac{\Delta v_i}{\Delta i} = R_1 + R_3$$ (Drawback)

High-Input-Impedance Differential-Mode Instrumentation Amplifier

- 提供很大的輸入阻抗

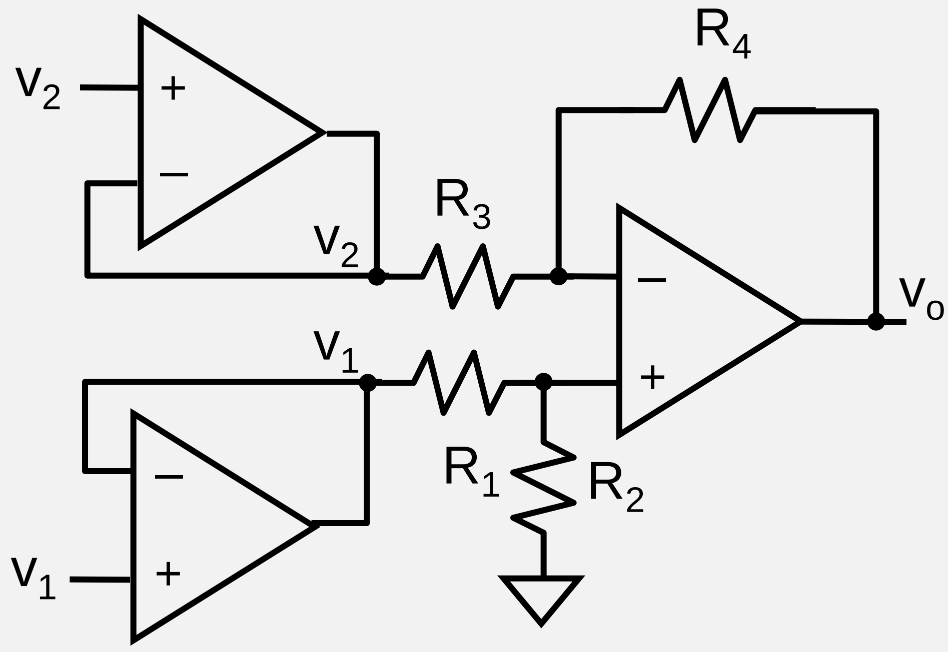
$$v_o = \left(1+\frac{R_4}{R_3}\right)\cdot\left(\frac{R_2}{R_1+R_2}\right)\cdot v_1 - \frac{R_4}{R_3}\cdot v_2$$

$$when\ \frac{R_4}{R_3} = \frac{R_2}{R_1}$$

$$v_o = \frac{R_2}{R_1}\cdot(v_1 - v_2)$$

$$when\ R_1 = R_2 = R_3 = R_4$$

$$v_o = v_1 - v_2$$



Variable-Gain Differential-Mode Instrumentation Amplifier (Design Problem IV)

The gain can be conveniently adjusted by tuning R_G

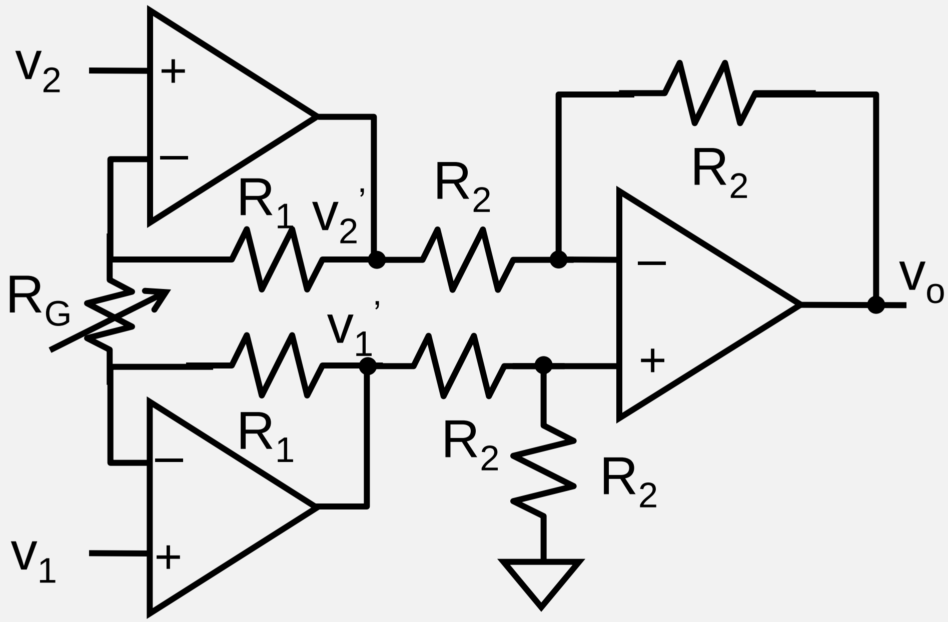
$$v_o = v_1' - v_2' \quad (1)$$

$$v_1' = \left(1 + \frac{R_1}{R_G}\right) \cdot v_1 - \frac{R_1}{R_G} \cdot v_2 \quad (2)$$

$$v_2' = \left(1 + \frac{R_1}{R_G}\right) \cdot v_2 - \frac{R_1}{R_G} \cdot v_1 \quad (3)$$

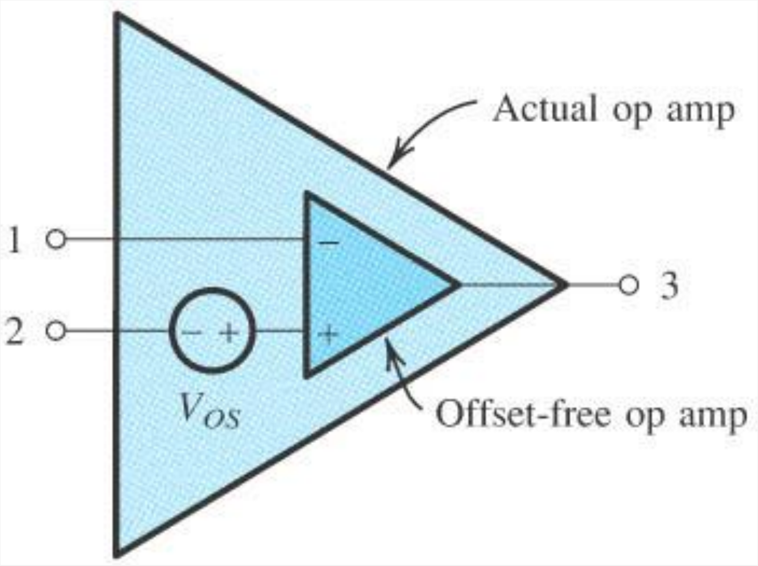
Substitute (2)(3) into (1):

$$v_o = \left(1 + \frac{2R_1}{R_G}\right) \cdot (v_1 - v_2)$$



Non-Ideal Effect in Op-amp: Offset Voltage

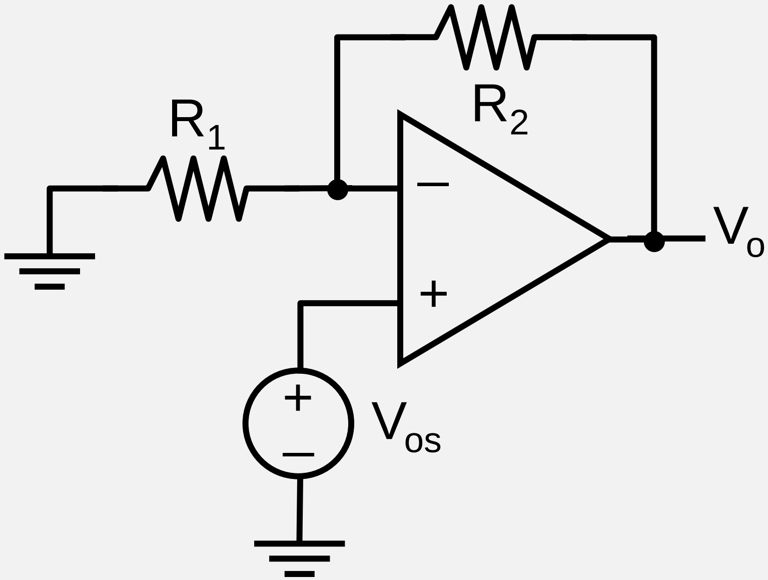
- Input offset voltage V_{os} occurs due to internal imbalances in the input differential amplifier
  - Result: Real op-amps have a non-zero output even if the DC level of two inputs are the same
  - How to reduce the non-zero output to avoid output saturation in an amplifier?



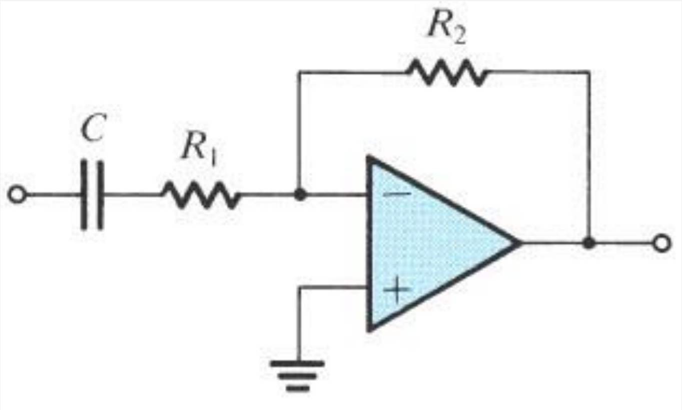
Reduce the Output Change due to V_{os}

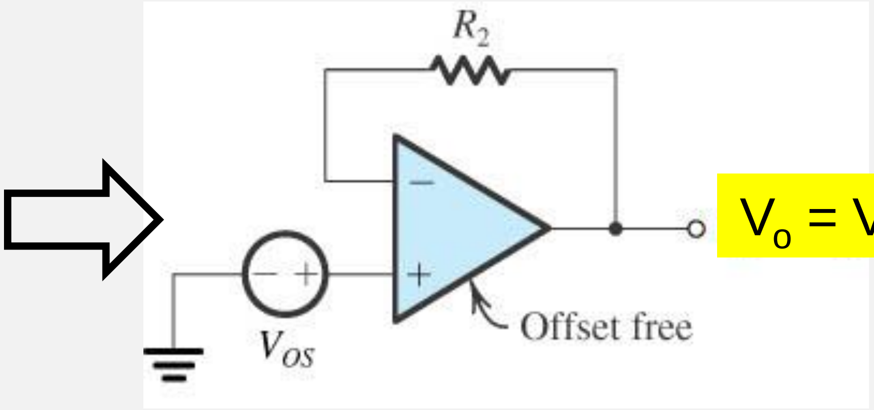
- In a non-inverting amplifier, the input dc offset is amplified by:

$$V_o = \left(1 + \frac{R_2}{R_1}\right) \cdot V_{os}$$



- Add a capacitor to reduce the output due to V_{os}

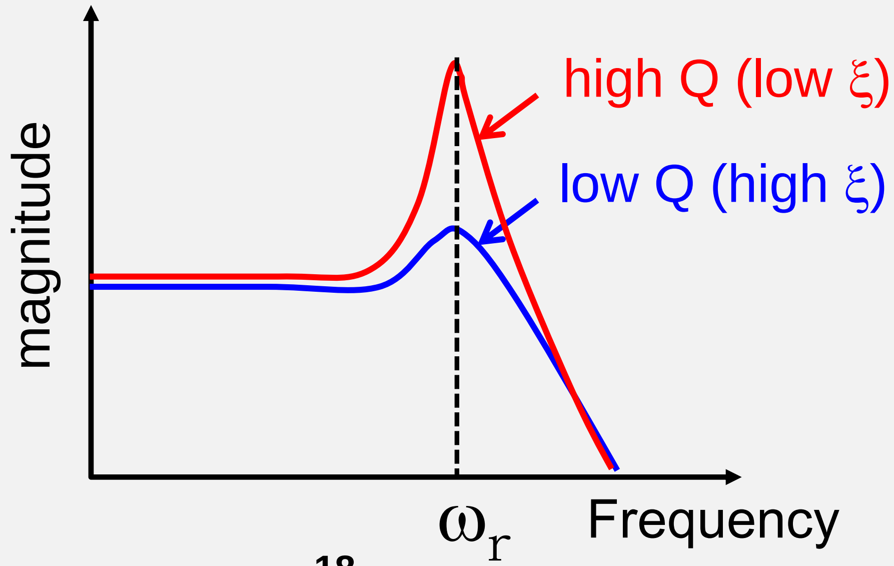




$V_o = V_{os}$

Quality Factor (Q) of a Frequency Response

- A parameter to evaluate the energy-dissipating behavior of a circuit near its resonant frequency
  - High Q: a large resonant peak and thus low energy dissipation
  - Low Q: a small resonant peak and thus high energy dissipation
- As shown, frequency responses of a low-Q and a high-Q filters:



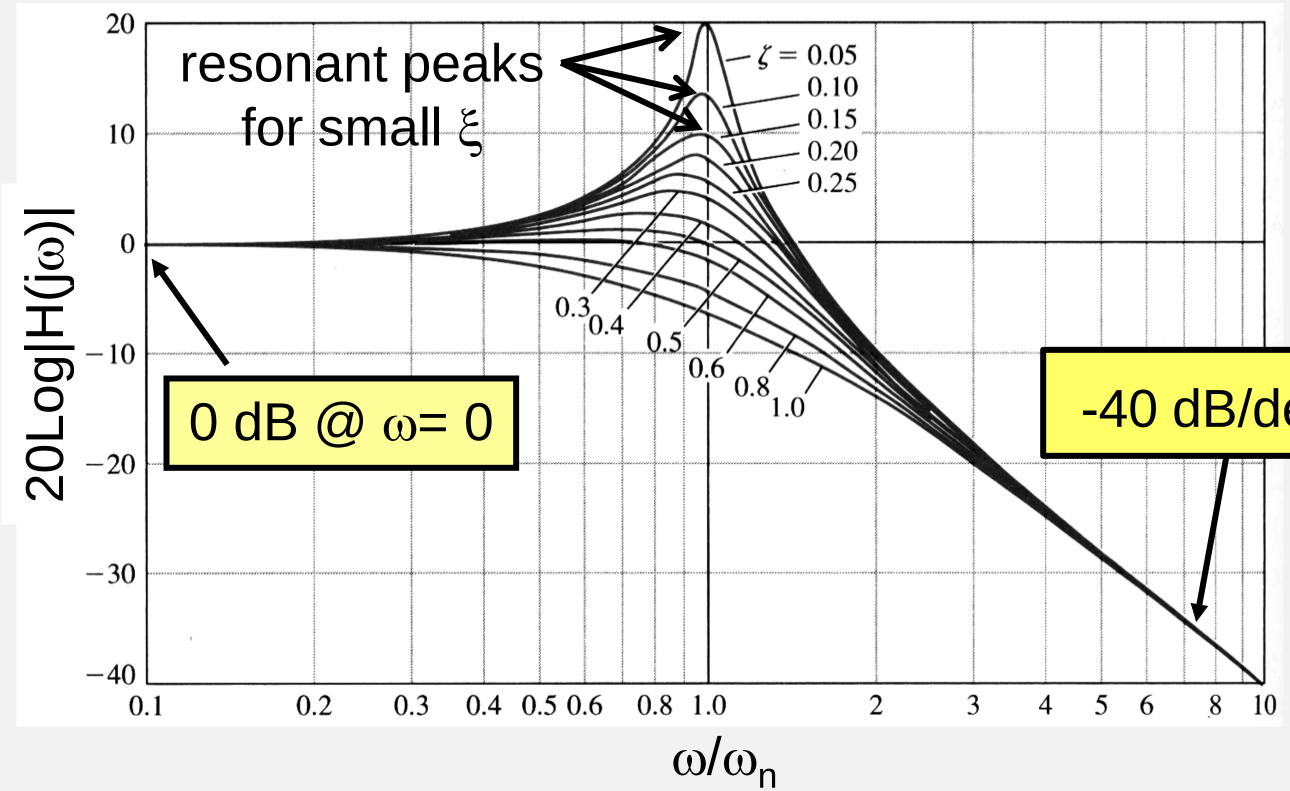
Reminder: the Frequency Response (Magnitude) of a 2nd-Order System for Different Damping Ratios ξ

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\left|H(j\omega)\right|_{\omega=0} = 1$$

$$\left|H(j\omega)\right|_{\omega=\omega_n} = \frac{1}{2\xi}$$

$$\left|H(j\omega)\right|_{\omega=\infty} = 0$$



Definition：Quality Factor (Q)

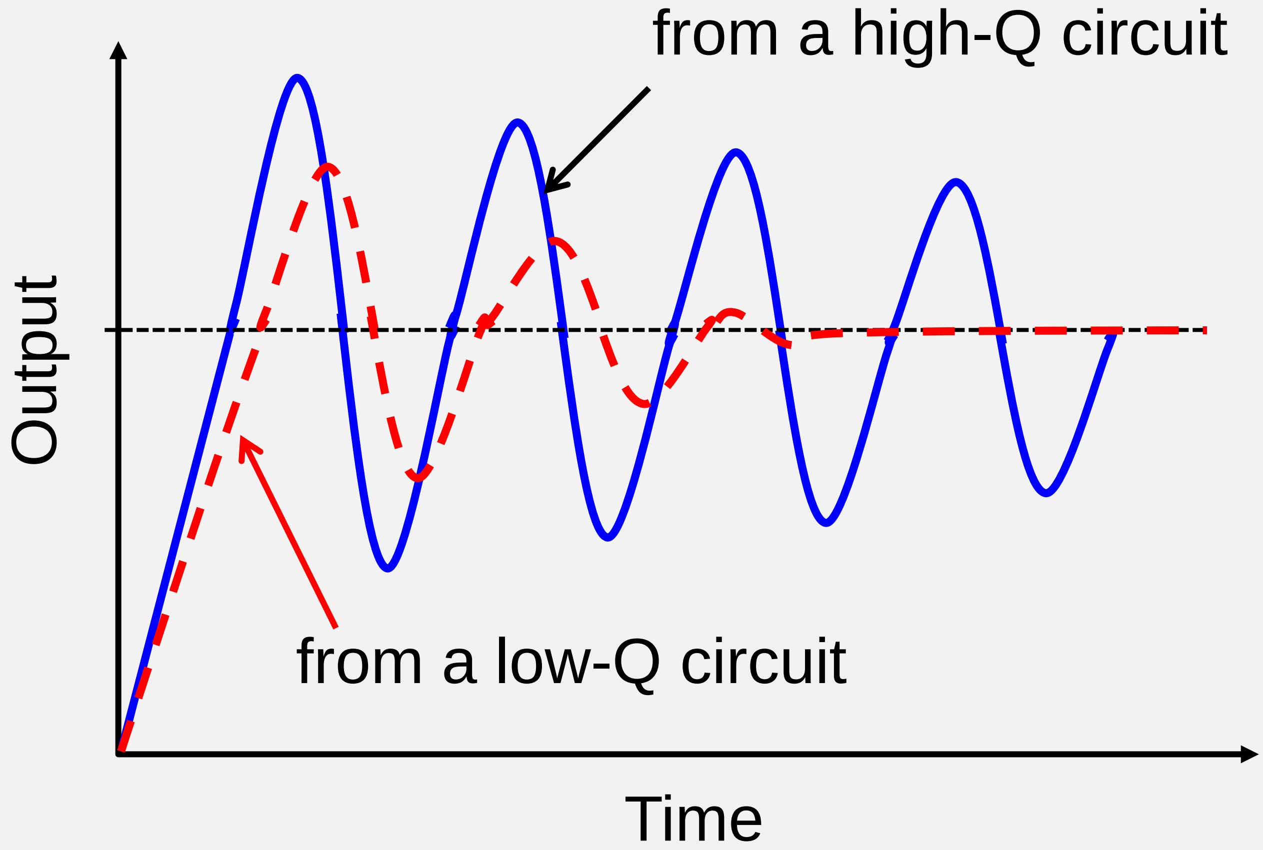
- The resonant peak magnitude is related to energy dissipation
- Definition:

$$Q \equiv \frac{\text{Energy stored}}{\text{Average energy dissipated per radian}}$$

  - Q is a dimensionless parameter and related to the damping ratio by $Q \approx 1/(2\xi)$
  - $Q = \infty$ implies there is no energy dissipation when a circuit oscillates

Step Response vs. Q

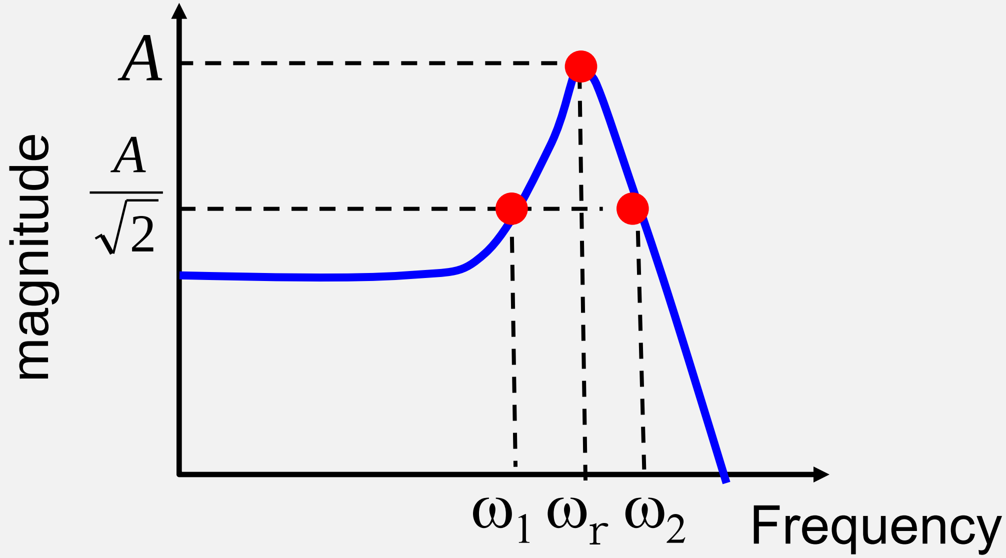
- A high-Q circuit has low energy dissipation, and long oscillation time in its step response



Obtain Q by Analysis or Experiment

- Q can be obtained by analysis:
  - For $\xi << 1$, Q is related to the damping ratio by $Q \approx 1/(2\xi)$。 Remember: Q and ξ are both dimensionless
- Q can be obtained by experiment using the half-power bandwidth (defined by ω_1 and ω_2):

$$Q = \frac{\omega_r}{\omega_2 - \omega_1}$$ （重要！你實驗demo Q值要用到）



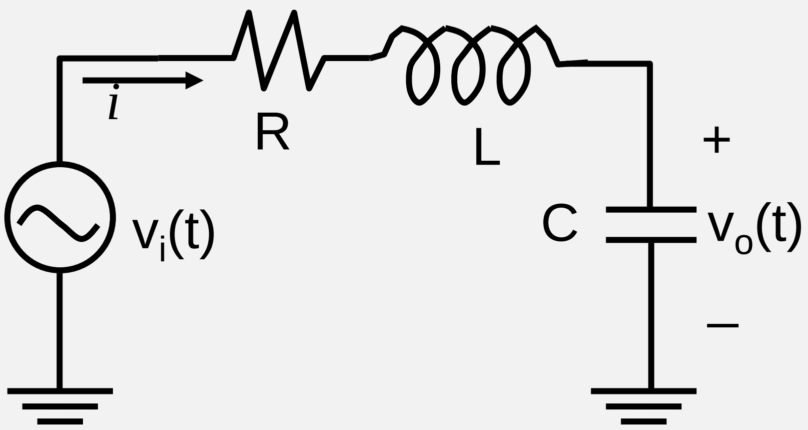
Example: Q of a Low-Pass Filter

Transfer function is given by:

$$\frac{v_o(s)}{v_i(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

where $\omega_n = \frac{1}{\sqrt{LC}}$ natural frequency

and $\xi = \frac{R}{L} \cdot \frac{1}{2\omega_n} = \frac{R}{2}\sqrt{\frac{C}{L}}$



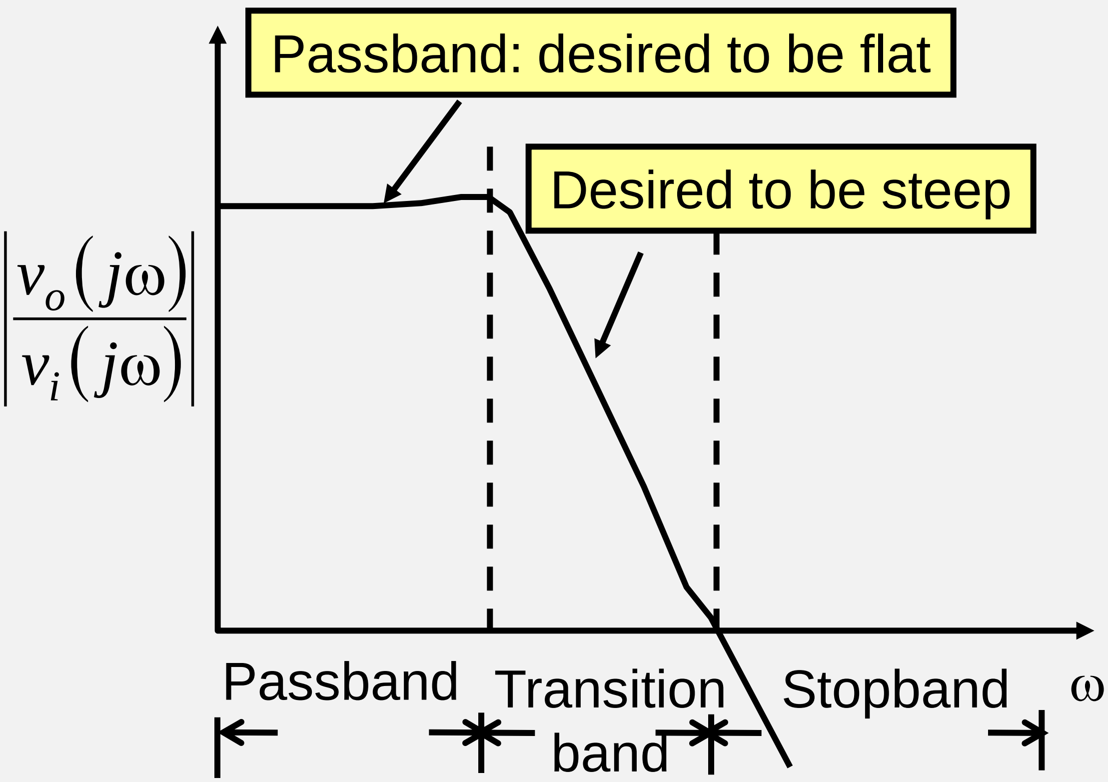
$$So\ Q = \frac{1}{2\xi} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Active Filters

- In addition to passive elements, operational amplifiers are used to build an **"active" filter**
- 特色：
  - A voltage gain more than one can be achieved
  - An active filter can achieve the frequency response of a standard 2nd-order R-L-C circuit without the use of an inductor
  - An active filter can achieve higher-order filters (> 2^{nd} order) with better attenuation
    - » Note: 1^{st}-order filter: -20 dB/decade, 2^{nd}-order filter: -40 dB/decade, nth-order = -20×n dB/decade

Filter Design and Specifications

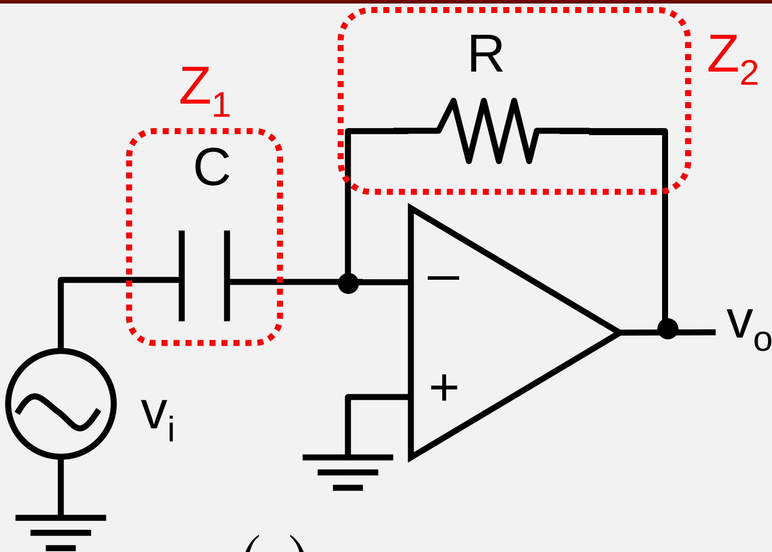
- It is usually desired to achieve a flat passband and a high roll-off rate (depending on the applications) in the transition band

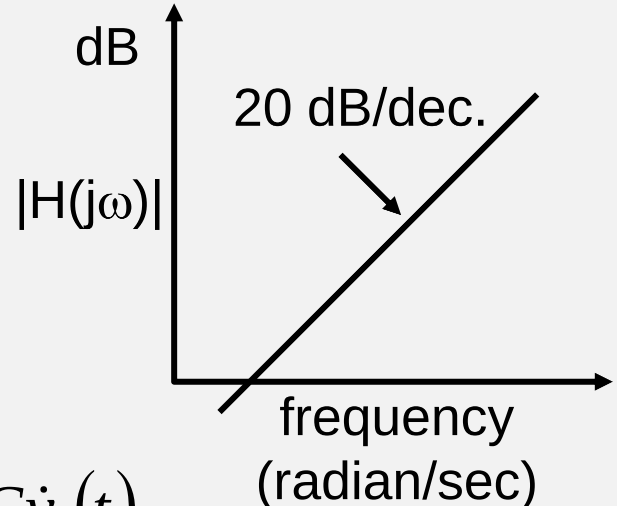


Types of Active Filters

- First-order active filters：
  - Low-pass, high-pass, band-pass, and band-reject filters
- Second-order active filters：
  - Sallen-Key filter
  - Biquad filter
  - State-Variable filter
  - …etc.
- Higher-order active filters:
  - Butterworth、Chebyshev、Cauer (elliptic)、Bessel filters, etc

First-Order Circuit: Differentiator





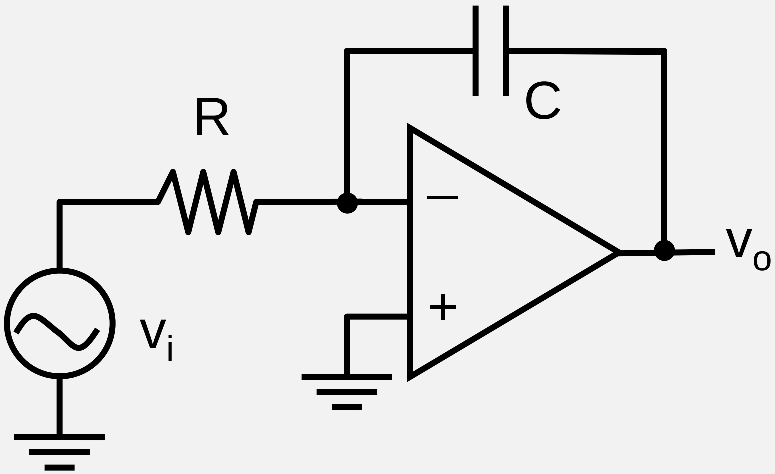
$$H(s)=\frac{v_o(s)}{v_i(s)}=-\frac{Z_2}{Z_1}=-sRC\,,v_o(t)=-RC\dot{v}_i(t)$$

$$\Rightarrow \mathrm{H}(\mathrm{j}\omega)=\frac{\mathrm{v_o}(\mathrm{j}\omega)}{\mathrm{v_i}(\mathrm{j}\omega)}=-\mathrm{j}\omega\mathrm{RC}$$

$$\therefore \left|\mathrm{H}(\mathrm{j}\omega)\right|=\omega\mathrm{RC}$$

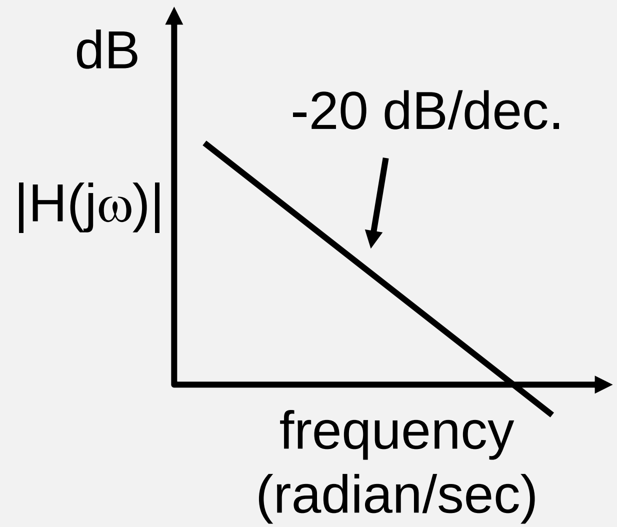
1. 頻率每增加十倍，增益也增為十倍（即20 dB/dec）
2. H(j0) = 0 = -∝ dB

Integrator



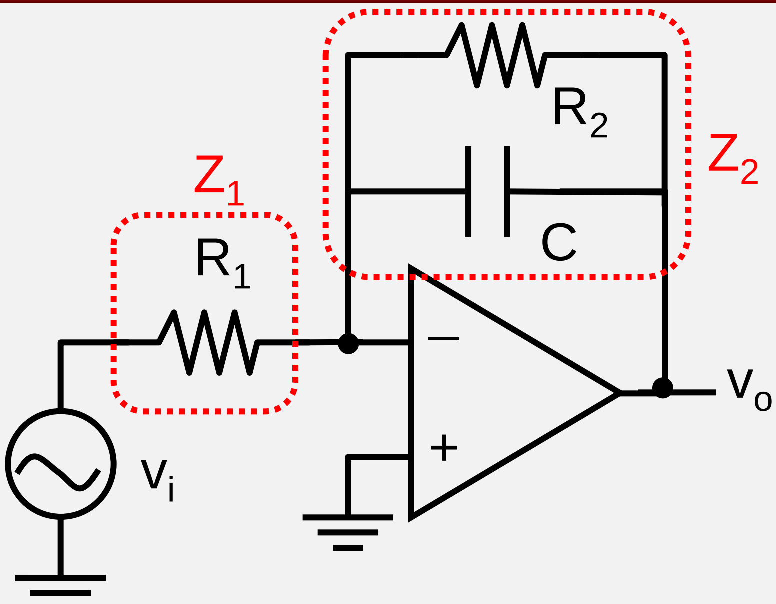
$$H(s)=\frac{v_o(s)}{v_i(s)}=-\frac{1}{sRC},\ v_o(t)=-\frac{\int_0^t v_i(t)dt}{RC}$$

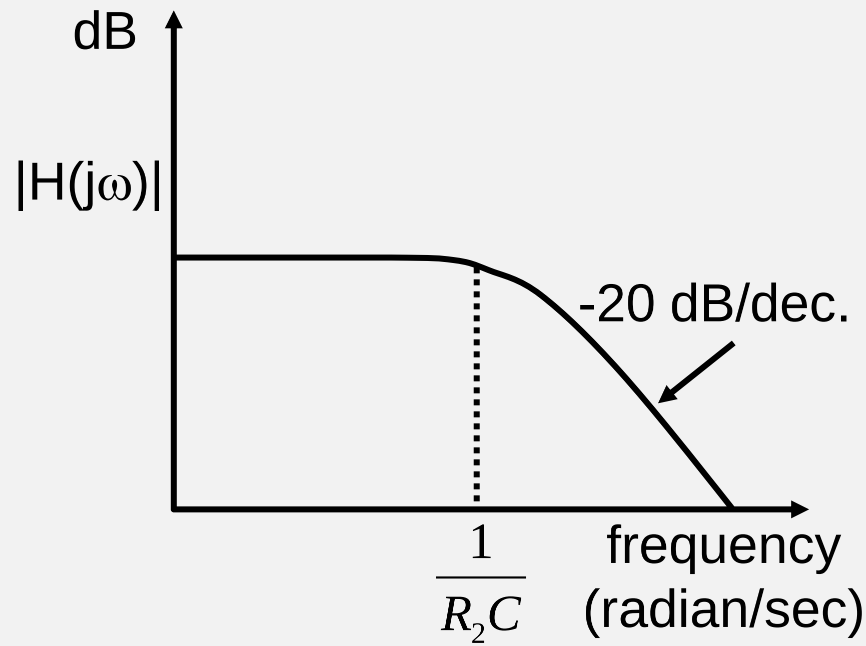
$$\therefore |H(j\omega)|=\frac{1}{\omega RC}$$



1. 頻率每增加十倍，增益減為十分之一（即-20 dB）
2. $|H(j0)|=\infty$

Low-Pass Filter with Gain





$$H(s)=\frac{v_o(s)}{v_i(s)}=-\frac{Z_2}{Z_1}=-\frac{R_2}{R_1}\frac{1}{sR_2C+1}$$

$$\therefore \left|H(j\omega)\right|=\frac{R_2}{R_1}\frac{1}{\sqrt{\omega^2R_2^2C^2+1}}$$

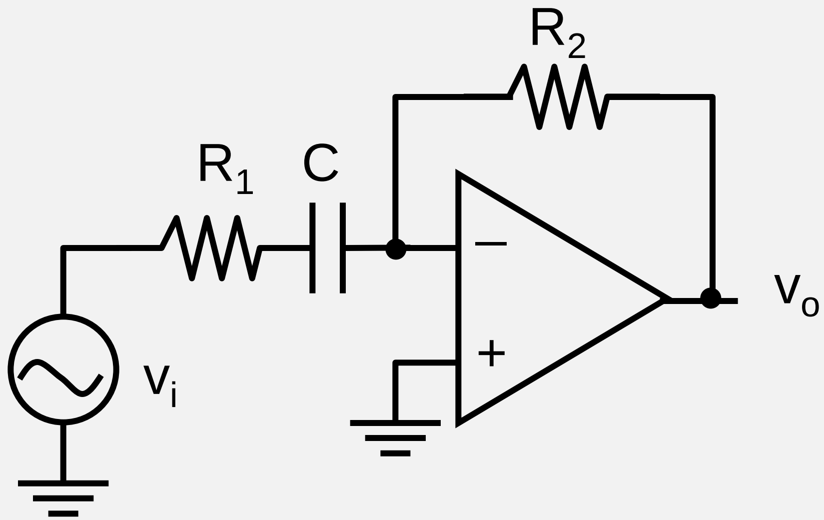
$\frac{1}{R_2C}$ is the -3dB frequency

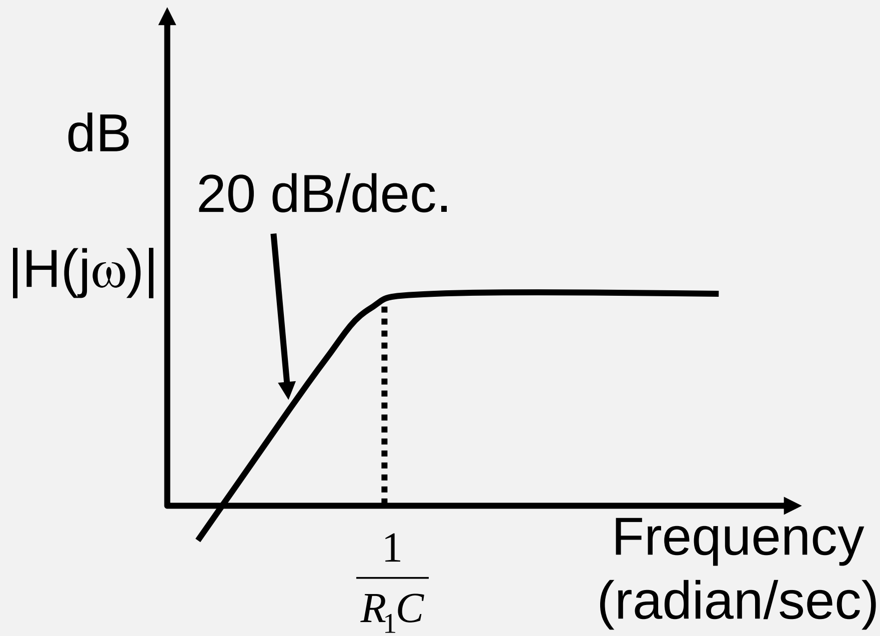
$$\left|H(j\omega)\right|_{\omega=0}=\frac{R_2}{R_1}$$

$$\left|H(j\omega)\right|_{\omega=\frac{1}{R_2C}}=0.707\frac{R_2}{R_1}$$

$$\left|H(j\omega)\right|_{\omega=\infty}=0$$

High-Pass Filter with Gain





$$H(s) = \frac{v_o(s)}{v_i(s)} = -\frac{R_2}{R_1}\frac{sR_1C}{sR_1C+1}$$

$$\therefore |H(j\omega)| = \frac{R_2}{R_1}\frac{\omega R_1 C}{\sqrt{\omega^2 R_1^2 C^2 + 1}}$$

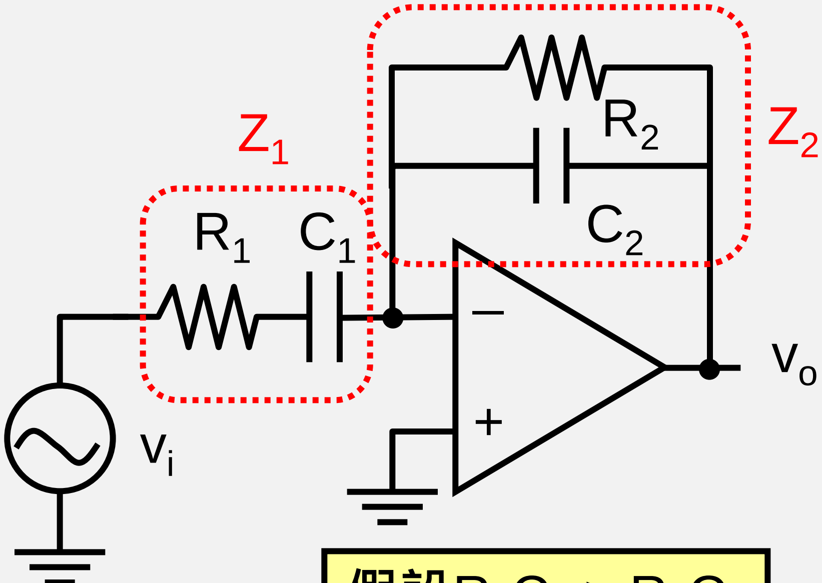
$\frac{1}{R_1C}$ is the -3dB frequency

$$|H(j\omega)|_{\omega=0} = 0$$

$$|H(j\omega)|_{\omega=\frac{1}{R_1C}} = 0.707\frac{R_2}{R_1}$$

$$|H(j\omega)|_{\omega=\infty} = \frac{R_2}{R_1}$$

Band-Pass Filter

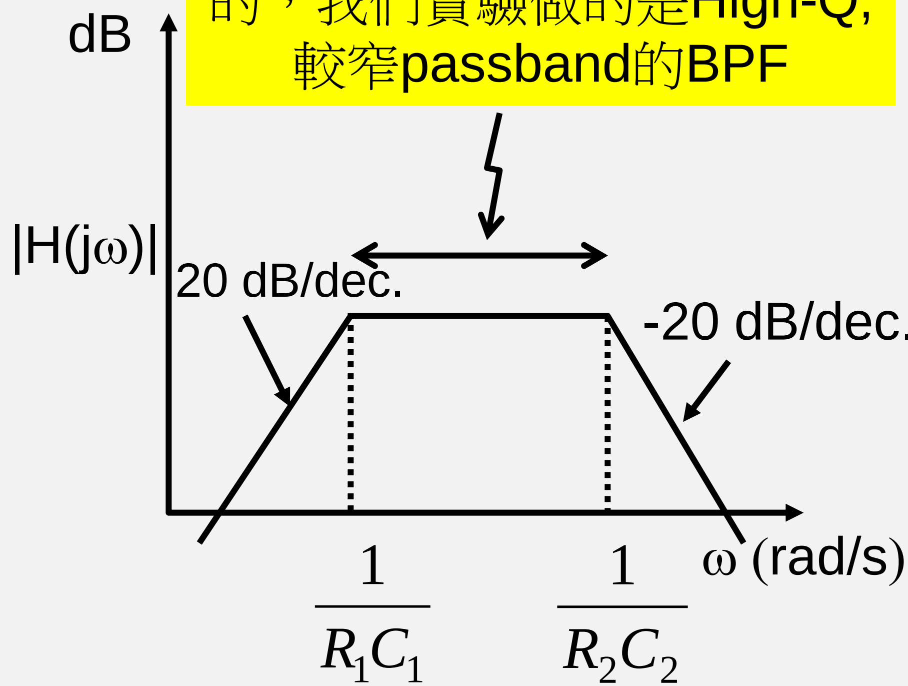


假設$R_1C_1 > R_2C_2$

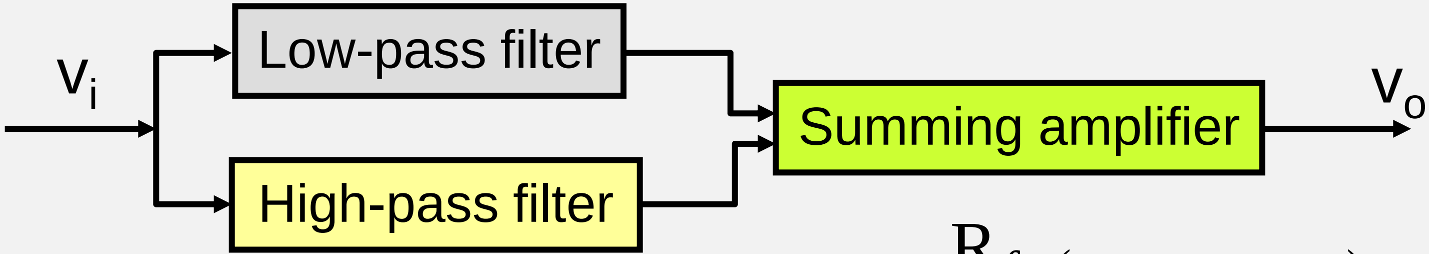
$$H(s) = \frac{v_o(s)}{v_i(s)} = -\frac{Z_2}{Z_1}$$

$$= -\frac{R_2}{R_1}\frac{sR_1C_1}{sR_1C_1+1}\cdot\frac{1}{sR_2C_2+1}$$

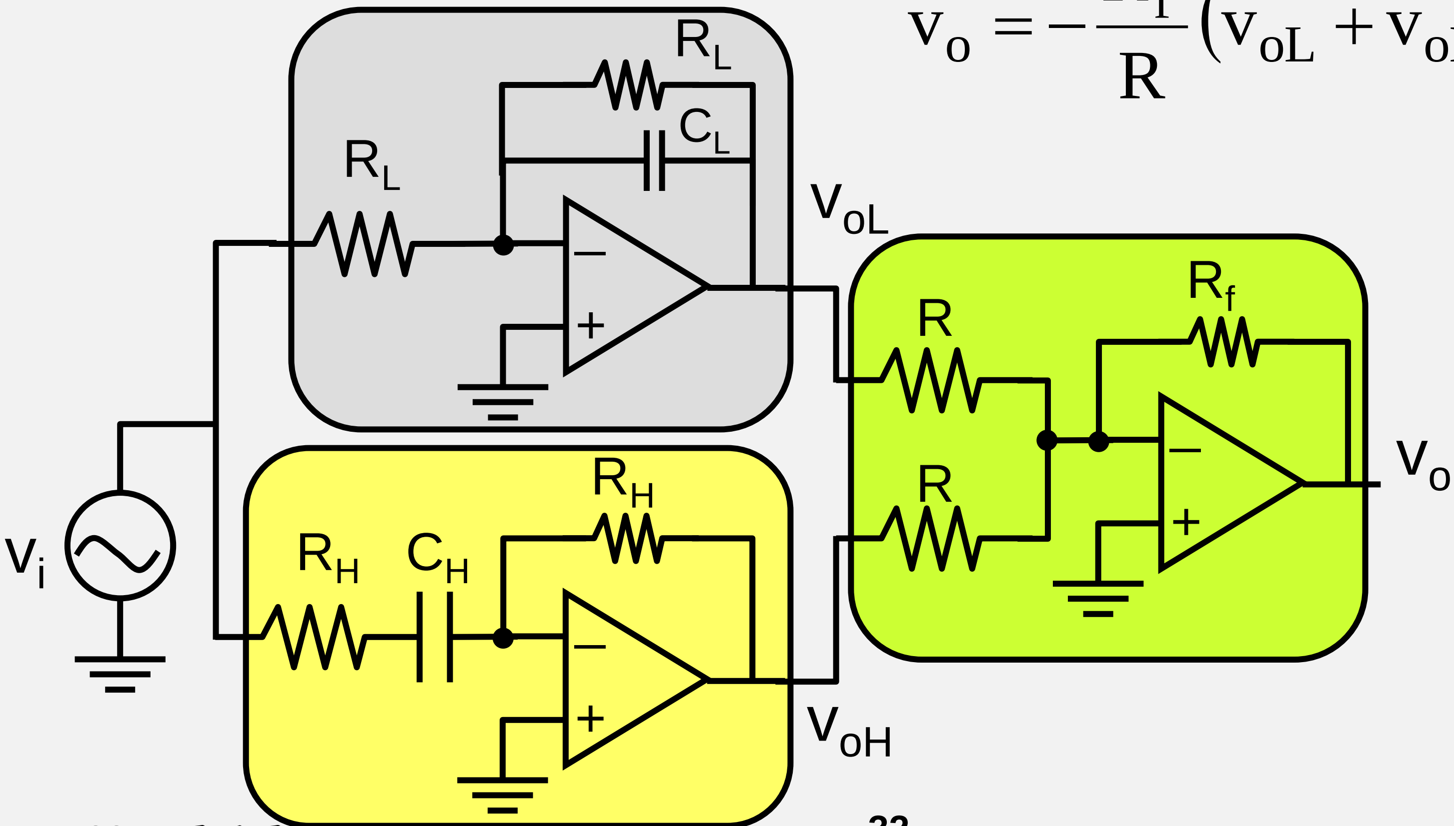
此BPF的passband是較寬的，我們實驗做的是High-Q,較窄passband的BPF



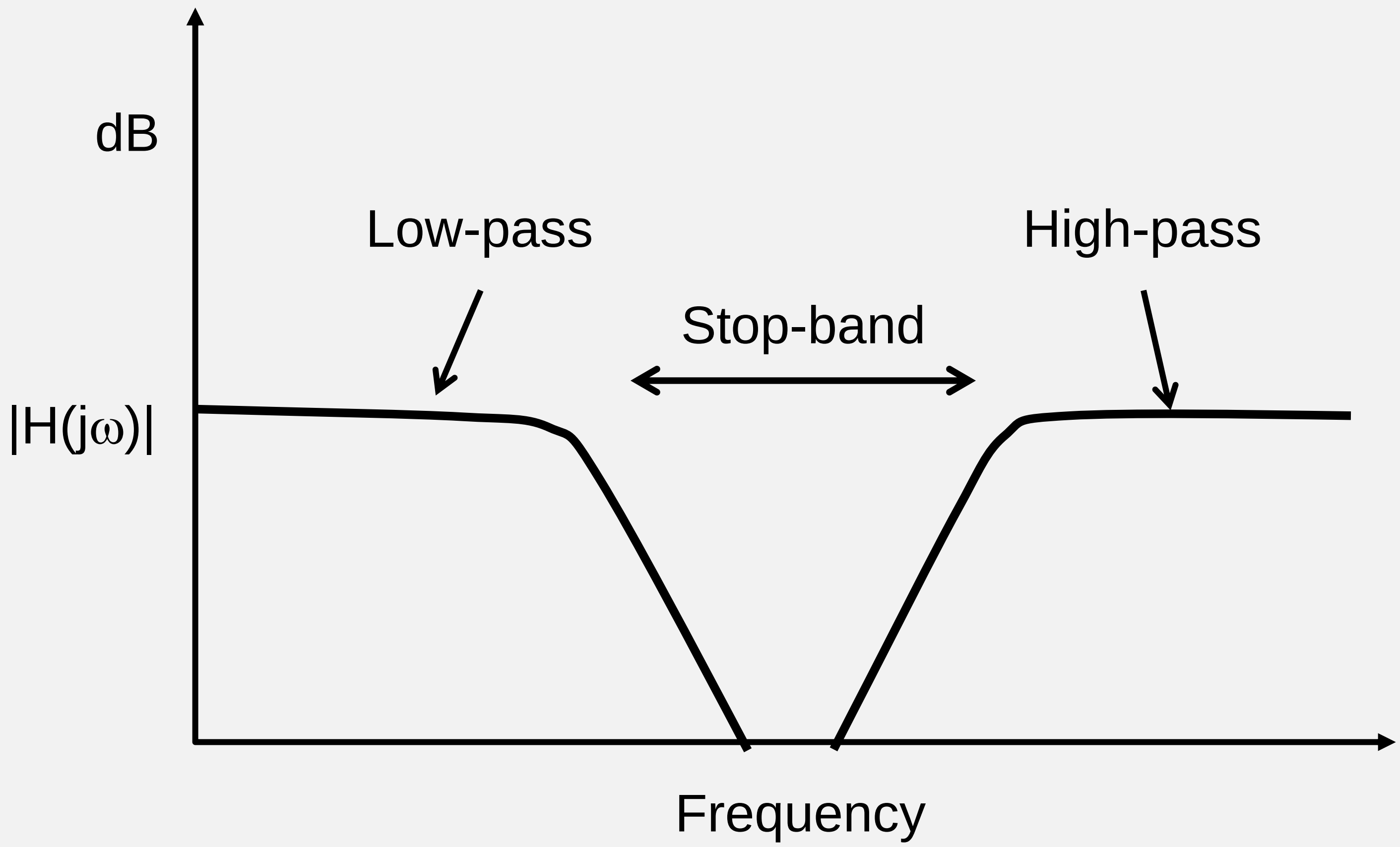
Band-Reject Filter



$$v_o = -\frac{R_f}{R}\left(v_{oL} + v_{oH}\right)$$



Cont'd: Frequency Response



2nd-Order Active Filter: Sallen-Key Low-Pass Filter

- Define the non-inverting gain:

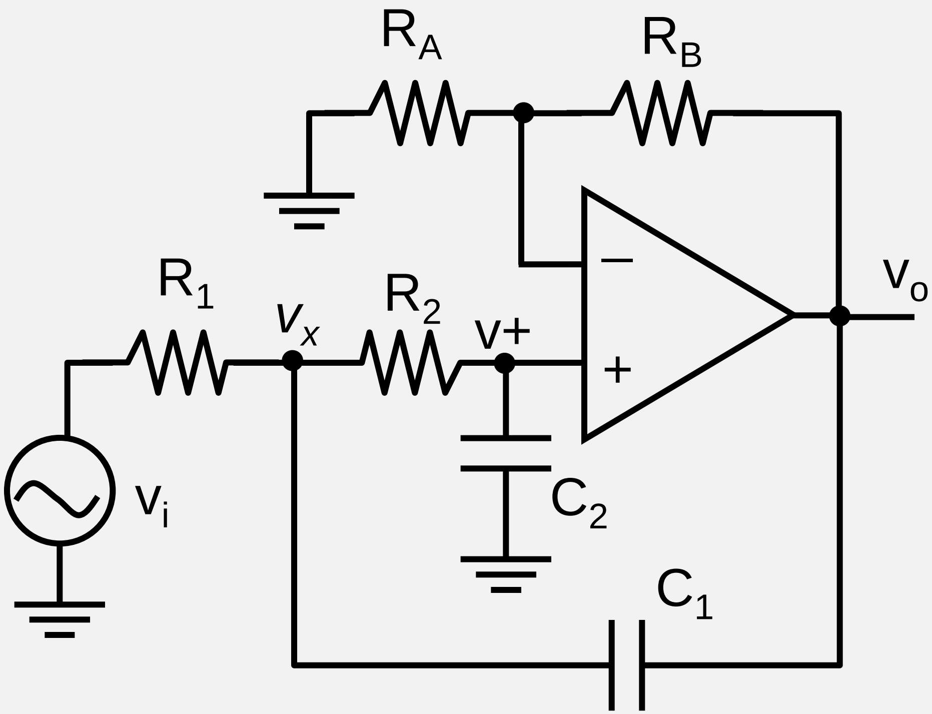
$$K = 1 + \frac{R_B}{R_A}$$

- The output v_o:

$$v_o = K v_+$$
$$= K\left(\frac{1}{R_2 C_2 s + 1} \cdot v_x\right)$$

- Summing currents at node v_x:

$$\frac{v_i - v_x}{R_1} + \frac{v_+ - v_x}{R_2} + \frac{v_o - v_x}{1/(sC_1)} = 0$$



Cont'd

- The transfer function:

$$\frac{v_o(s)}{v_i(s)} = \frac{K}{R_1C_1R_2C_2s^2 + [(1-K)R_1C_1 + R_1C_2 + R_2C_2]s + 1}$$

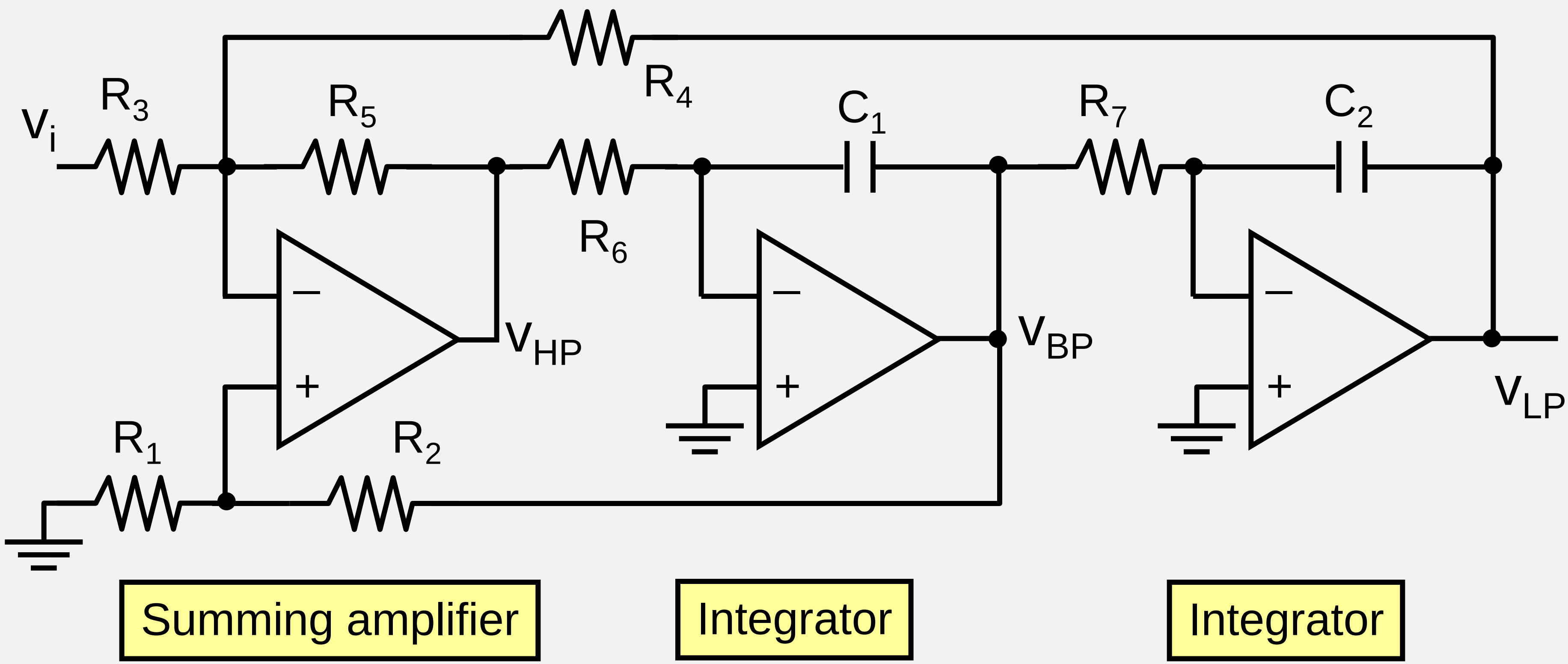
For solving ω_n and ξ (i.e., Q), remember to use the form:

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

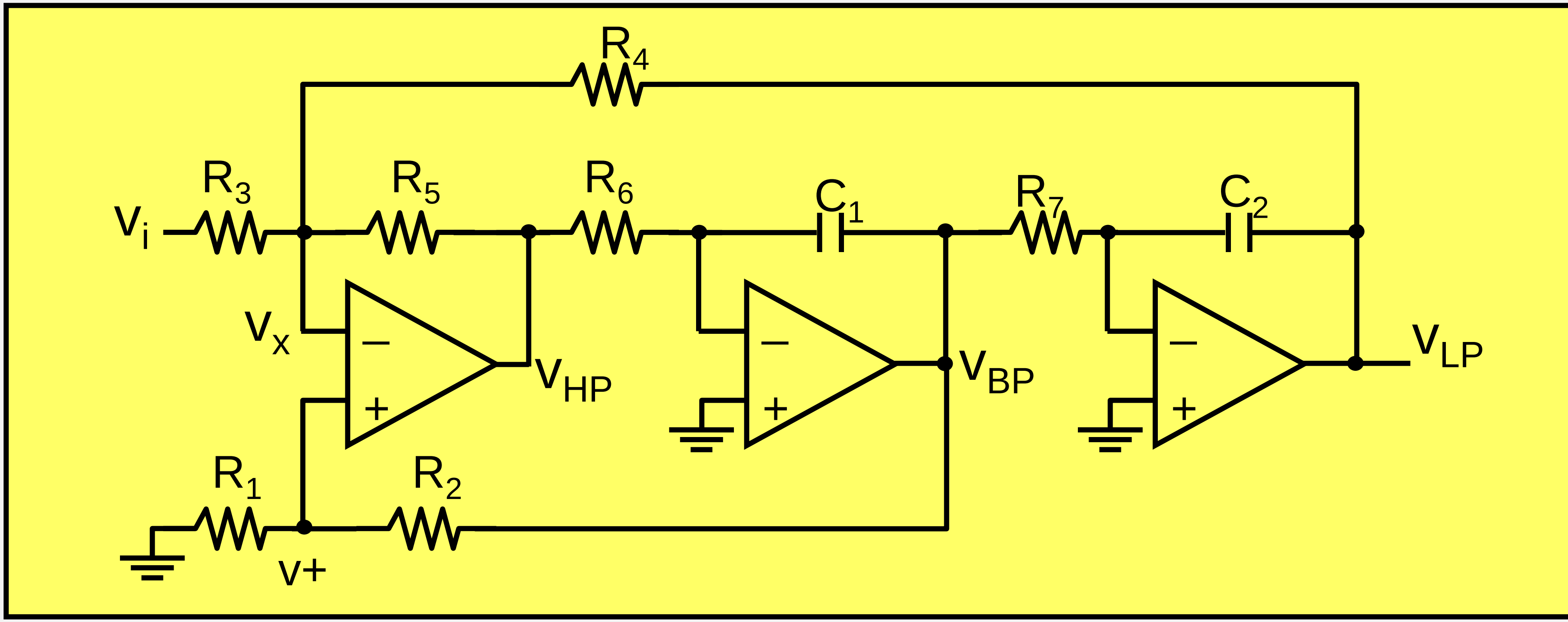
- The gain rolls off after the natural frequency:

$$\omega_n = \frac{1}{\sqrt{R_1C_1R_2C_2}} \qquad Q = \frac{1}{2\xi} = \frac{\sqrt{R_1C_1R_2C_2}}{(1-K)R_1C_1 + R_1C_2 + R_2C_2}$$

State-Variable Filter (KHN Filter)



- It simultaneously realizes the high-pass (v_{HP}), band-pass (v_{BP}) and low-pass (v_{LP}) frequency responses

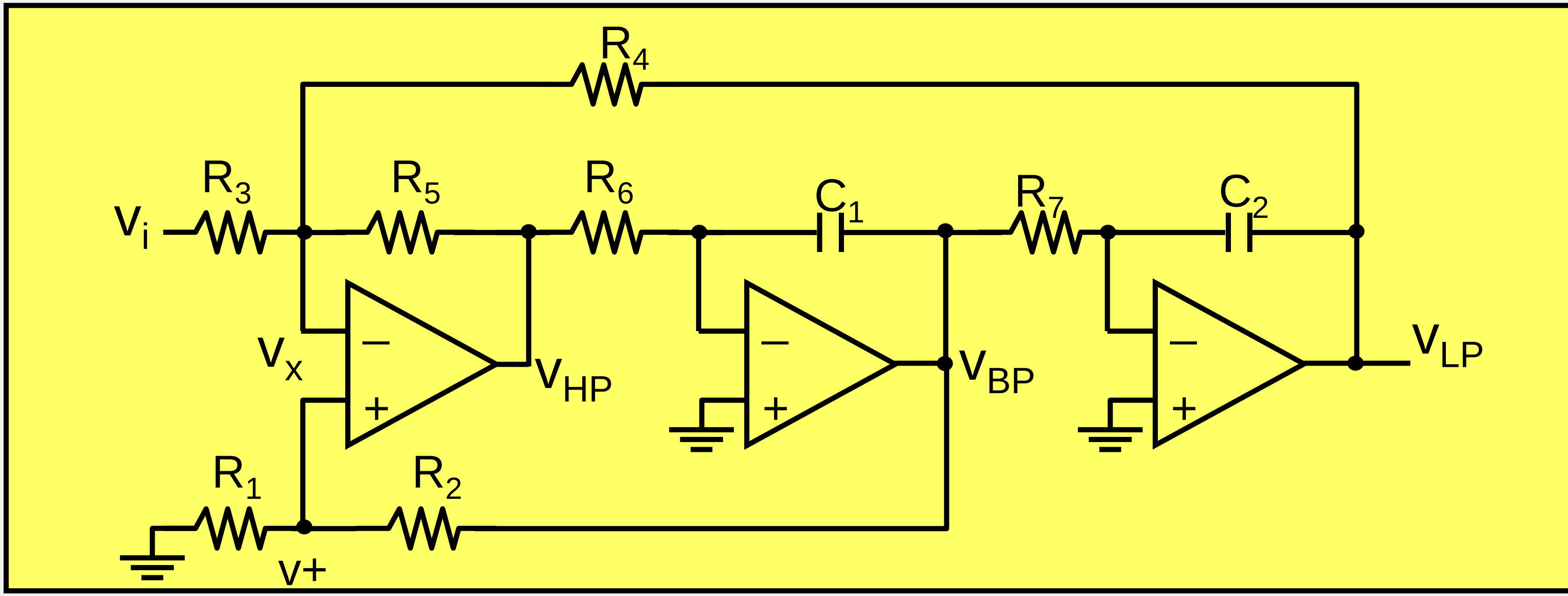


By KCL at V_x: $$\frac{v_i - v_x}{R_3} + \frac{v_{LP} - v_x}{R_4} + \frac{v_{HP} - v_x}{R_5} = 0 \quad (1)$$

$$v_x = \frac{R_1}{R_1 + R_2} v_{BP} \quad (2) \qquad v_{BP} = -\frac{1}{sR_6C_1} v_{HP} \quad (3)$$

$$v_{LP} = -\frac{1}{sR_7C_2} v_{BP} = \frac{1}{s^2 R_6 R_7 C_1 C_2} v_{HP} \quad (4)$$

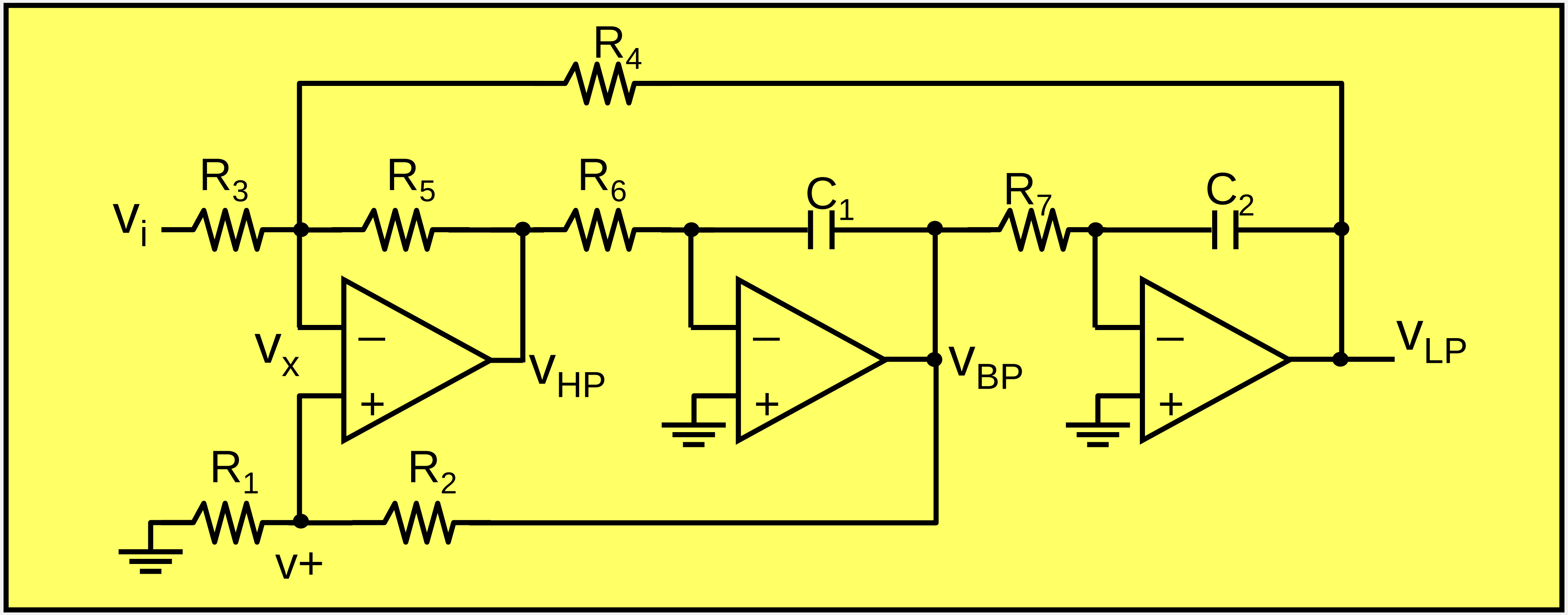
(3)代入(2)，其結果再與(4)一起代入(1)



$$\therefore \frac{v_{HP}(s)}{v_i(s)} = -\frac{1}{R_3} \cdot \frac{R_4R_6R_7C_1C_2s^2}{\frac{R_4R_6R_7C_1C_2}{R_5}s^2 + \frac{R_4R_7C_2\left(1+R_5/R_3+R_5/R_4\right)}{\left(1+R_2/R_1\right)R_5}s+1}$$

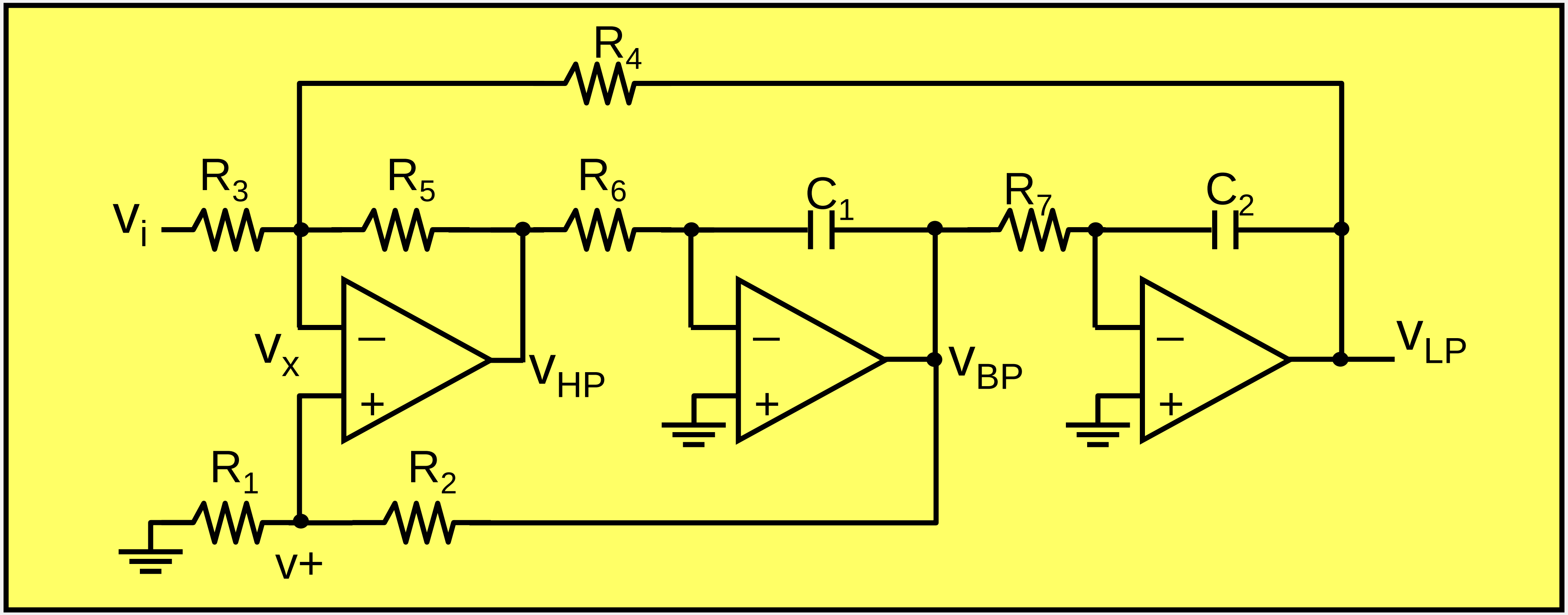
$$\therefore \omega_n = \frac{\sqrt{R_5/R_4}}{\sqrt{R_6C_1R_7C_2}} \qquad Q = \frac{1}{2\xi} = \frac{\left(1+R_2/R_1\right)\sqrt{R_5R_6C_1/R_4R_7C_2}}{1+R_5/R_3+R_5/R_4}$$

記得如何得到ξ嗎？



$$\therefore \frac{v_{BP}(s)}{v_i(s)} = -\frac{1}{sR_6C_1}\cdot\frac{v_{HP}(s)}{v_i(s)}$$

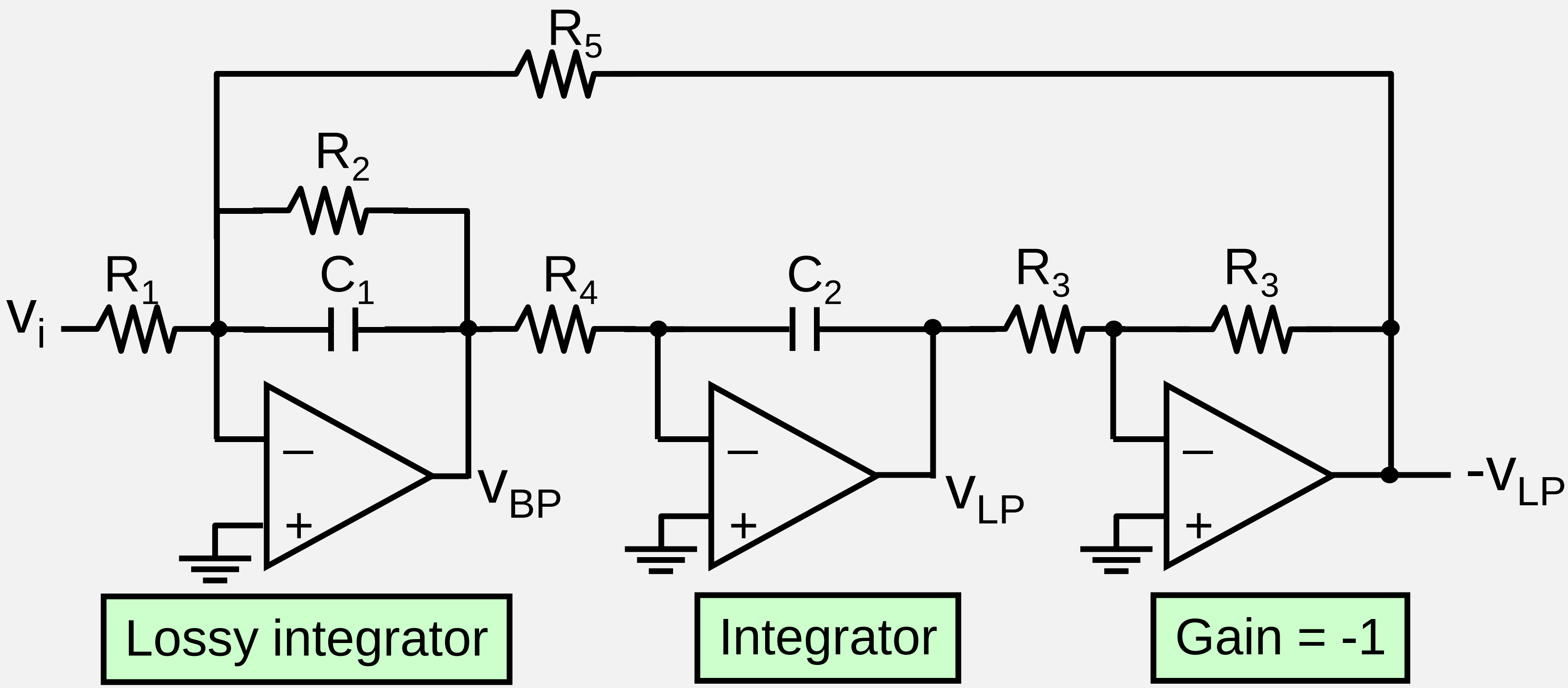
$$= \frac{1}{R_3}\cdot\frac{R_4R_7C_2s}{\frac{R_4R_6R_7C_1C_2}{R_5}s^2 + \frac{R_4R_7C_2\left(1+R_5/R_3+R_5/R_4\right)}{\left(1+R_2/R_1\right)R_5}s+1}$$



$$\therefore \frac{v_{LP}(s)}{v_i(s)} = -\frac{1}{sR_7C_2} \cdot \frac{v_{BP}(s)}{v_i(s)}$$

$$= -\frac{R_4}{R_3} \cdot \frac{1}{\frac{R_4R_6R_7C_1C_2}{R_5}s^2 + \frac{R_4R_7C_2(1+R_5/R_3+R_5/R_4)}{(1+R_2/R_1)R_5}s+1}$$

Biquad Filter (Tow-Thomas Filter) (Design Problem I)



- It simultaneously realizes the band-pass (v_{BP}) and low-pass (v_{LP}) frequency responses

Cont'd: Please Derive $\frac{v_{LP}(s)}{v_i(s)}$ on Your Own

At the inverting node of first op amp, by KCL:

$$\frac{v_i}{R_1} - \frac{v_{LP}}{R_5} + \frac{v_{BP}}{R_2} + \frac{v_{BP}}{1/sC_1} = 0 \qquad (1)$$

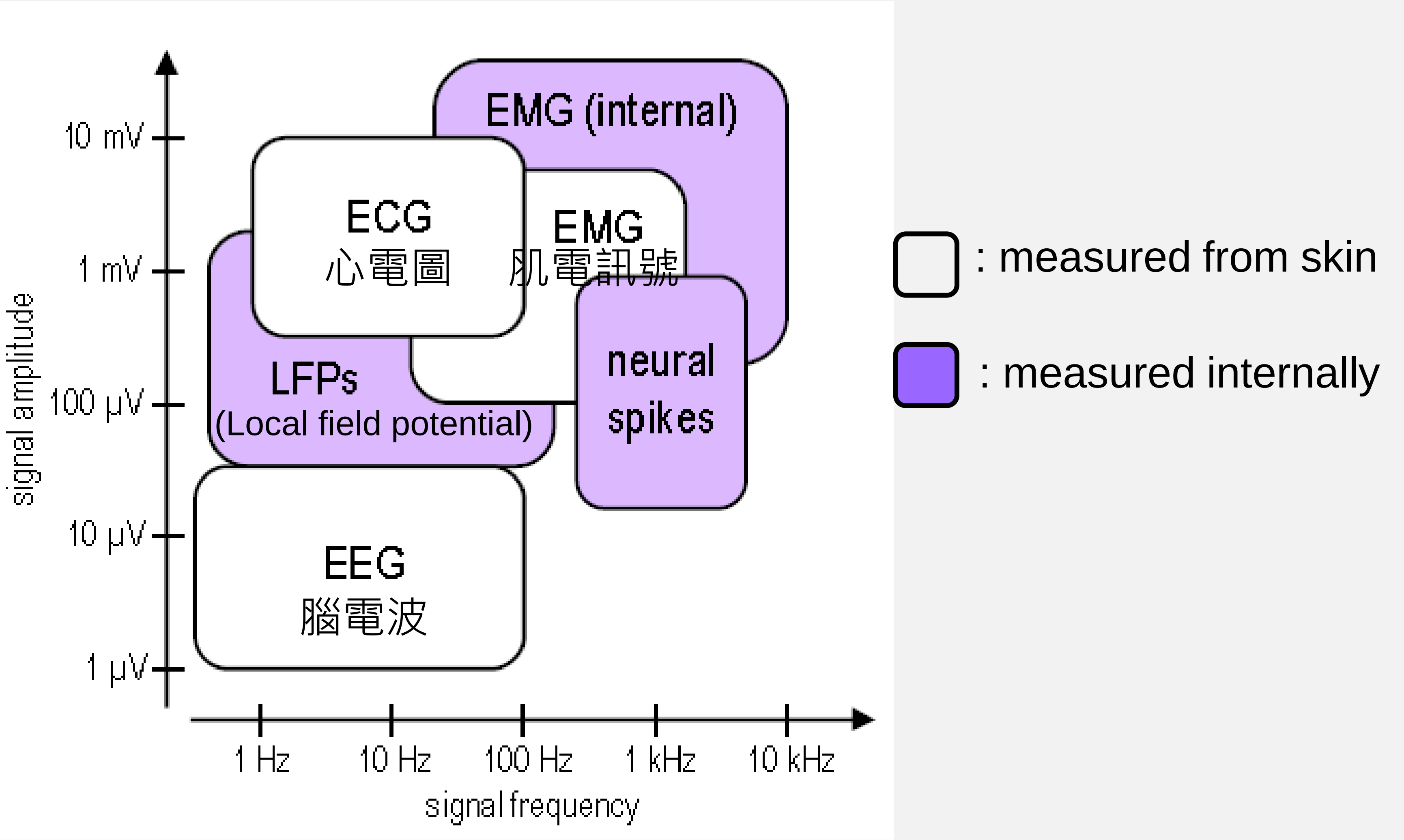
$$\text{and } v_{LP} = -\frac{v_{BP}}{R_4 C_2 s} \qquad (2)$$

Solve (1) and (2):

$$\therefore \frac{v_{LP}(s)}{v_i(s)} = \frac{R_5}{R_1} \cdot \frac{1}{R_4 R_5 C_1 C_2 s^2 + \frac{R_4 R_5 C_2}{R_2} s + 1}$$

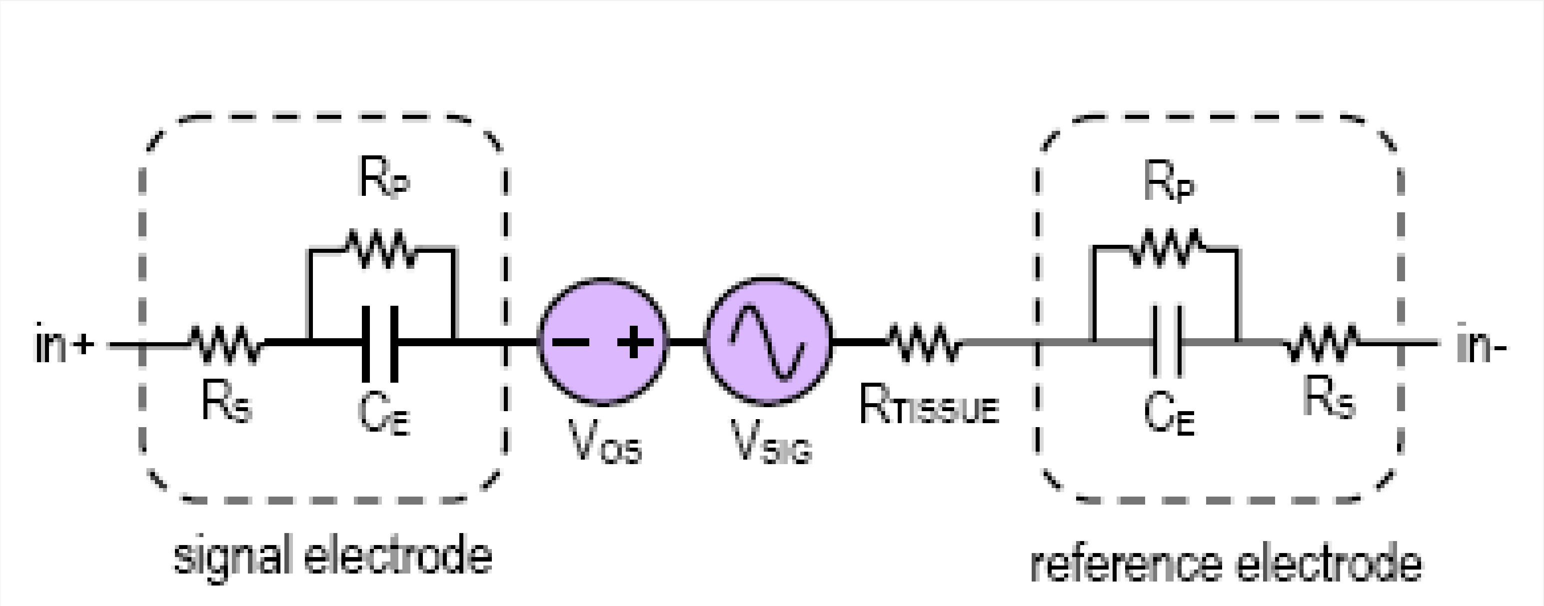
$$\therefore \omega_n = \frac{1}{\sqrt{R_4 C_1 R_5 C_2}} \qquad Q = \frac{R_2 \sqrt{C_1}}{\sqrt{R_4 R_5 C_2}}$$

ECG: Figure Out Your Design for the Specific Application First!



R.R. Harrison 2007 IEEE CICC

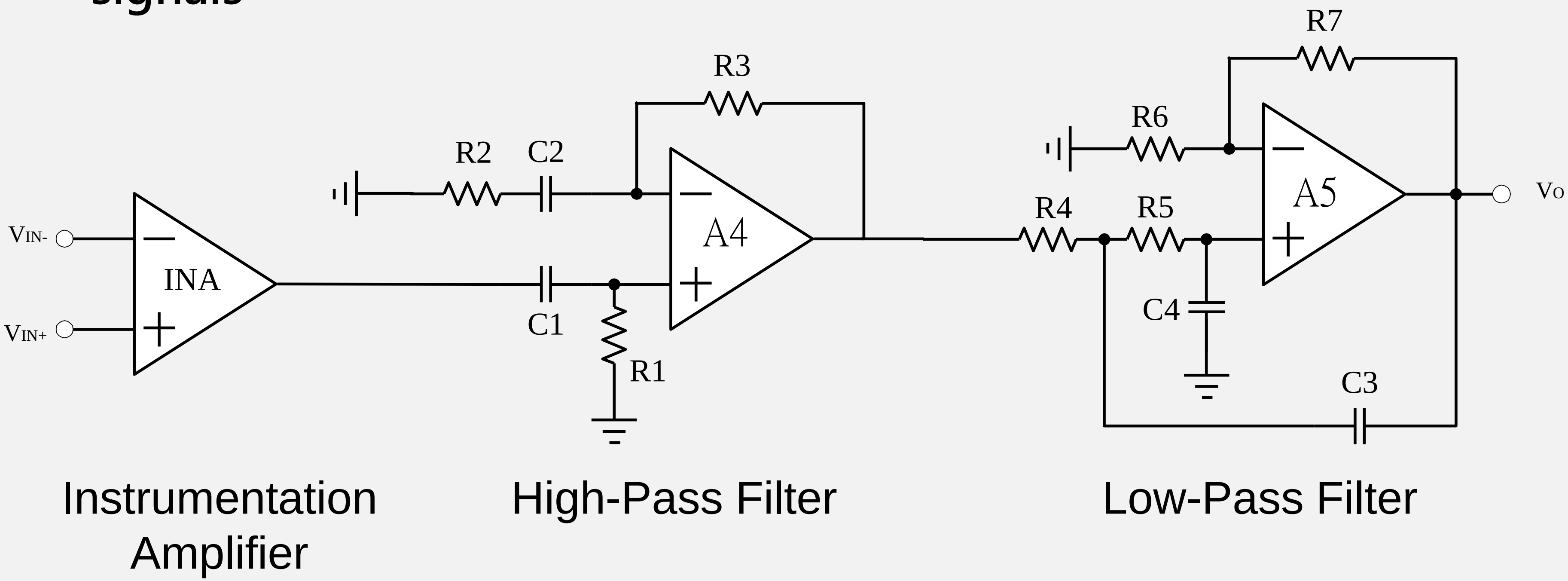
訊號與阻抗：Model of Electrode-Electrolyte Interface

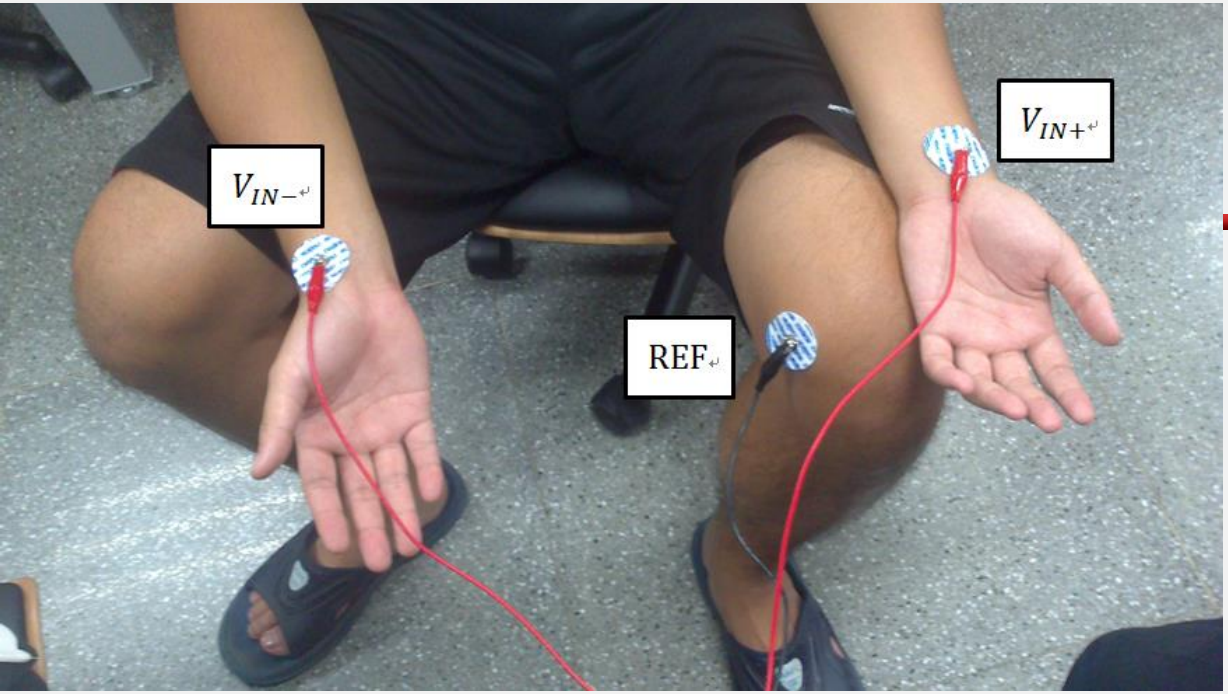


來自於身體的生醫訊號會經過身體以及電極的阻抗，再被電路放大

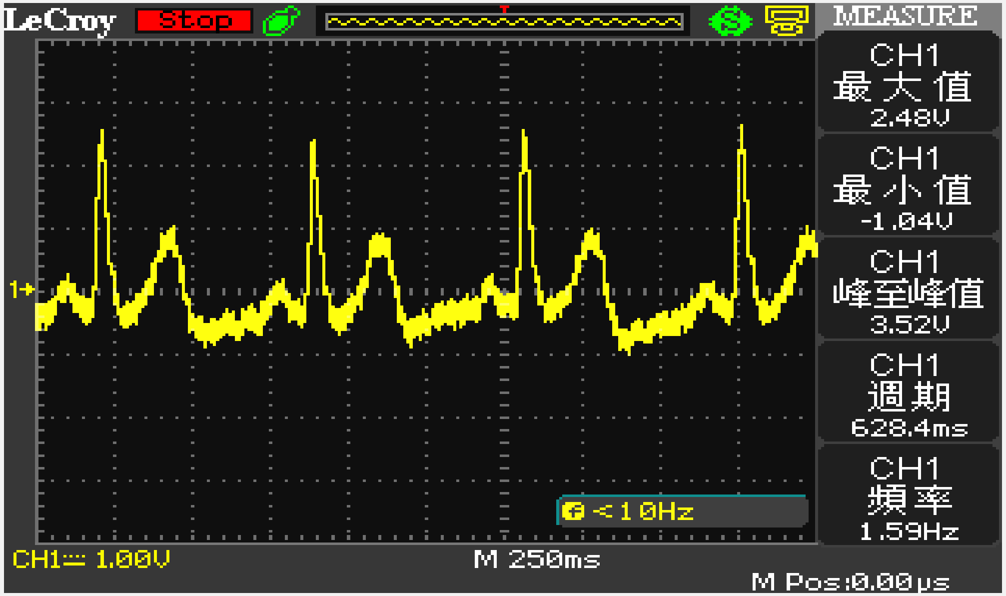
ECG Measurement (Design Problem II to IV)

- 針對ECG量測，要慎選filter的-3dB frequency,才能濾掉interference signals





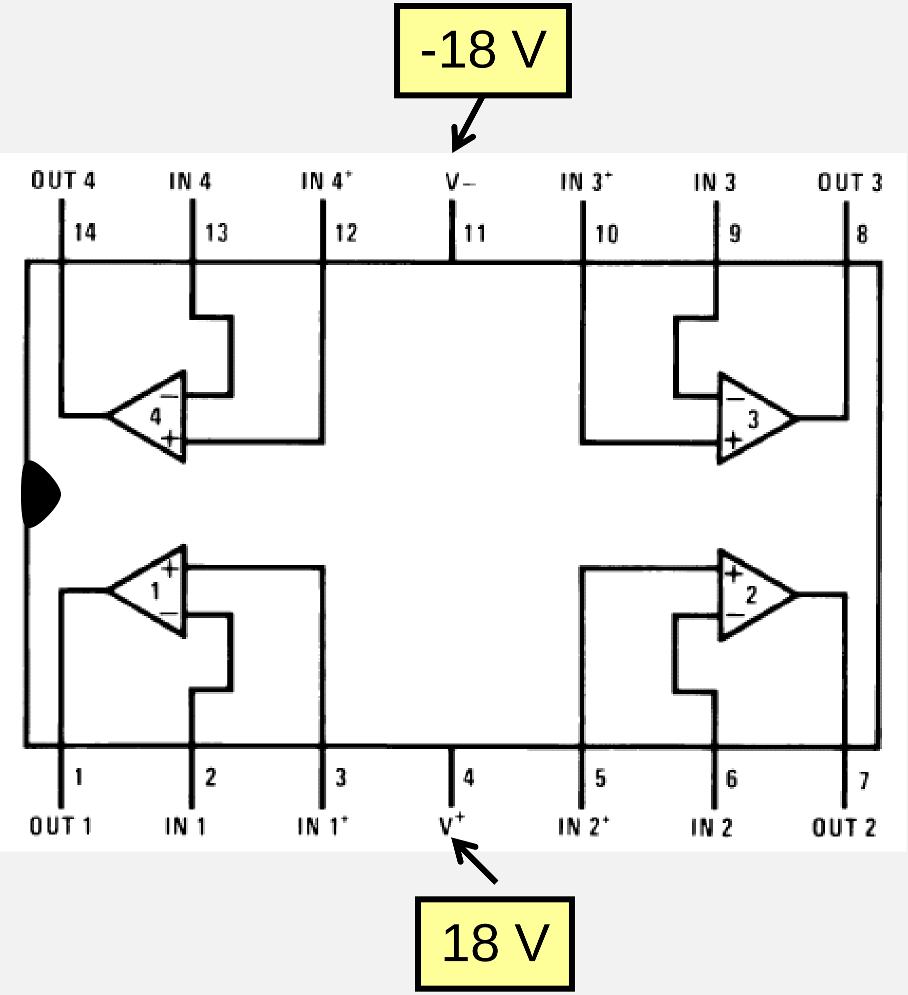
V_{IN-}
V_{IN+}
REF



LeCroy
Stop
MEASURE
CH1
最大值
2.48V
CH1
最小值
-1.04V
CH1
峰至峰值
3.52V
CH1
週期
628.4ms
CH1
頻率
1.59Hz
<10Hz
CH1 1.00V
M 250ms
M Pos:0.00μs

The LM348 Chip for Design Problem I

- Contains FOUR op amps
- Power supply = ±18 V
- Caution: avoid touching the metal pins of the chip to reduce damage from electrostatic charges



The TLC2264 Chip for Design Problems II to IV

Power supply = ±8 V

Compared to LM348, TLC 2264 has lower noise and input offset, and higher input impedance, suitable for biomedical signal processing