



EECS1010 Logic Design

Lecture 6 Registers and Counters

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Outline

- Digital systems and information
- Boolean algebra and logic gates
- Gate-level minimization
- Combinational logic
- Sequential circuits
- Registers and counters
- Memory



Chapter Outline

- Registers
- Counters



Registers



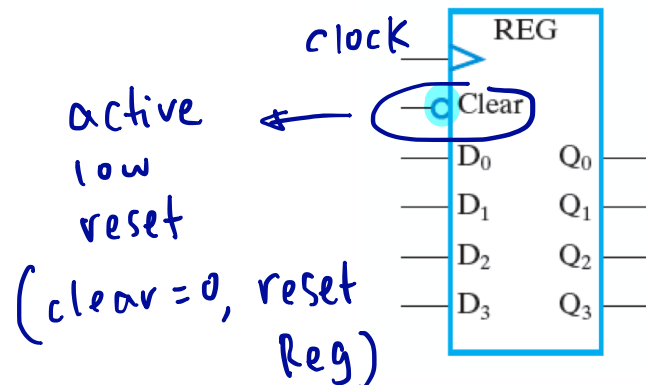
Registers (1/2)

- Register: a collection of binary storage elements
- An n-bit register stores n bit binary information.
- Registers are commonly used to perform data storage and processing.
- A register may have combinational gates to control when and how the new information is transferred into the register.
- Transfer new information into a register is called loading the register.
- Counter: a register that goes through a predetermined sequence of states.

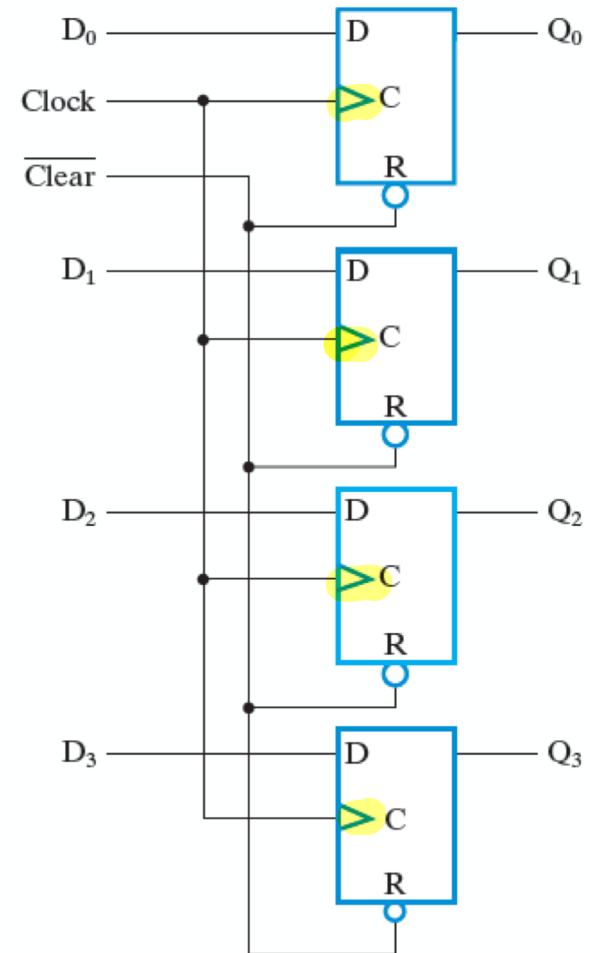


Register (2/2)

- A register can be viewed as a bitwise extension of a flip-flop.
 - The simplest storage component with n inputs, n outputs, and a clock signal.
 - All flip-flops are driven by a common clock signal.



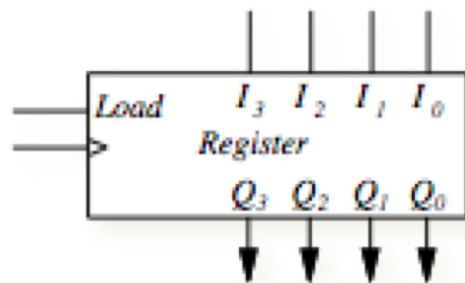
4-bit register



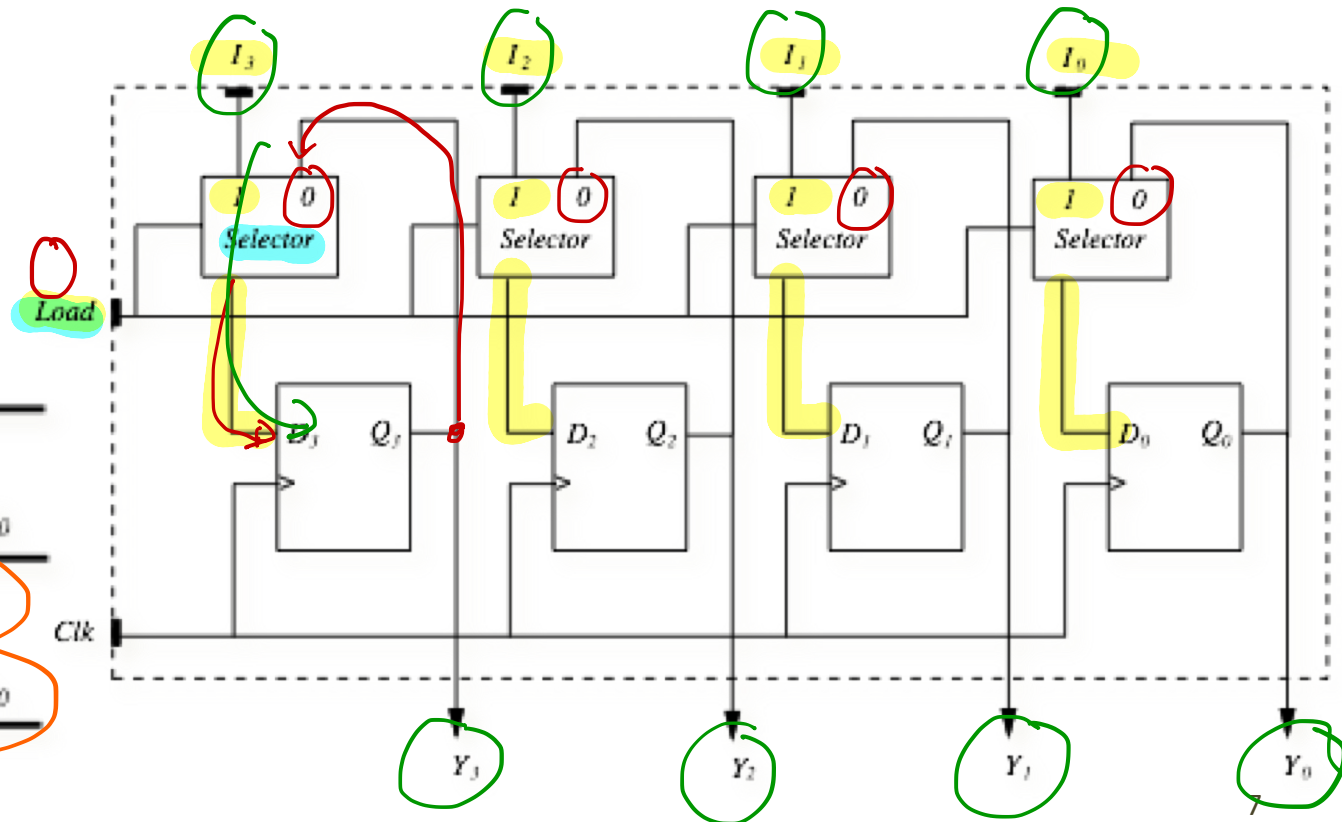


Register with Parallel Load

- Load: control when the data is transferred into a register, and how long it will be stored.

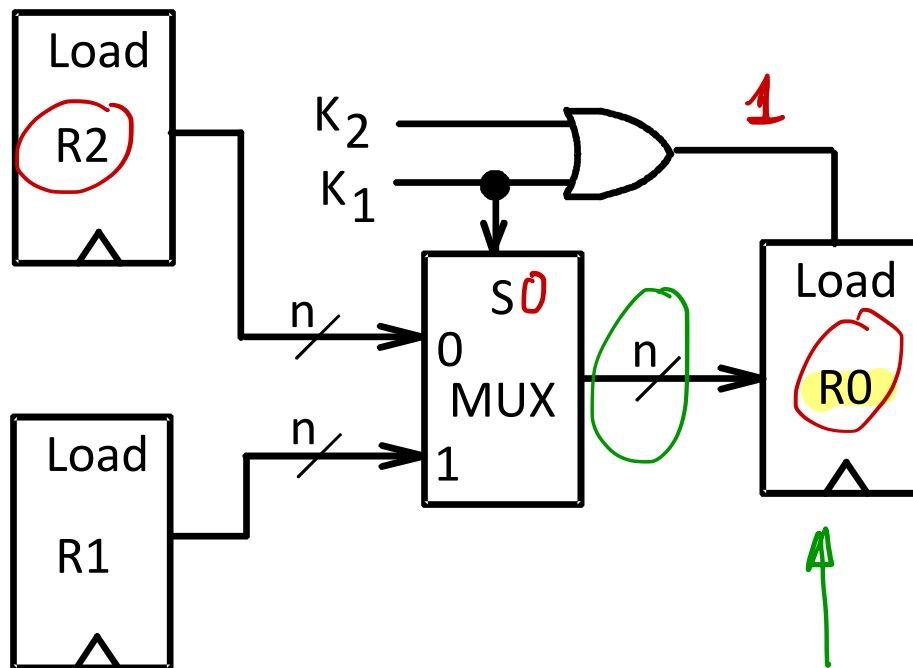


Present state	Next state			
	Q_3	Q_2	Q_1	Q_0
Load				
0	No change			
1	I_3	I_2	I_1	I_0



Multiplexer-Based Transfers

- Register may receive data from many different sources at different times.



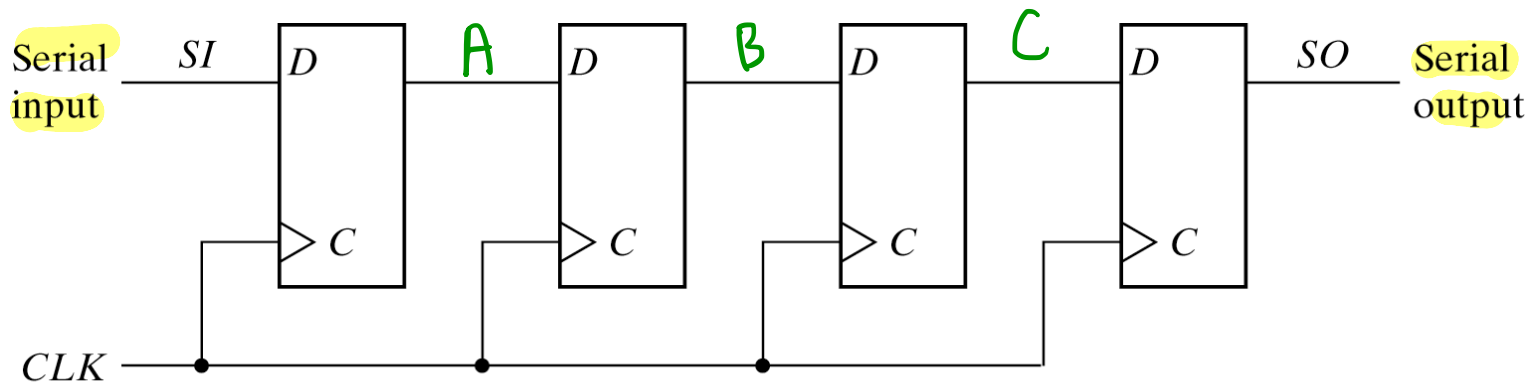
$$k_1 = 1, \quad R_0 \leftarrow R_1$$

$$\bar{k}_1 \cdot k_2 = 1, \quad R_0 \leftarrow R_2$$



Shift Registers

- Shift registers move data within the register toward its MSB or LSB position. It shifts one bit at a time.
- In the simplest case, the shift register is simply a set of D flip-flops connected in a row like this:



- Data output, SO, is often called *Serial output*
- The vector (A, B, C, SO) is called *parallel output*
- Generally, input and output can be serial or parallel.

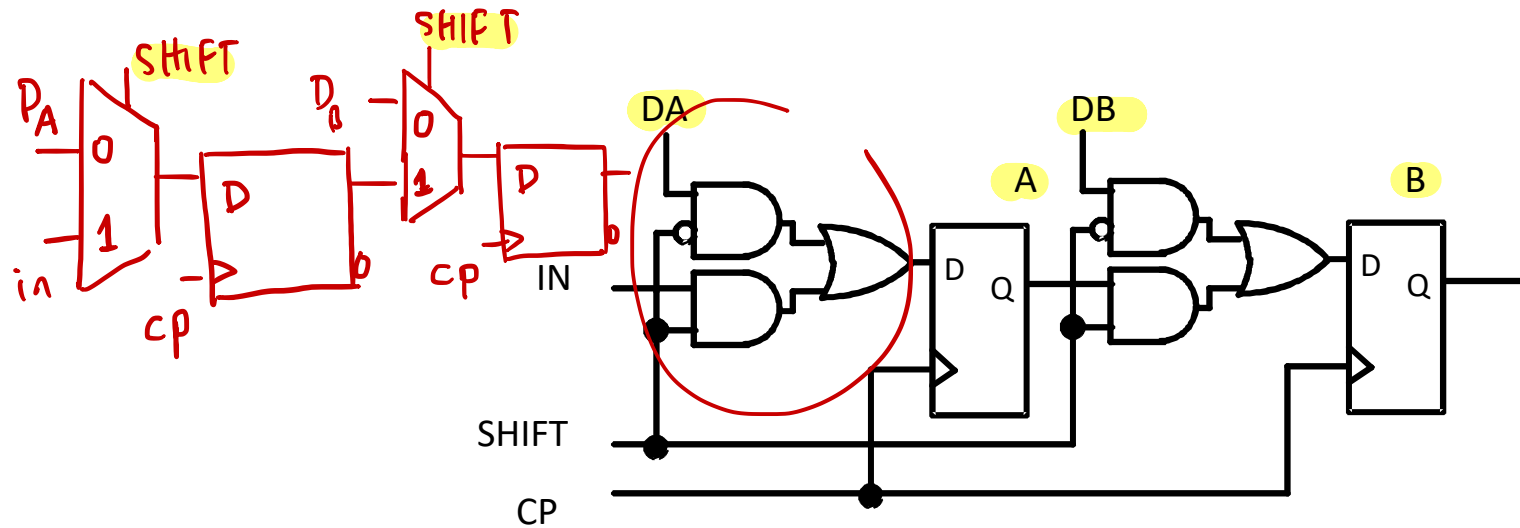


Shifter Types

- Logical shifter: shift the number to the left or right and fills empty spots with 0's.
- Arithmetic shifter: same as logical shifter but on right shift fills empty MSBs with the sign bit (sign extension).
- Barrel shifter: rotate numbers in a circle such that empty spots are filled with the bits shifted off the other end.



(p.7+p.9) Parallel Load Shift Registers



- Add a MUX between each shift register.
- Data can be shifted or loaded.
- When $\text{SHIFT} = 0$, D_A and D_B are loaded into DFF_A and DFF_B .

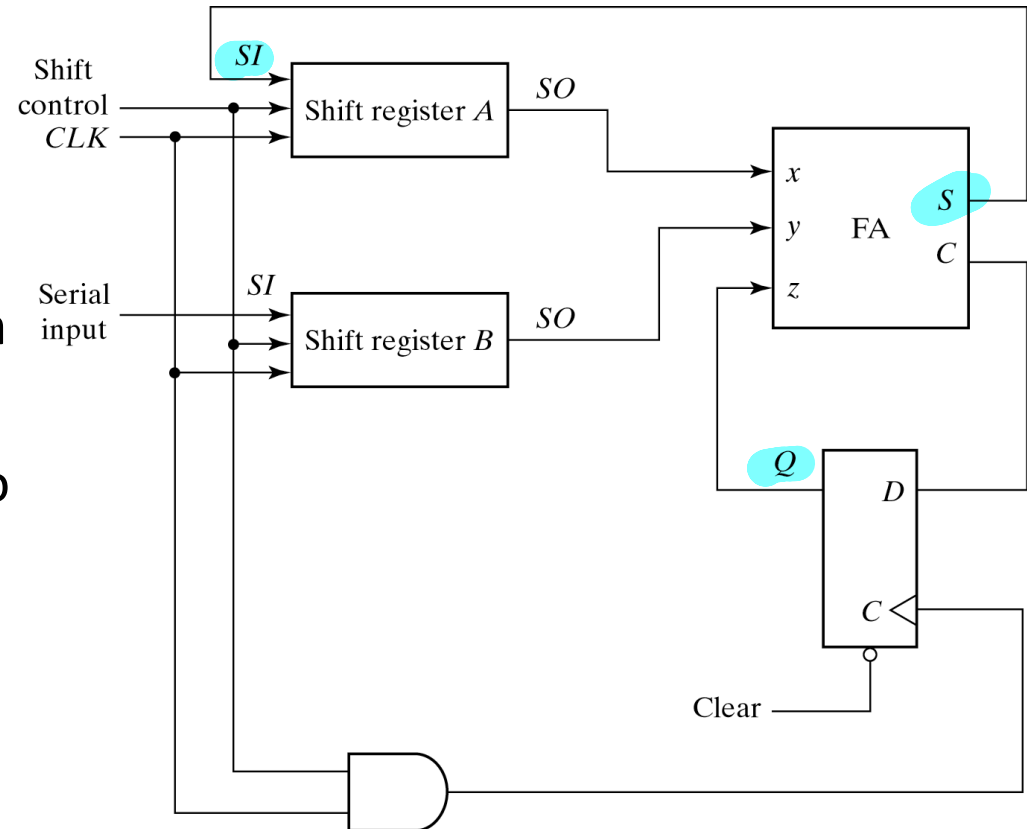
$\text{SHIFT} = 1$, data shift right.

$$\begin{array}{r}
 0101 \\
 + 0101 \\
 \hline
 1010
 \end{array}$$

Serial Adder using DFFs



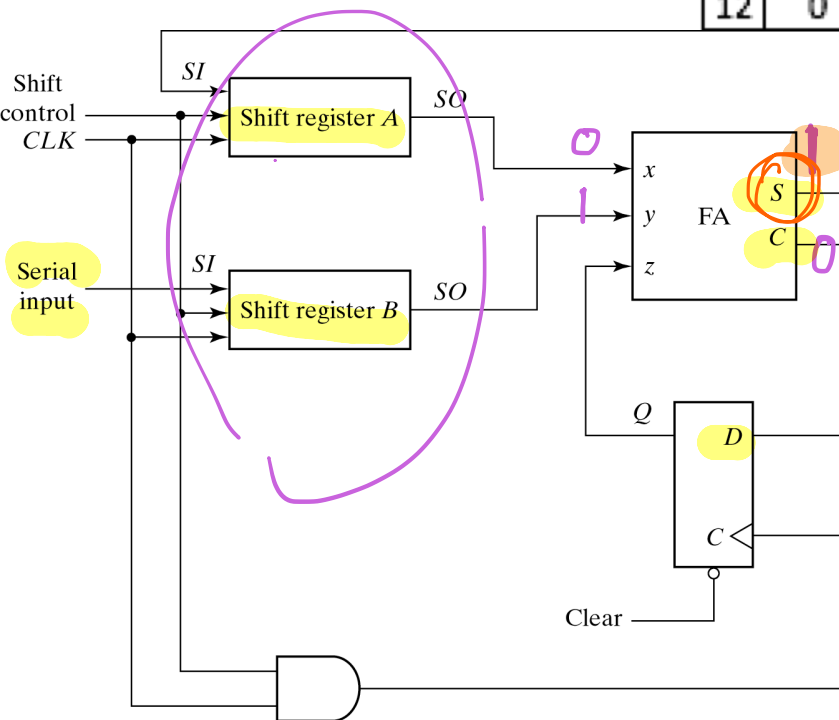
- Initially, augend in register A and addend in register B.
- Shift control enables the triggering of clock and 1-bit addition of two operands from LSB to MSB.
- A new sum bit is transferred to shift register A.
- A carry-out is transferred to Q.
- Finally, shift control is disabled and the sum is stored in shift register A.



o Example:

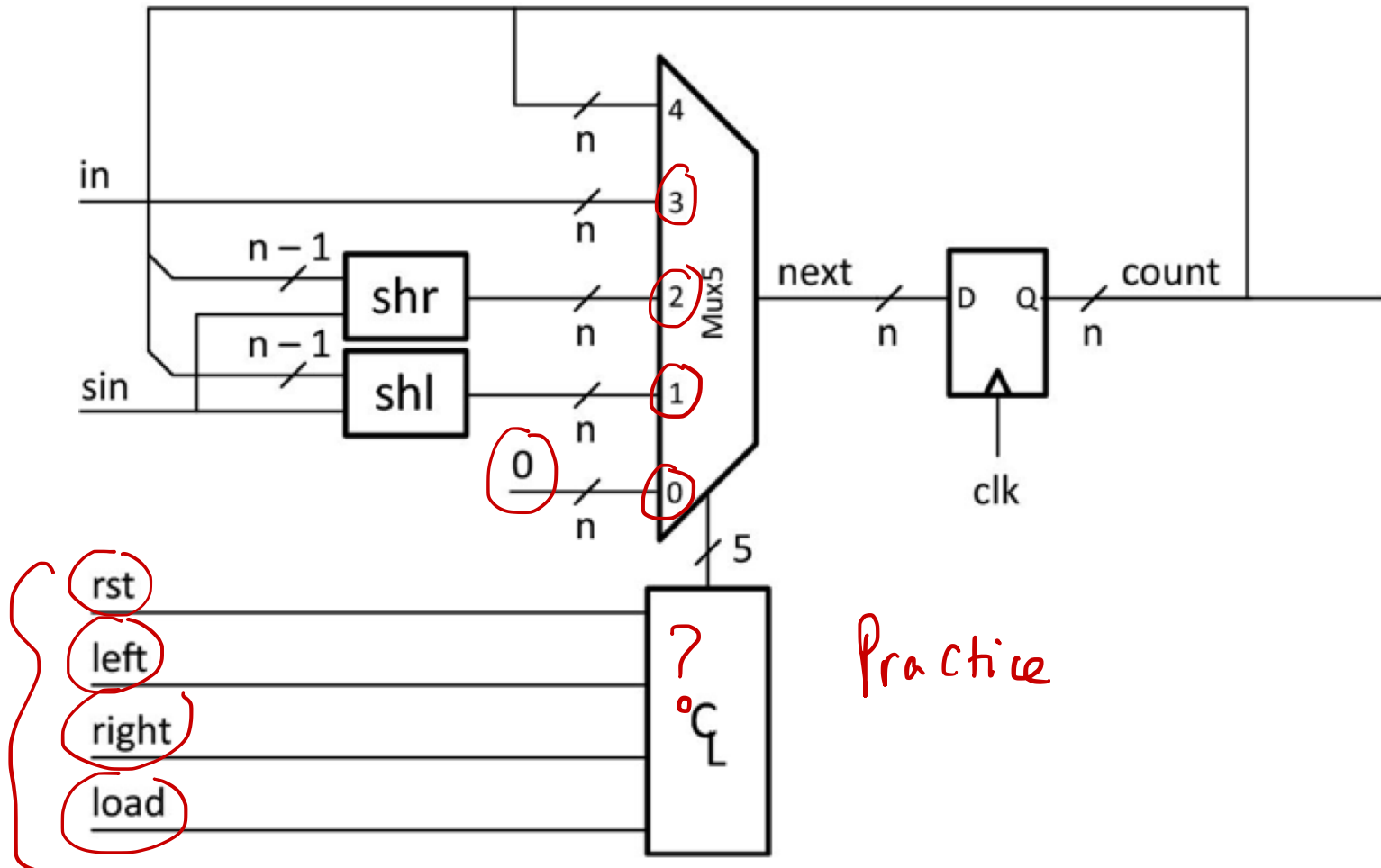
01010
+ 1010

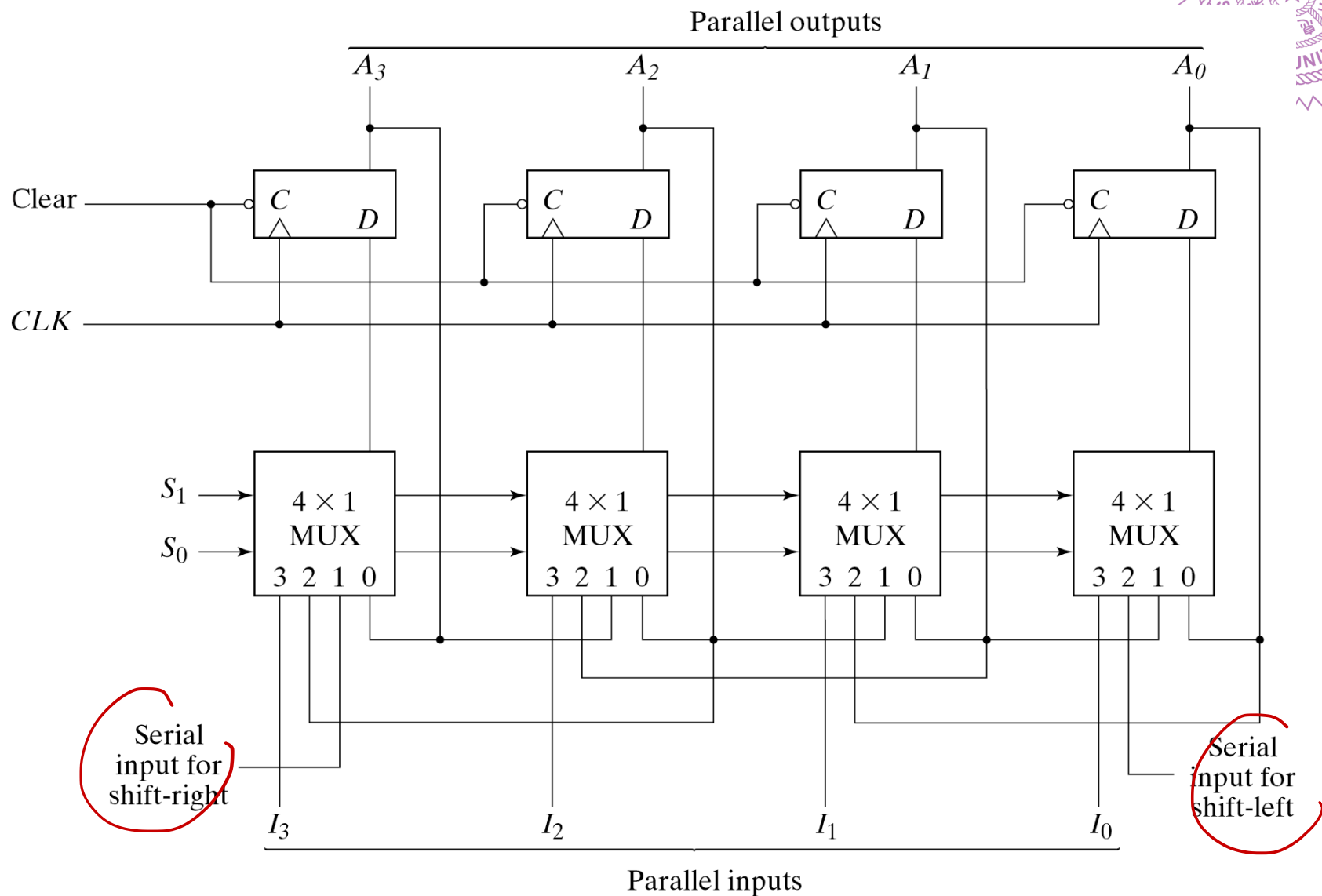
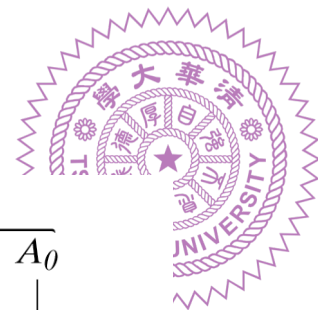
t	SI (B)	B3	B2	B1	B0	SI(A),s	A3	A2	A1	A0	x (A0)	y (B0)	z	c	s
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
4	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
5	0	0	1	0	1	1	0	0	0	0	0	1	0	0	1
6	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0
7	0	1	0	0	1	1	0	1	0	0	0	1	0	0	1
8	1	0	1	0	0	0	1	0	1	0	0	0	0	0	0
9	0	1	0	1	0	1	0	1	0	1	1	0	0	0	1
10	0	0	1	0	1	1	1	0	1	0	0	1	0	0	1
11	0	0	0	1	0	1	1	1	0	1	1	0	0	0	1
12	0	0	0	0	1	1	1	1	1	0	0	1	0	0	1
							1	1	1	1	1				
								1	1	1	1				
								1	1	1	1				
									1	1	1				
										1	1				





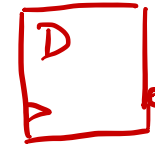
Left/Right/Load Shift Register





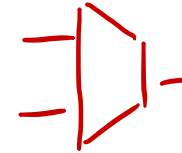


Edge-triggered



- DFF_s

- Parallel load, reset/enabling



count
load



1. Ripple

2. Synchronous

3. BCD

4. Ring

5. Johnson

Counters



Counters

- Counters are sequential circuits which "count" through a specific state sequence.
- Two distinct types are in common usage:
 - Ripple Counters
 - The output transition of flip-flop serves as a source for triggering other flip-flops
 - Synchronous Counters
 - The clock inputs of all flip-flops receive a common clock



↓

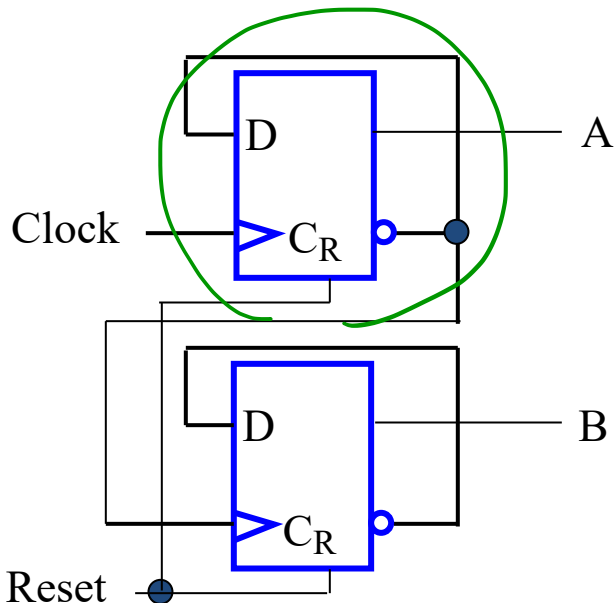
Ripple Counter

up counting :

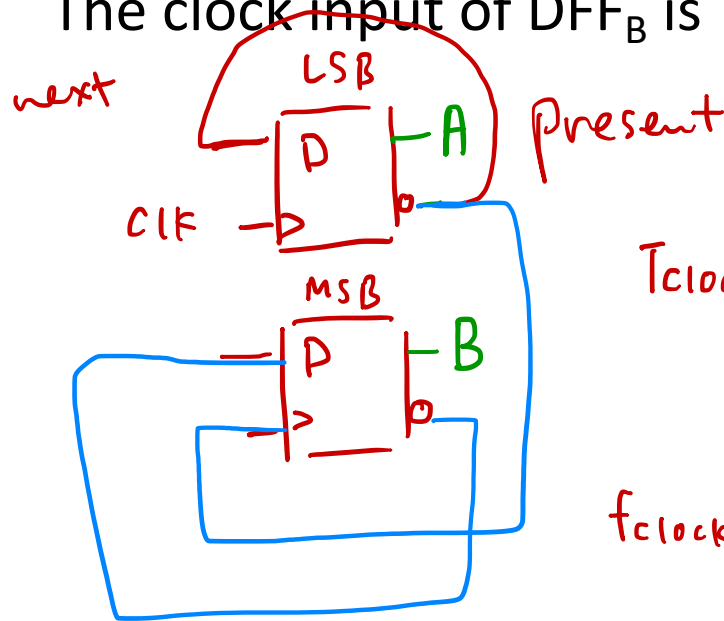
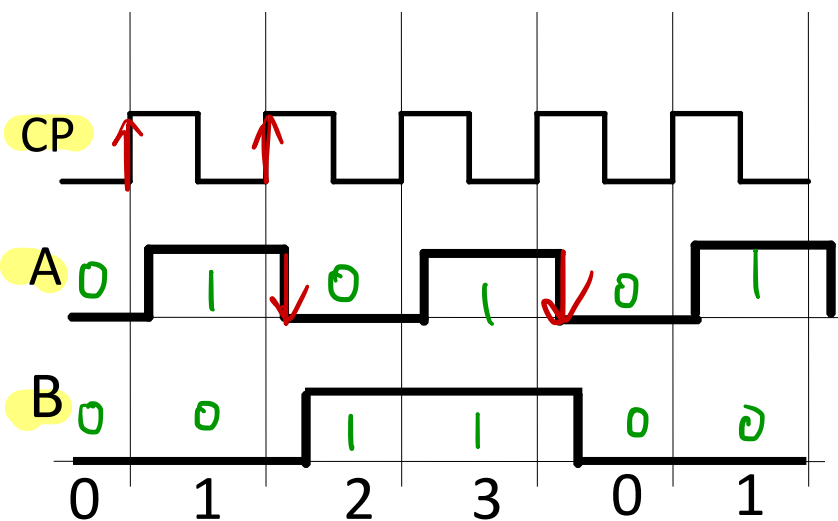
$00 \rightarrow 01 \rightarrow 10 \rightarrow 11 \rightarrow 00 \rightarrow \dots$

down counting

$11 \rightarrow 10 \rightarrow 01 \rightarrow 00 \rightarrow 11 \rightarrow 10 \rightarrow \dots$



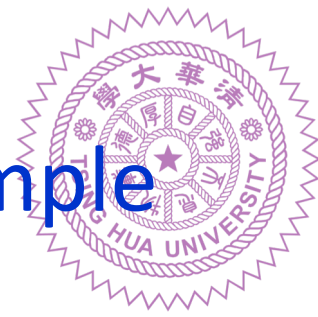
- Clock only connects to the LSB FF.
- At positive edge, A complements ($A^+ = \bar{A}$)
- The clock input of DFF_B is \bar{A}



$$T_{\text{clock}} = \frac{1}{2} T_A$$

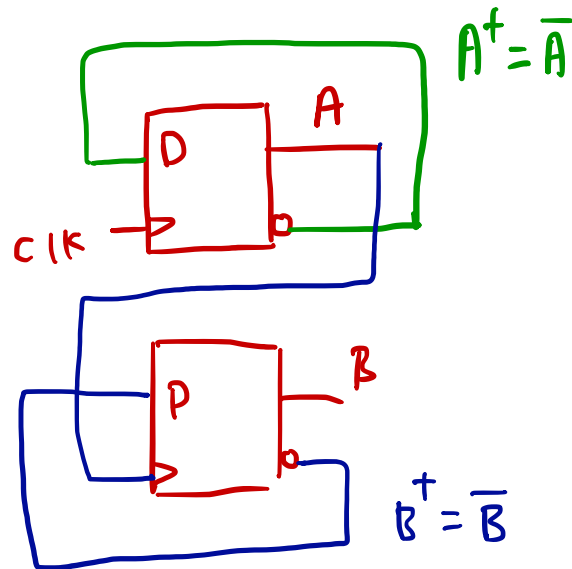
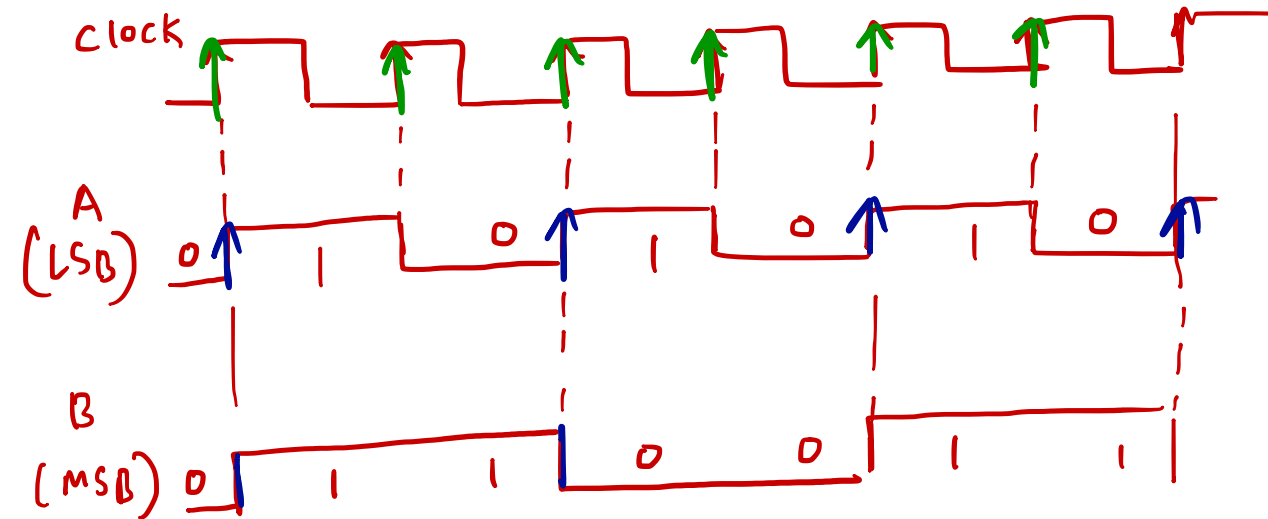
$$= \frac{1}{4} T_B$$

$$f_{\text{clock}} = 2f_A = 4f_B$$



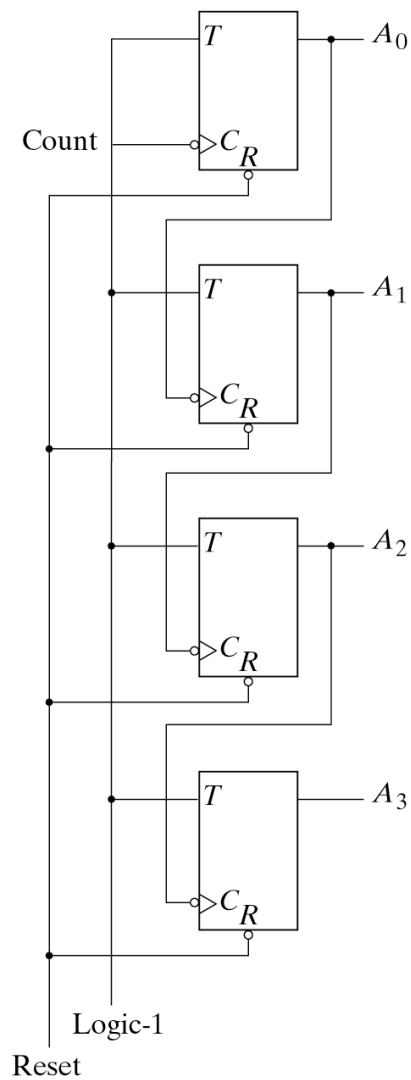
2-Bit Downward Ripple Counter Example

11 → 10 → 01 → 00 → 11...

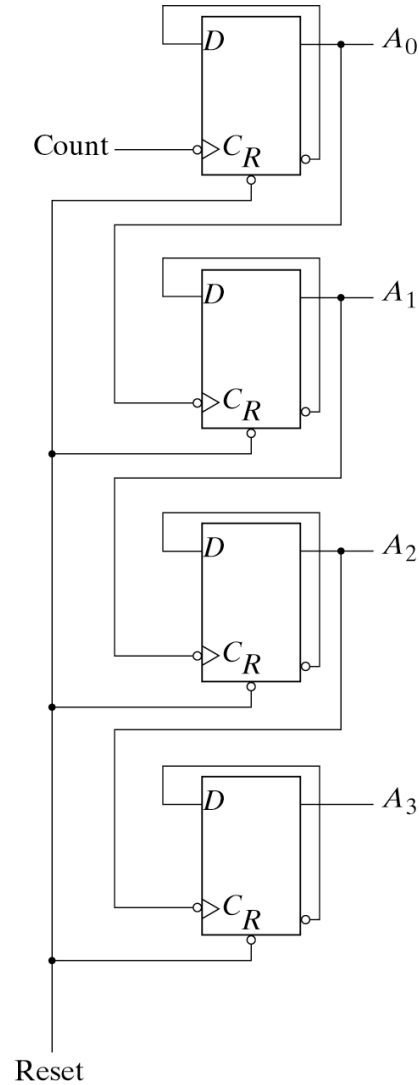




4-Bit Ripple Up Counter (1/2)



(a) With T flip-flops

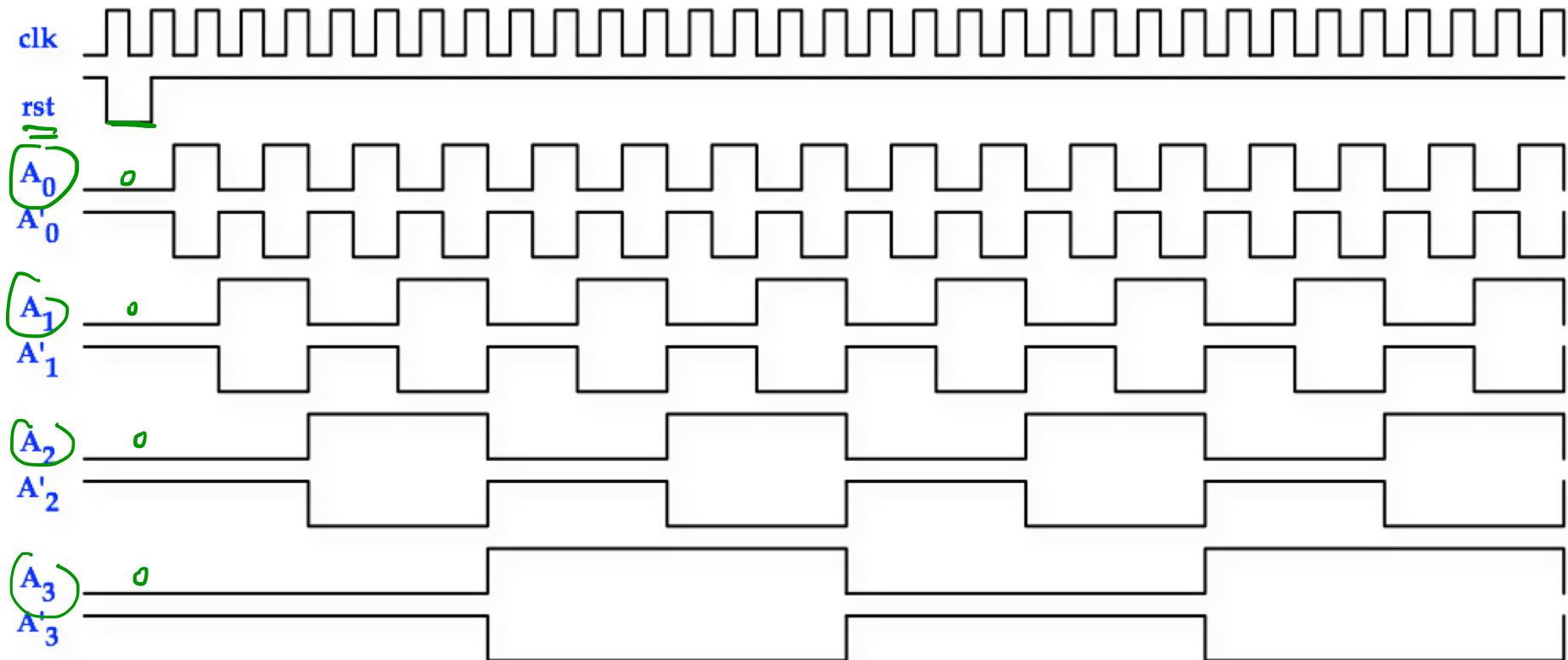


(b) With D flip-flops



4-Bit Ripple Up Counter (2/2)

- Use DFFs





2.

Synchronous Counters (1/2)

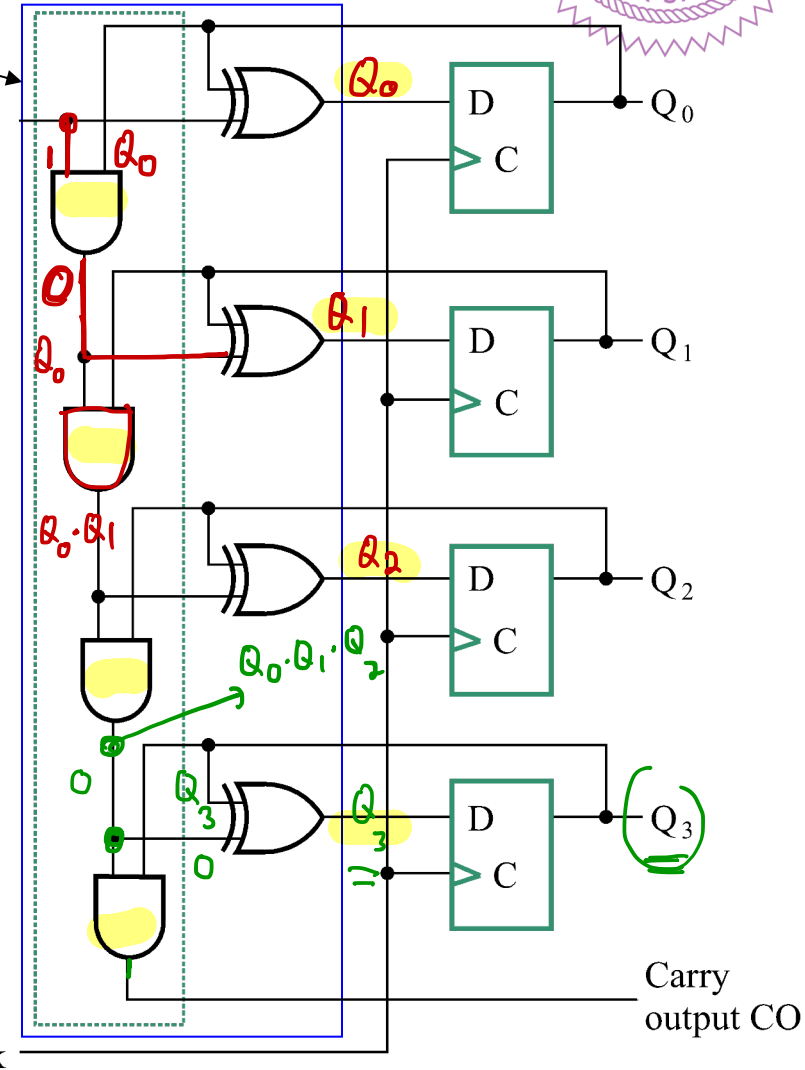
- Clock is applied to all FFs.
- Formed by incrementer and DFFs.
- AND chain causes complement of a bit if all bits toward LSB from it equal 1.
- XOR complements each bit.
- Count enable (EN)

$EN=1$, count

$EN=0$, hold

0000
0001
0010
0011
0100
0101
0110
0111
1000
⋮

Incrementer
Count enable EN



Serial gating: large path delay toward MSB due to serial AND gates.

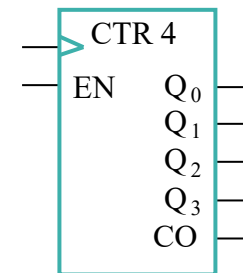
(a) Logic Diagram-Serial Gating



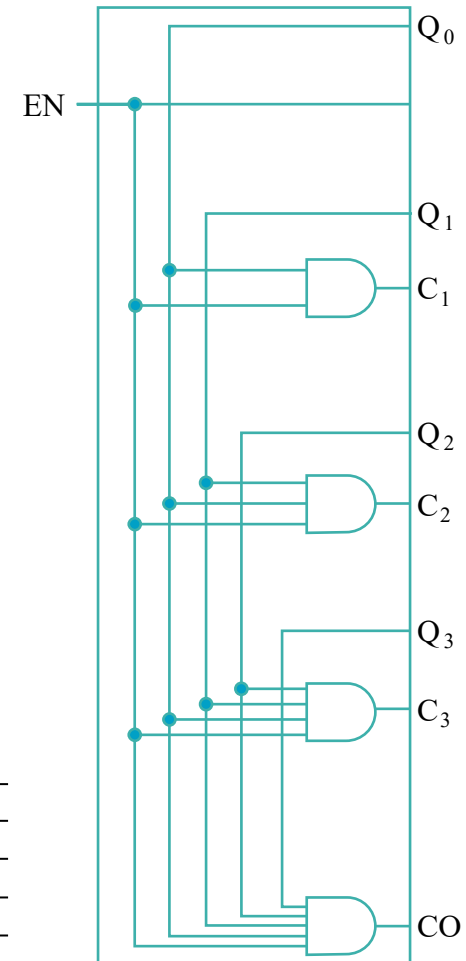
Synchronous Counters (2/2)

- Replace AND carry chain with ANDs in parallel.
- EN applied at each bit.
- Parallel gating reduce path delay.

Practice. sync 2-bit up counter (5^{chapter})



Symbol



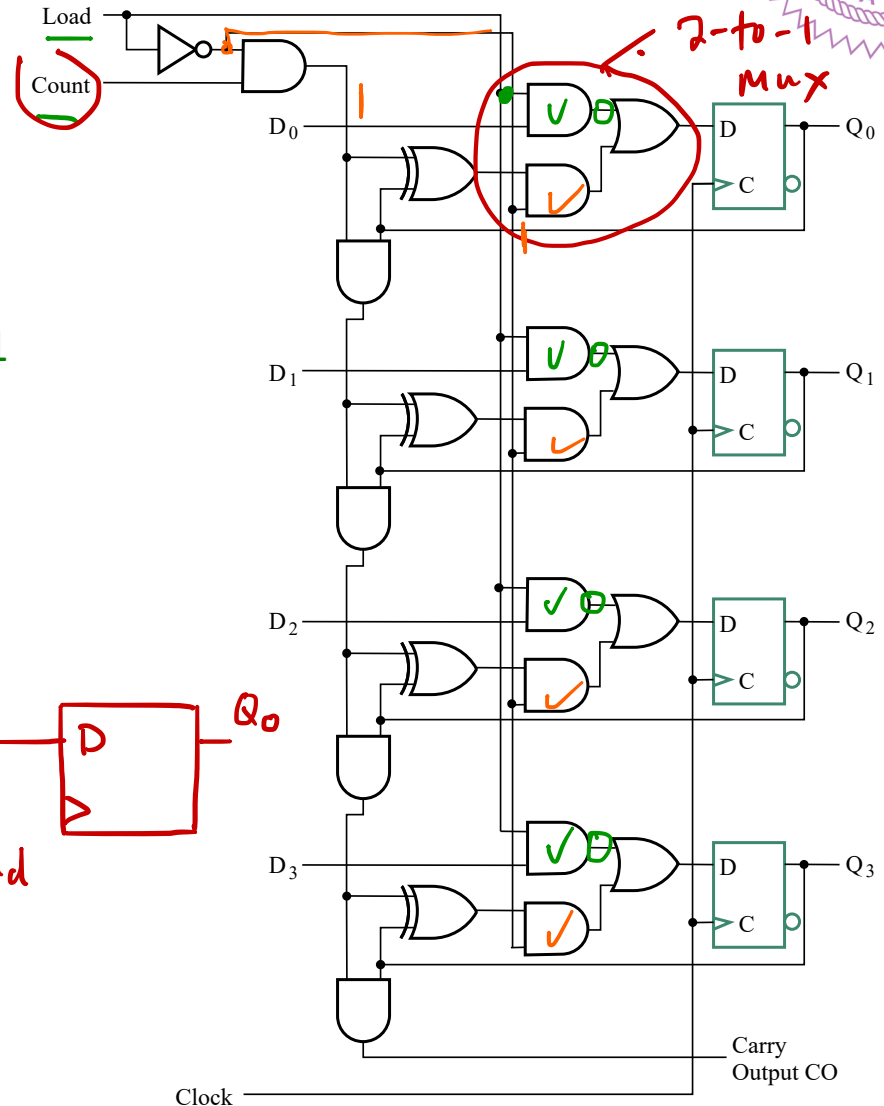
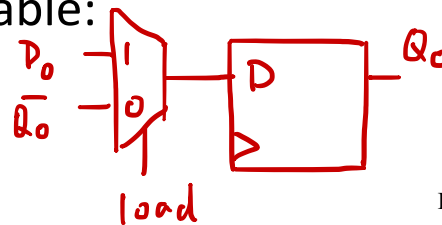


Counter with Parallel Load

- Add path for input data.
 - Enabled for Load = 1
- Add logic to
 - Disable count logic for Load = 1
 - Disable feedback from outputs for Load = 1
 - Enable count logic for Load = 0 and Count = 1

- The resulting function table:

Load	Count	operation
1	x	<u>load</u>
0	1	count
0	0	<u>hold</u>

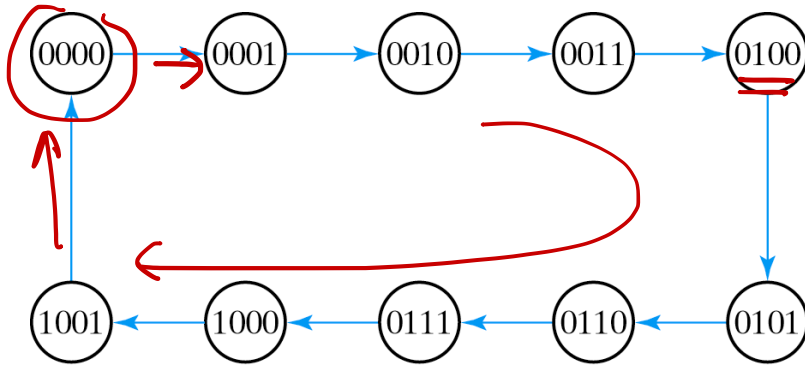




3.

state diagram

BCD Counter (1/2)



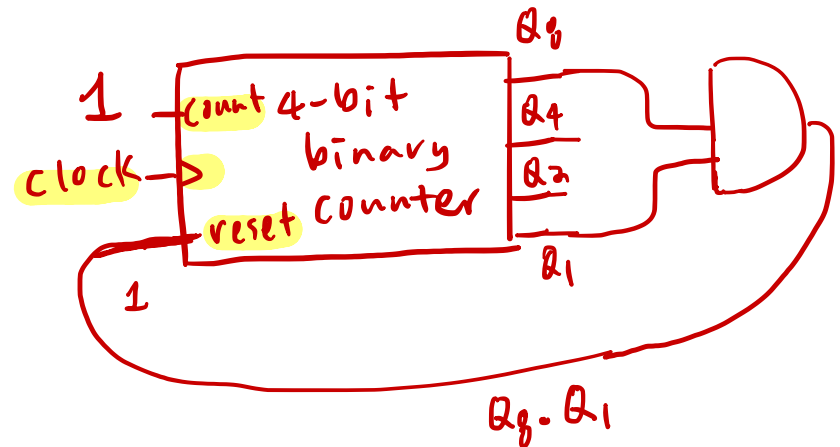
Present State

Next State

state
table

<u>Q8</u>	<u>Q4</u>	<u>Q2</u>	<u>Q1</u>	<u>Q8</u>	<u>Q4</u>	<u>Q2</u>	<u>Q1</u>
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0

o If a 4-bit up binary counter is available,





BCD Counter (2/2)

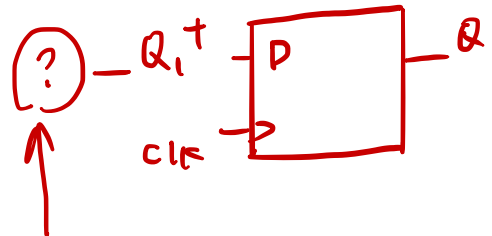
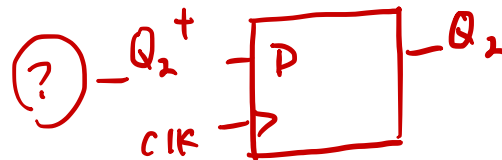
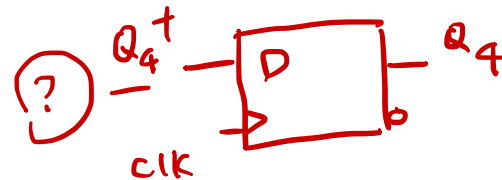
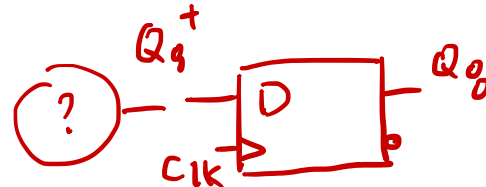
Practice .

$$Q_0^+ =$$

$$Q_4^+ =$$

$$Q_2^+ =$$

$$Q_1^+ =$$



$$Q_1^+ = f(Q_0, Q_4, Q_2, Q_1)$$

Sync.
BCD counter

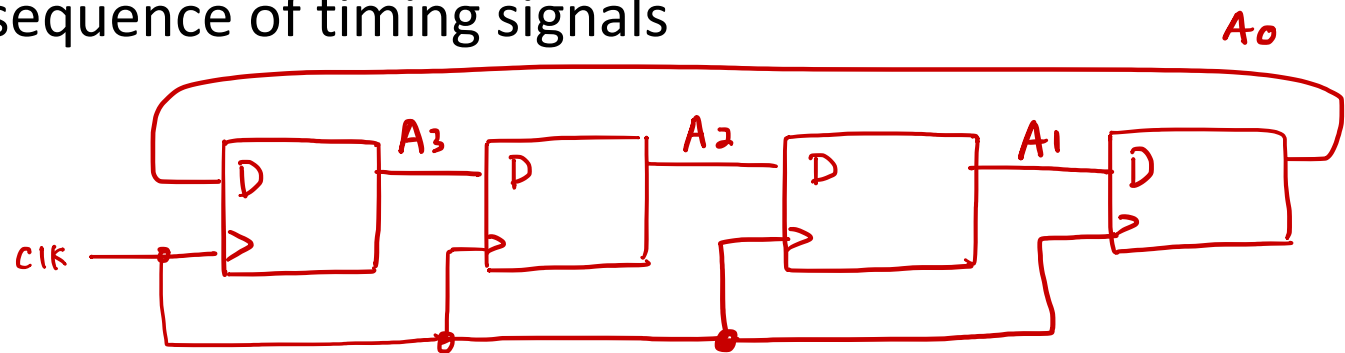


4.

Ring Counter

- A circular shift register with only one flip-flop being set at any particular time, all others are cleared. (initial value 1000...000)
- The single bit is shifted from one flip-flop to the next to produce the sequence of timing signals

	A ₃	A ₂	A ₁	A ₀
#1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1
#1	1	0	0	0

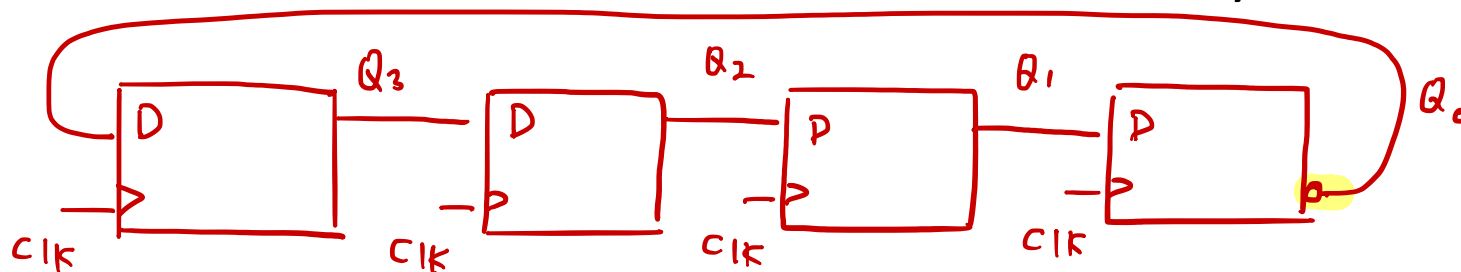


- Advantage: simple implementation
- Disadvantage: only represents n states with n FFs.



5. Johnson Counter

- Johnson counter: modified ring counter that the last output is inverted and fed back as the input of the first FF. Johnson counter circulates a stream of ones followed by zeros.

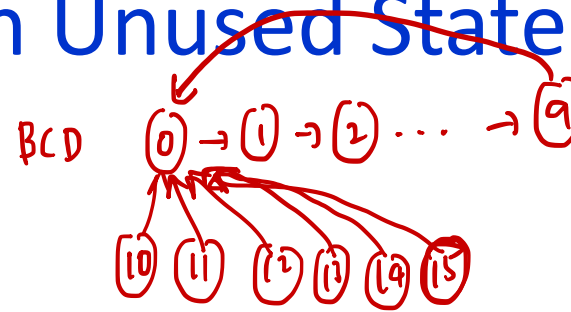


Q_3	Q_2	Q_1	Q_0
0	0	0	0
1	0	0	0
1	1	0	0
1	1	1	0
1	1	1	1
0	1	1	1
0	0	1	1
0	0	0	1
0	0	0	0

8 states.
($2w$ states with w FFs)



Counters with Unused States



- n flops $\Rightarrow 2^n$ states
- Unused states
 - States that are not used in specifying the FSM, may be treated as don't-care conditions or may be assigned specific next states.
- Self-correcting counters
 - Ensure that when a circuit enter one of its unused states, it eventually goes into one of the valid states after one or more clock pulses so that it can resume normal operation.
 - Analyze the circuit to determine the next state from an unused state after it is designed.